DISCRETE STRUCTURES

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TOPIC:

METHODS OF PROOF

1.

$P_1 \wedge P_2 \wedge P_3 \wedge ... \wedge P_n$ => Q ……..This implication shows that if all P's are true then Q is true

Example 1:

If P=>Q and Q=>R it implies P=>R I.e. (P=>Q) ∩ (Q=>R) ≡ P=>R

Example 2:

P
P=>Q

Q/P …………..this is valid as evident from the truth table

Truth table

P	Q	P=>Q	(P∩(P=>Q))	(P∩(P=>Q))=>P	(P∩(P=>Q))=>Q
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	T	T

Smoking is healthy
If smoking is healthy physicians prescribe cigar

Physicians prescribe cigar

Example 3:

P=>Q
Q

P

Truth table

P	Q	P=>Q	((P=>Q)∩Q)	((P=>Q)∩Q)=>P
T	T	T	T	T
T	F	F	F	T

F	T	T	T	F
F	F	T	F	T

Hence this is not a tautology. Hence ((P=>Q) ∩ Q) =>P is not valid

If taxes are lowered, income rises
Income rises

Taxes are lowered ……………this is not true

Example 4:

Q) If n be an integer prove, n is odd if n^2 is odd.

A) Let P - : n^2 is odd
Q - : n is odd

Therefore we have to prove P=>Q
We can prove (P=>Q) ≡ (~Q) => (~P) first

<u>Truth table</u>

P	Q	~P	~Q	P=>Q	~Q=>~P
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

As the values obtained for (P=>Q) is same for all cases to (~Q=>~P), Therefore we now try to prove ~Q=>~P i.e. If n is not odd then n^2 is not odd

Proof:
Let n be even (not odd): n=2k (k is arbitrary)
$N^2= (2k)^2=4k^2$=even (not odd)

Hence ~Q=>~P
Therefore P=>Q

2. Proof by contradiction
This method is based on the tautology ((P=>Q) ∩ (~Q)) =>~P

Q) Prove that there is no rational number p/q such that the square is 2
A) We prove by method of contradiction

Let us assume $(p/q)^2=2$
Therefore $p^2=2q^2$
=>p^2 is even
=>p is even
p=2k (k is arbitrary)
Therefore
$4k^2=2q^2$
=>$q^2=2k^2$
=>q^2 is even
=>q is even
As p, q are even they have a common divisor and hence are not co prime.

We get a contradiction, hence our assumption was wrong.
Hence proved that there is no rational number p/q such that the square is 2.

Q) Let m, n be integers prove that $n^2=m^2$ iff n=m or n=-m
A) Let p be the statement $n^2=m^2$
Q be the statement n=m
R be the statement n=-m

Hence we wish to prove P<=>Q υ R
We know if
P=>Q υ R
Q υ R=>P
Are simultaneously true then P<=>Q υ R
Part 1:
To prove Q υ R=>P
If n=m then $n^2=m^2$
If n=-m then n2= $(-m)^2=m^2$
Hence Q υ R=>P
Part 2:
To prove P=>Q υ R
$N^2-m^2=(n-m)(n+m)$
Let R1 be the statement (n-m) (n+m) =0
Therefore it is proved that P=>R1
To prove R1=>Q υ R is same as proving ~ (Q υ R) =>~R1
Now
~ (Q υ R) is same as (~Q) ∩ (~R)
If n! =m and n! =-m therefore (n-m) (n+m)! =0
Or (~Q) ∩ (~R) =>~R1
It therefore implies R1=>Q υ R
As P=>R1
R1=>Q υ R
Therefore
P=>Q υ R........ (Proved)