Lecture 31

DISCRETE STRUCTURES

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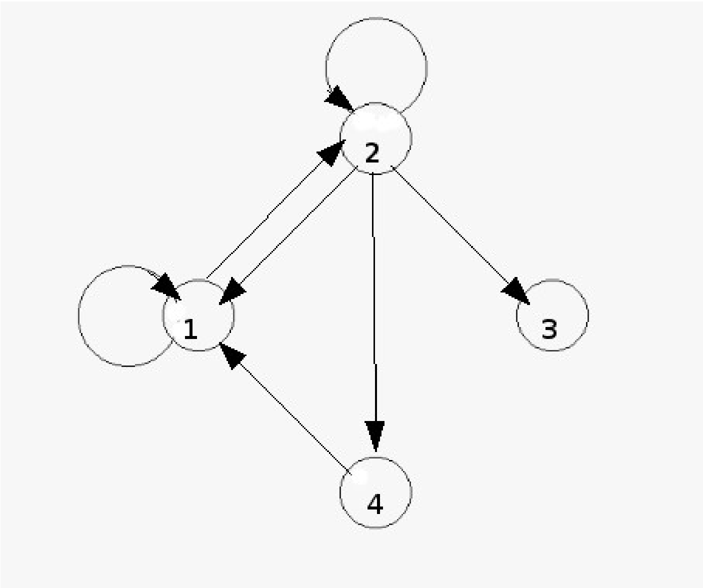
The Digraph Of a Relation

Let R be a relation on A. Draw an **edge**,from vertex a_i to vertex a_j if and only if a_i R a_j . The resulting pictorial representation of R is called a directed graph or digraph of R .

EXAMPLE: Let A= {1,2,3,4}

R = { (1,1),(1,2),(2,1),(2,2),(2,3),(2,4),(3,4),(4,1) }.

Then the digraph of R is as shown below.

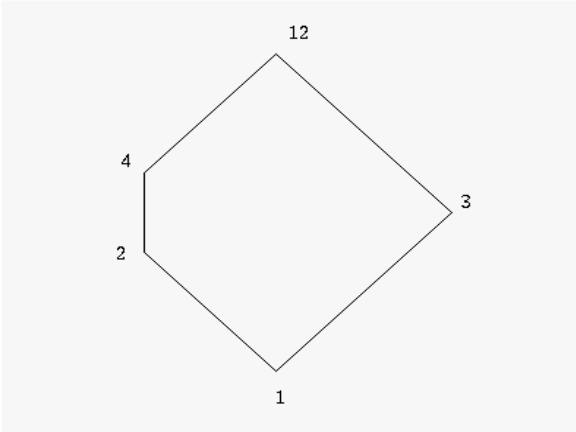


Hasse Diagrams

Draw the digraph of a partial order with all edges pointing upward,so that arrows may be omitted from the edges.Replace the circles representing the vertices by dots.The resulting diagram of a partialorder is called the **Hasse diagram** of the partial order of the poset.

EXAMPLE: Let **A** = {1,2,3,4,6}. Consider the partial order of divisibility on **A**. That is , if a and b Є A, a ≤ b if and only if a|b. Draw the Hasse diagram of the poset (A,≤).

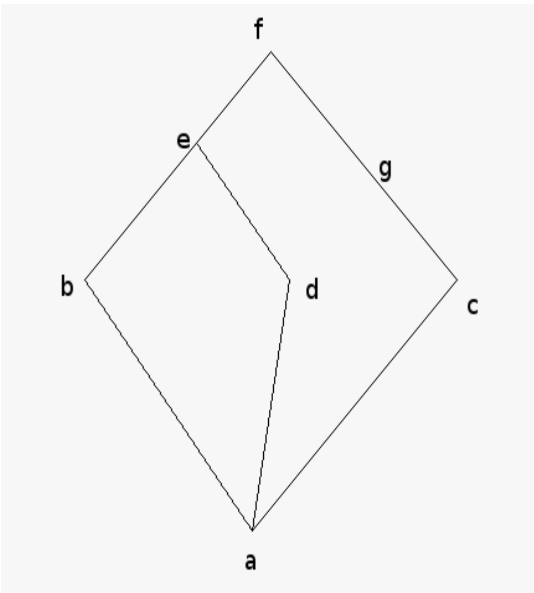
SOLUTION: The **Hasse diagram** is shown below.

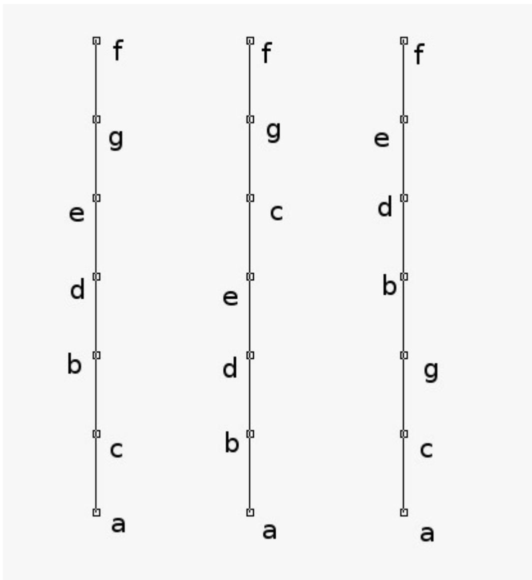


Toplogical Sorting

If **A** is a poset with partial order ≤, we sometimes need to find a linear order **<** for the set **A** that will merely be an extension of the given partial order in the sense that if a ≤ b, then a < b. The process of constructing a linear order such as is called **Toplogical Sorting** .

EXAMPLE: Give a topological sorting for thr poset whose Hasse diagram is as shown below.





SOLUTION: The partial order < whose Hasse diagram **(a)** is clearly a linear order. It is easy to see that every pair in ≤ is also in the order < ,so < is a topological sorting of the partial order ≤.Figures **(b)** and **(c)**show two other solutions of this problem

Isomorphism

Let (A,**≤**) and (**A',≤'**) be posets and let f **: A**→ **A'** be a one to one correspondence between **A** and **A**′. The function f is called an **isomorphism** from (**A ,≤**) to (**A',≤'**) if, for any a and b in A,

$$a \le b \quad \text{if and only if} \quad f(a) \le f(b).$$

If f: **A→A'** is an isomorphism, we say that (**A,≤**) and (**A',≤'**) are **isomorphic** posets.

Example:

Let T be the set of all eve integers.Show that the semigroups {Z,+} and {T,+} are isomorphic.

Solution:

Define the function f : Z → T by f(a) = 2a.
Suppose that $f(a_1)=f(a_2)$ then $2a_1 = 2a_2$, so $a_1=a_2$ .Hence f is one-one.
Suppose that b is any even integer .Then a=b/2 ϵ Z and
f(a)=f(b/2) =2(b/2) =b
f(a+b)=2(a+b) = 2a +2b = f(a) +f(b)

Hence (Z,+) and (T,+) are isomorphic semigroups.