Pigeonhole Principle and Probability

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Contents

1 Pigeon Hole Principle

The pigeonhole principle is also known as Dirichlet's box (or drawer) principle

1.1 Basic Principle

If n pigeons are present in m pigeonholes where m<n , then atleast one pigeon hole will have more than one pigeon

1.2 Examples

Example 1:- Show that if any five numbers are chosen from 1 to 8, two of them will add upto 9
Solution :-
Now the sets which add upto 9 are {1,8} {2,7} {3,6} {4,5} .By Pigeonhole principle , if we have to choose five numbers then we must take atleast two numbers belonging to one set....thus two of five numbers will definitely add upto 9

Example 2 :- Show that any 11 numbers chosen from the set {1,2,3,4.....18,19,20} then one will be multiple of other
Solution :-
Each number can be represented by $(2^m) \times n$ where n is odd.
So now as there are only 10 odd numbers between 1 to 20 , if we select 11 numbers from the set 1,2,3,......,18,19,20 then by Pigeon Hole Principle two of them are bound to have same odd part...Hence one of them can be divided by other.Hence proved.

Example 3 :- If seven points are chosen in a regular hexagon each of side 1 units then two of them must be no further apart than 1 unit
Solution :-
Now we must find the pigeons and the pigeonholes...So divide the hexagon into six equal regions.Each will be an equilateral trangle of length of each side as one unit.Now these are our pigeonholes and the points as our pigeons. So by Pigeon Hole Principle two points will be in the same triangle. Thus they will be at a distance less than one unit.Hence Proved

Example 4 :- Shirts are numbered 1 to 20 are to be worn by the 20 members of a bowling league.When any three of these shirts are chosen , the league proposes to use the sum of their shirts as a code number for the team. Show that if any eight are chosen from the 20 shirts, then from these eight shirts one may form atleast two different teams having the same code number

Solution :-
Number of ways of choosing a code for any selected 8 shirts is ${}_8C_3 = 56$ different teams.Now these are our pigeons and the pigeonholes are the number of options we have.That is it will range from $(1 + 2 + 3) = 6$ to $(18 + 19 + 20) = 57$.Thus we have only 52 options .So by Pigeon Hole Principle, atleast two of them will have the same team code.

2 The Extended Pigeon Hole Principle

Now if there are more than 2m pigeons and m pigeonholes then obviously one will have atleast three pigeons.This can be easily visualized by considering the most uniform distribution.So extending this principle

2.1 Statement

If n pigeons are assigned m pigeonholes then atleast one contains $\lfloor (n-1)/m \rfloor + 1$ pigeons

2.2 Proof

Let each pigeonhole contain at max. $\lfloor (n-1)/m \rfloor$ pigeons
Then Total Number of Pigeons $<= m \times \lfloor (n-1)/m \rfloor <= m \times (n-1)/m = n-1$
This is a contradiction as there are n pigeons . So one of the pigeonholes contains $\lfloor (n-1)/m \rfloor + 1$ pigeons
Hence Proved

3 Probability

Probability is the likelihood or chance that something is the case or will happen

3.1 Sample Space

A set A contains of all the outcomes of an experiment is called a sample space of the experiment. With a given experiment , we can often associate more then one sample space depending on what observer chooses to record as an outcome. For example if two coins are tossed together the we have the sample space as $\{HH, HT, TH, HH\}$ where H and T represents heads and tails respecively. Now if the observer decides to record the number of heads as the outcome then our sample space becomes $\{0, 1, 2\}$ Thus we can easily see that sample space also depends on the criteria the observer sets for recording an outcome.

3.2 Events

An event is basically a subset of the sample space.
A statement about the outcome of an experiment, which for a particular outcome will be either true or false, is said to describe an event For example , if a set is exhaustive and mutually exclusive events :-
$P(n_i) >= 0$
$P(n_1) + P(n_2) + P(n_3) + \ldots\ldots\ldots + P(n_m) = 1$

3.3 Examples

Example 1:- If there is a Graph G = (V,E) where V corresponds to vertices and E corresponds to edges and the probability of two vertices being joined is p then find the average degree of the tree
textbfSolution :-
no. of pairs of vertices = ${}_nC_2$
no. of edges = ${}_nC_2 \times p = n \times (n-1)/2$
Average Degree = $2 \times TotalEdges/n$
$= 2 \times n(n-1)p/(2n)$
$= (n-1)p$
$\approx np$