CS21001:Discrete Structures

Autumn semester 2009-10

Tutorial: **Methods of proof, permutations and combinations**

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1. **Prove**

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \binom{n}{2}\binom{m}{r-2} + ... + \binom{n}{r}\binom{m}{0}$$

2. Let F_n, n ϵ N, denote the sequence of Fibonacci numbers. Prove by induction on n that

$$F_{2n} = F_n(F_{n+1} + F_{n-1}) \text{ and } F_{2n+1} = F_{n+1}^2 + F_n^2 \text{ for all } n \geq 1.$$

3. Show that $2^n > n^3$ for $n \geq 10$.

4. (a) Show that the total number of permutations of p red balls and 0, or 1, or 2 ..., or q white balls is

$$\frac{p!}{p!} + \frac{(p+1)!}{p!1!} + \frac{(p+2)!}{p!2!} + ... + \frac{(p+q)!}{p!q!}$$

   (b) Show that the sum in part (a) is

$$\frac{(p+q+1)!}{(p+1)!q!}$$

5. **Prove or disprove**

   (a) Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

   (b) $x - y$ divides $x^n - y^n$ for $n \geq 1$.

6. Prove the triangle inequality, which states that if x and y are real numbers then

$$|x| + |y| \geq |x + y|$$

   (where $|x|$ represents the absolute value of x)

7. Five fair coins are tossed and the results are recorded.

   (a) How many different sequences of heads and tails are possible?

   (b) How many of the sequences in part (a) have exactly one head recorded?

(c) How many of the sequences in part (a) have exactly three heads recorded?

8. How many zeroes are there at the end of 12!? at the end of 26!? at the end of 53!?

9. Prove by mathematical induction that if $A_1, A_2, ..., A_n$ and B are any $n+1$ sets, then

$$\left(\bigcup_{i=1}^{n} A_i\right) \bigcap B = \bigcup_{i=1}^{n}(A_i \cap B)$$