Lecture 1

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Contents

Abstract

Some of the most important sets arise in connection with sequences.A sequence is a list of objects arranged in a definite order.
The list my stop after n steps,n$\in$ N or it may go on forever.

1 Sequence Formula

Sequence formula are of 3 types:
1) Recursive Formula
$C_n = 2C_{n-1}$
2)$Explicit Formula$
$C_n = n^2$
3) Non-Mathematical Formula
s,t,u,d,y$\rightarrow$letters of word 'study'
h,e,l,o$\rightarrow$ unique letters of word 'hello'

2 Characteristic Function

If A is a subset of a universal set U, the characteristic function f(A) is defined for each x$\in$U as follows:
f_A(x)= 1 if x$\in$A
0 if x$\notin$ A

2.1 Theorem 1

a) $f_{A\cap B}=f_A f_B$
b) $f_{A\cup B}=f_A+f_B-f_A f_B$
c) $f_{A+B}=f_A+f_B-2f_A f_B$

3 Sets

A set is countable if the members of the set can be arranged in a list.All finite sets are countable.
A set that is not countable is called uncountable.
An example of an uncountable set is the set of all real numbers that can be represented by an infinite decimal of the form 0.a $_1a_2a_3$....

,where a_i is an integer and $0 \leq a_i \leq 9$ .We shall prove that it is uncountable by contradiction.Let it be countable.Then we can build the foll. list:
d_1=0.$a_1a_2a_3$...
d_1=0.$b_1b_2b_3$...
d_1=0.$c_1c_2c_3$...
.
.
Each infinite decimal must appear on the list.
But we can form an infinite decimal which is not in the list.
Let it be x=0.$x_1x_2x_3$... where $x_1$is 1 if a_1=2 else x_1=2;x_2=1 if b_2=2 else x_2 is 2.
This can be continued indefinitely.The number thus is an infinite decimal of 1's and 2's but x is in the list at some position.
Thus x is not in the list by contradiction.Hence it is an uncountable set.

4 Regular Expressions

Given a set A,we can construct the set A* consisting of all finite sequences of A.We assume that A* containsthe empty sequence,without any symbols,and we denote it by $\wedge$.
A regular expression over A is a string built from the elements of A and the symbols (,),$\vee, *, \wedge$.

RE1:The symbol$\wedge$is a regular expression.
RE2:If x$\in$A,the symbol x is a regular expression.
RE3:If α andβ are regular expressions,then $(\alpha\beta)$ is regular.
RE4:If α and β are regular expressions,then $(\alpha \vee \beta)$ is regular.
RE5:If α is a regular expression,then (α)* is regular.
Associated with each regular expression over A,there is a corr. subset of A*.Such sets are called regular sets.