Lecture 32
DISCRETE STRUCTURES

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Maximal and Minimal elements

1. Maximal Element
   An element a Є A is called a maximal element of A if there is no element c in A such that $a < c$.

2. Minimal Element
   An element b Є A is called minimal element of A if there is no element c in A such that $c < b$.

NOTE-

a Є A is a maximal element of $(A,\leq)$ if and only if a is the minimal element of $(A,\geq)$.

Theorem-

Let A be finite nonempty poset with partial order ≤, Then A has at least one maximal element and one minimal element.

Theorem-

A poset has atmost one greatest element and atmost one least element.

NOTE-

The greatest element of a poset if it exists is denoted by I called unit element.
The least element of the poset if it exists is denoted by 0 called zero element.

Upper/Lower Bounds

Consider a poset and a subset B of A. An element a Є A is called an upper bound of B if $b \leq a$ for all b Є B.
An element a Є A is called the lower bound of B if $b \geq a$ for all b Є B.

An element a Є A is called a least upper bound of B LUB(B) is a is an upper bound of B and $a \leq a'$ whenever a’ is a upper bound of B.

An element a Є A is called a greatest lower bound of B GLB(B) if a is a lower bound of B and a’ ≤ a whenever a’ is a lower bound of B.

NOTE-

The GLB LUB of two elements should be comparable to them.

Theorem-

Let (A, ≤) be a poset. Then a subset B of A has at most one LUB and at most one GLB.