INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

*Date* 22.09.2008 AN *Time:* 2 Hrs. *Full Marks* **25** *No. of Students:* **65**

Autumn Semester:, 2008 Department: Computer Science and Engineering

Sub. No: CS 21001 2nd**Yr. B. Tech.(Hons.) Sub. Name: Discrete Structures**

Question 1 [10 x 1.5]

1. Prove that if m and n are relatively prime and m.n is a perfect square, then m and n are each perfect squares.
2. Show that if A and B are  matrices and ,  both exist, then .
3. Let  be a sequence of positive integers defined recursively as:



 for all 

Prove the following assertions. You may use induction on n, whenever necessary

**i).**  for all 

**ii).**  for all 

**iii).** for all 

1. Prove that if the product of integers *p* and *q* is odd, then both *p* and *q* must be odd.
2. Show that in any set X of people there are two members of X who have the same number of friends in X. (It is assumed that |X| is at least 2, and if x is a friend of x’ then x’ is a friend of x.)
3. The set Y consists of the following numbers: Y = {1, 31/2, 3, 33/2, …, 319/2, 310}. In how many ways can a pair of distinct numbers be selected from the set Y so that their product is greater than or equal to 310?
4. Find a recurrence relation for the number of bit strings of length *n* that have three consecutive 0s. Use this relation to find the number of such bit strings of length 7.
5. Given that |*A*| = 24 and an equivalence relation **q** on *A* partitions it into three distinct equivalence classes *A*1, *A*2, and *A*3, where |*A*1| = |*A*2| = |*A*3|, what is |**q**|?

Question II [2 + 2 + 1 +1]

1. Let A = {1,2,3,4,5}, R and S are equivalence relations, R = {(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4), (5,5)}, and S = {(1,1),(2,2),(3,3),(4,4),(4,5),(5,4),(5,5)}. Find the smallest equivalence relation containing R and S and compute the partition of A it produces.



1. The figure defines two relations on the set {a,b,c,d}. Find the list and matrix representations of those relations.
2. A relation on the set {a,b,c,d} is defined by the following list: {(*a,c*), (*c,c*), (*a,a*), (*b,b*), (*c,a*), (*d,b*), (*d,a*)}. Draw its directed graph representation.
3. What is wrong with the following “proof” that every symmetric and transitive relation is reflexive? If (*a,b*)*R* then (*b,a)* *R* by symmetry. By transitivity (*a,a)**R.* Therefore *R* is reflexive.
4. Backtrack to find an explicit formula for the sequence defined by the recurrence relation

bn = 2.bn-1 = 1 with initial condition b1 = 7.

Question III [2 + 2]

Prove

1. Let R be a relation on a set A. Then R∞ is the transitive closure of R.
2. Let R be an equivalence relation on a set A and let a € A and b € A. Then

a R b if and only if R(a) = R(b) .