CS21001:Discrete Structures

Autumn semester 2009-10

Tutorial: **Relations and Digraphs**

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1. Let $A = 1, 2, 3, 4$ and let R and S be the relations on A described by

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Use Warshall's algorithm to compute the transitive closure of $R \cup S$.

2. Let R and S be relations on A.

   (a) If R is symmetric, so are R^{-1} and $\overline{R}$.

   (b) If R and S are symmetric, so are $R \cap S$ and $R \cup S$.

3. Let R and S be relations on A.

   (a) If R is reflexive, so is R^{-1}.

   (b) If R and S are reflexive, so are $R \cap S$ and $R \cup S$.

   (c) R is reflexive if and only if $\overline{R}$ is irreflexive.

4. Let R and S be relations on A.

   (a) $(R \cap S)^2 \subseteq R^2 \cap S^2$.

   (b) If R and S are transitive, so is $R \cap S$.

   (c) If R and S are equivalence relations, so is $R \cap S$.

5. Prove that the number of partitions of a set with n elements into k subsets satisfies the recurrence relation

$$S(n, k) = S(n-1, k-1) + k.S(n-1, k)$$

6. Let $P_1 = \{A_1, A_2, ..., A_k\}$ be a partition of A and $P_2 = \{B_1, B_2, ..., B_m\}$ a partition of B. Prove that

$$P = \{A_i \times B_j, 1 \leq i \leq k, 1 \leq j \leq m\}$$

is a partition of $A \times B$.

7. Prove by induction that if a relation R on a set A is symmetric, then R^n is symmetric for $n \geq 1$.

8. Let $A = \{a, b, c, d, e\}$ and $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$, give the relation R defined on A and its digraph.

9. Let R be a relation from A to B. Prove that for all subsets A_1 and A_2 of A

$$R(A_1 \cap A_2) = R(A_1) \cap R(A_2) \qquad \text{if and only if}$$

$$R(a) \cap R(b) = \{\} \qquad \text{for any distinct } a, b \text{ in } A$$