Hyp

Lecture 7
LOGIC

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p	$\neg p$
T	F
F	T

Table 1: Truth table for negation

Abstract

Logic is the discipline that deals with the method of reasoning.On an elementary level,logic provides rules and techniques for determining whether a given arguement is valid.Logical reasoning is used in mathematics to prove theorems,in computer science to verify the correctness of programs and prove theorems,and in everyday life to solve a multitude of problems.In this lecture,a few basic ideas are discussed.

1 Introduction

A statement or proposition is a declarative sentence that is either true or false but not both.

Example:-
i)The earth is round.
ii)2+3=5.
iii)Do you speak English?
iv)2+x=5.

Out of these i) and ii) are statements or propositions, iii) is a question and iv) is an equation true for $x = 3$ and false for other values of x.

2 Operations on statements

2.1 Negation

$p : 2 + 3 = 5$
$\neg p : 2 + 3 \neq 5$
Table 1 gives the truth table for negation of a statement.

2.2 Conjunction

p: It is snowing.
q: I am cold.
$p \wedge q$: It is snowing and I am cold.
Table 2 gives the truth table for conjunction of two statements.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 2: Truth table for conjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Table 3: Truth table for disjunction

2.3 Disjunction

p: 2 is a positive integer.
q: 3 is a negative integer.
$p \vee q$: 2 is a positive integer or 3 is a negative integer. Since p is true ,the disjunction of $p \vee q$ is true,even though q is false.

Disjunction can be used in two senses - i)exclusive and ii)inclusive.

In the exclusive sense both p and q can not be both true.
Example:I will go on Friday or I will go on Monday.
In the inclusive sense both p and q can be both true.
Example:I will pass in Maths or I will fail in French.
In computer science disjunction is used in inclusive sense.
Table 3 gives the truth table for disjunction of two statements.

3 Quantifiers

$\{x \mid P(x)\} \rightarrow$ set of all x for which $P(x)$ is true.

3.1 Universal Quantifier

$P(x) : -(-x) = x$
$p : P(x)$ is true. Hence,we say that p is a universal quantification of $P(x)$.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 4: Truth table for conditional statement

3.2 Existential Quantifier

$P(x) : x + 4 < 7$
$q : \exists x P(x)$ is true. Hence,we say that q is an existential quantification of $P(x)$.

For universal quantifier order doesn't matter.
$Example : \forall A \forall B, A * B = B * A \equiv \forall B \forall A, B * A = A * B$.
However, for existential quantifier order matters.
$Example : p : \forall A \exists B, A + B = I$ is true.
But,$q : \exists B \forall A, A + B = I$ is false.
$r : \exists B \forall A, A + B = A$ is true.

We can write the universal quantification of a statement as the existential quantification of the negation of that statement and vice-versa.
$Example$:
$\forall x P(x) = \neg(\exists x \neg P(x))$
$\exists x P(x) = \neg(\forall x \neg P(x))$

4 Conditional Statement

$p \Rightarrow q$ means "If p then q."
Here p is the hypothesis and q is the conclusion.
Table 4 gives the truth table for conditional statement.

If $p \Rightarrow q$ is an implication ,we say $q \Rightarrow p$ is the converse and $\neg q \Rightarrow \neg p$ is the contrapositive.
$Example$:
$Statement$:If it is raining,then I get wet.
$Converse$:If I get wet,then it is raining.
$Contrapositive$:If I do not get wet,then it is not raining.

5 Biconditional Statement

$p \Leftrightarrow q$ means $p \Rightarrow q$ and $q \Rightarrow p$
Table 5 gives the truth table for biconditional statement.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Table 5: Truth table for biconditional statement

p	q	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$	$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Table 6: Truth table for a tautology

6 Tautology , Contradiction & Contingency

A statement that is true for all possible values of its propositional variables is called a **tautolgy**.A statement that is always false is called a **contradiction** or an **absurdity**,and a statement that can be either true or false,depending on the truth values of its propositional variables,is called a **contingency**.

The truth table of a tautology contains all true entries.Table 6 shows the truth table of a tautology.

7 Properties of Operations

The operations for propsitions have the following properties:-
Commutative Properties
1.p∨q≡q∨p
2.p∧q≡q∧p

Associative Properties
3.p∨(q∨r) ≡ (p∨q)∨r
4.p∧(q∧r) ≡ (p∧q)∧r

Distributive Properties
5.p∨(q∧r) ≡ (p∨q) ∧ (p∨r)
6.p∧(q∨r) ≡ (p∧q) ∨ (p∧r)

Idempotent Properties
7.p$\vee$p$\equiv$p
8.p$\wedge$p$\equiv$p

Negation Properties
9.$\neg$($\neg$p) $\equiv$p
10.$\neg$(p$\vee$q) $\equiv (\neg p) \wedge (\neg q)$
11.$\neg$(p$\wedge$q) $\equiv (\neg p) \vee (\neg q)$

Properties 10 and 11 are also called De Morgan's laws.