Prediction of performance of exterior beam-column connections with headed bars subject to load reversal

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\textbf{ABSTRACT}

Given that both ACI 318-08 provisions and 352R-02 recommendations have been developed based on quite limited experimental data, an extensive database was assembled by Kang et al. [12], which contains most of the available test data of reinforced concrete exterior beam-column connections with headed bars subject to load reversal. In this study, the database has been further expanded by adding the recent data focusing on the investigation of design parameters of clear bar spacing and head size, and re-evaluated using a variety of statistical and empirical techniques. An effort has been made to find a statistical model linking quantitative design parameters and qualitative connection response. In this study, binomial logistic regression methodology has been applied. The statistical methodology quantifies the effect of each design parameter in determining the performance of the connection. A reliable and robust goodness-of-fit test, the log-likelihood ratio test, was performed to evaluate the developed logistic regression model. Finally, the recent connection data were used to validate the predictive capability of the developed statistical model.

\section{1. Introduction}

The use of headed bars is becoming more popular since it provides a solution to the constructional problem of steel congestion, particularly in reinforced concrete beam-column connections (see Fig. 1) [22,24,6,9,18,12]. Relevant provisions and limitations have been provided in the 2008 edition of ACI 318 (Sections 12.6.1 and 12.6.2). The limitations or restrictions include bar strength, bar and head size, clear cover and bar spacing, and concrete weight. Prior to this, design guidelines for headed bars in beam-column connections were incorporated into the 2002 edition of the ACI 352 report based on both monotonic and cyclic tests. This ACI-ASCE Committee 352 report recommends the development length for headed bars along with some other details such as the location of heads and the amount of head-restraining reinforcement for preventing the prying action of headed bars placed near a free surface of concrete.

In the last two decades, significant amounts of experimental investigation have been carried out to determine the suitability of each parameter restriction and limitation as imposed by the ACI documents (compiled by Kang [10,11] and Kang et al. [12]). Three-quarters of the data were available only in Japanese. There is still a need for additional data for various design parameters such as headed bar clear spacing, number of layers of beam reinforcement and head size of headed bars, particularly from well-documented studies published in English. Also, no study has yet been done to investigate the coupled effect of these design parameters which probably influence connection response. Quantification of these design parameters that probably influence connection response has also not been dealt with in any previous study. Furthermore, no previous study has provided a statistical prediction of satisfactory seismic connection performance over unsatisfactory performance given a set of design parameters as specified by the code recommendations.

To bridge this gap, a thorough data analysis was conducted using a variety of statistical and empirical techniques. A brief summary of the two recent testing programs conducted as part of this analysis has been provided. A statistical model has been developed to predict satisfactory over unsatisfactory performance of the connections with headed bars given a set of design parameters that might influence connection responses. For this, binomial logistic regression methodology has been applied. The statistical methodology quantifies the effect of each design parameter in determining the performance of the connection. Subsequently, the methodology has been evaluated by using a goodness-of-fit test. Such comprehensive investigation is made as part of Joint ACI-ASCE Committee 352 task group efforts. Currently, Joint ACI-ASCE Committee 352 recommendations on headed bars in beam-column connections are included...
connections are under revision by the Task Group within the committee, and the authors are spearheading the Task Group’s efforts.

2. Prior database of exterior connections with headed bars

An extensive database spanning a wide range of design parameters for reinforced concrete exterior and knee beam-column connections with headed bars had been assembled by Kang et al. [12] (see Table 1). All the included specimens were beam-column connection subassemblies subject to load reversals. This dataset includes classifications of satisfactory and unsatisfactory seismic connection performance based on the following performance indices: (1) the ratio of measured peak moment to nominal moment capacity; (2) drift ratio at the point of 20% drop from the peak lateral load; (3) ratio of strain in the headed bar at the joint-member interface to yield strain; and (4) joint shear distortion during about 3.0% drift cycles. Connection behavior was assumed unsatisfactory if the ratio of peak to nominal moments was less than 1.0 and no bar yielding was monitored by strain gauges [8,15]. If the specimen exhibited more than 20% reduction in strength until 3.5% drift and exceeded 1.2% of joint shear distortion until 3.5% drift cycles, the connection was also considered to have exhibited unsatisfactory seismic performance. More details are available in the papers by Kang [10,11] and Kang et al. [12]. Only the interstory exterior connections of the database are considered in this study.

3. Recent data of exterior connections with headed bars

Reversed cyclic tests of two full-scale exterior beam-column connection subassemblies were carried out to evaluate the applicability of headed bars with small heads in exterior connections [13]. One specimen had headed deformed bars with a small head size of $A_{bgr/A_b} = 2.6$, while the other specimen had 90-degree hooked bars, where $A_{bgr}$ is the net head bearing area and $A_b$ is the bar area. The performance of the connection with headed bars was satisfactory as demonstrated by modest joint shear distortions ($\leq 0.01$), and generally complied with the ACI 374.1-05 [1], Acceptance Criteria for Moment Frames Based on Structural Testing and Commentary, even with the lack of joint confinement (Fig. 2). More details on the specimen design and test procedures and results are provided elsewhere [13].

Recently, two more exterior beam-column subassemblies were subjected to cyclic load reversals: one with a horizontal layer of closely-spaced headed bars ($c_s = 2.1d_b$) and the other with two layers of headed bars with heads touching each other ($c_s = 1.3d_b$) [14]. The connections were designed in adherence to ACI 318-08 [2] and ACI 352R-02 [3], except for the minimum bar clear spacing requirement. Both specimens exceeded the values of $M_{pl}$ by 15–20%, exhibited limited joint shear distortions, and dissipated substantial energy by beam flexural hinging (Fig. 3). Furthermore, neither strength degradation nor anchorage failure was signaled until the end of testing (up to 5% drift). As a result, the test results generally met the ACI 374.1-05 [1] acceptance criteria.

4. Discussion of new data in connection with trends

Fig. 4 presents the relationship of the provided embedment depth and head size for headed bars used in the satisfactory specimens (‘o’ marks), and unsatisfactory specimens (‘x’ marks) that were affected by improperly short bond development. Note that these marks are consistently used throughout the paper. Additionally, the three new data discussed in the preceding section were plotted using ‘v’ marks in Fig. 4. Based on the test results [13,14], all three ‘v’ marked specimens are classified as “satisfactorily performed.” A new data point [13] for headed bars with small heads was obtained, giving a valuable indication of the relationship between a small head size and development length. Fig. 5 signals that a minimum head bearing area ($A_{bgr}$) of three times the bar area ($A_b$) may be feasible for headed bars terminating in beam-column connections, provided that the development length of the bar complies with ACI 318-08 [2], Chapter 12. Additional test data would be helpful to confirm this.

After examining the two new data of beam-column connections with closely-spaced headed bars [14], it was found that the clear bar spacing smaller than $4d_b$ did not adversely affect the connection performance due to significant confinement provided by joint transverse reinforcement and a longitudinal beam, where $d_b$ is the bar diameter. For the satisfactory connections (see ‘o’ and ‘v’ marks in Fig. 5), the small clear bar spacing hardly affected the drift ratio measured at a drop to 80% of the peak lateral load, which is considered a seismic performance indicator. Particularly, the new test results [14] showed that both the connection having a single layer of beam top (or bottom) longitudinal reinforcement with $c_s$ of 2.1$d_b$ and the connection having two layers of beam top (or bottom) longitudinal reinforcement with heads touching each other ($c_s = 1.3d_b$) performed excellently (see Fig. 3) and generally satisfied the ACI 374.1-05 [1] acceptance criteria. The new data strengthen the proposal to increase the minimum clear bar spacing limit in a horizontal layer of beam longitudinal reinforcement from $4d_b$ to $2d_b$ for the design of exterior beam-column connections.

5. Methodological approach used for statistical model

Experimental observations provide a qualitative measure of the impact of various design parameters on connection response. There is an obvious need to relate the quantitative measures with quantitative design parameters such that the effect of the independent variables can be quantified and the designer acquires an
Table 1
Test data of beam-column connections with headed bars under cyclic loads (1 ksi = 6.895 MPa; 1 in. = 25.4 mm; 1 kip = 4.45 kN).

<table>
<thead>
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<th>Author</th>
<th>Type</th>
<th>F.M.</th>
<th>$f_y$, ksi</th>
<th>$d_h$, in.</th>
<th>$l_p$, in.</th>
<th>$l_o$, in</th>
<th>$A_{hy} / A_{ho}$</th>
<th>$c_0$, dp</th>
<th>$P_j / (A_{hy})$</th>
<th>$V_u$, kip</th>
<th>$V_{u,p}$, kip</th>
<th>$\rho_p (\psi)^{0.12}$</th>
<th>$\bar{d}_{f,peak}$</th>
<th>$\gamma$</th>
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<td>4.0</td>
<td>Trans. 0</td>
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<td>1.19</td>
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<td></td>
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<td>Knee</td>
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<td>190.2</td>
<td>104.9</td>
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<th>l_d, in.</th>
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<th>c_{sh}, in.</th>
<th>d_{p}, in.</th>
<th>( F/(A_f) )</th>
<th>( V_n ), kip</th>
<th>( V_u ), kip</th>
<th>( \rho_c ), ( \rho^{1/2} ), d_{peak}</th>
<th>( \gamma )</th>
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F.M. = Failure Mode; I = Category-I (member flexural hinging followed by modest joint deterioration); II = Category-II (member flexural hinging followed by joint failure); III = Category-III (joint failure prior to member flexural hinging); Trans. = with Transverse beams; A_{arb} = net bearing area of head; A_{sh} = area of joint transverse reinforcement in principal direction within hoop spacing (\( s_h \)); c_{sh} = clear cover to bar; d_{p} = bar diameter; \( f_y \) = specified yield stress of headed bars; \( b_{peak} \) = drift ratio at a drop to 80% from peak lateral load; \( c_j \) = maximum joint shear distortion during about 3.5% drift cycles.

\[ q_h = \frac{A_{sh}}{s_h s_h} \]

a Eccentric connection with an offset of a half of the beam width.

b Average values for staggered headed bars.

c At least (i.e., testing was stopped prior to 20% drop from the peak).

d One bearing plate was used for a group of heads; A_{sh} = bar area.

References:
understanding of how the parameters are to be varied to obtain the desired response. Linear and/or nonlinear regression is one possible approach for developing such a model; however, this is not ideal because it requires assigning a quantitative measure to the qualitative connection response parameter. Conversely, logistic regression is ideally suited for developing this type of model. This method allows for quantification of the conditional probability of a qualitative measure based on quantitative data. In fields other than structural engineering, such as in medicine, clinical studies, sociology, economics and transportation, the logistic regression is an established technique for developing relationships between qualitative response variables and a set of independent quantitative parameters. In structural engineering, the method of logistic regression has been applied by Mitra et al. [21], Mitra and Samui [20] and Mitra [19] to determine the failure initiation mechanism of reinforced concrete interior and exterior beam column connections utilizing different types of independent quantitative parameters. For the current study, the logistic regression was used to develop a relationship between the performance of beam-column connections with headed bars under load reversal (a qualitative measure) and a set of independent design parameters (quantitative measures). In comparison with linear/nonlinear regression, the logistic regression model has less stringent requirements, since it does not assume linearity (or nonlinearity) of relationship between the independent variables and the dependent variable nor does it require normally distributed variables. Details regarding the methodological approach of application of logistic regression models

![Fig. 2. Moment versus lateral drift ratio relationship for specimen tested by Kang et al. [13].](image2)

![Fig. 3. Moment versus lateral drift ratio relationship for specimens tested by Kang et al. [14].](image3)

![Fig. 4. Head size versus provided-to-required development length per ACI 352.](image4)

![Fig. 5. Drift ratio at 20% drop from the peak versus headed bar clear spacing.](image5)
can be obtained from Mitra et al. [21], Mitra and Samui [20] and Mitra [19]; however, for sake of thoroughness, the methodology has been described briefly in the following subsection.

5.1. Development of binomial logit model

The logistic models [42,4] employ a regression relationship between independent quantitative variables and discrete qualitative events. In the present model, the two discrete qualitative events are “satisfactory performance” (referred to as Event 1) and “unsatisfactory performance” (referred to as Event 0). The likelihood of observing a discrete event of satisfactory performance is defined by the log of the odds ratio for that event. The odds ratio for Event 1 is the ratio of the probability of occurrence of Event 1, \( P_{E=1} \), to the probability of occurrence of Event 0, \( P_{E=0} \). Thus,

\[
Y = \log \left( \frac{P_{E=1}}{1 - P_{E=1}} \right) = \log \left( \frac{P_{E=1}}{P_{E=0}} \right) = \beta_0 + \sum_{k=1}^{K} \beta_k X_k \tag{1}
\]

where \( \beta_k \)'s are the logistic regression parameters, \( X_k \)'s are the covariates or connection design parameters and \( K \) is the total number (\( \approx 8 \)) of design parameters considered. As described in the book by Ben-Akiva and Lerman [5], Eq. (1) may be manipulated to define the probability of occurrence of Events 1 and 0 such that:

\[
P_{E=1} = \frac{e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}}{1 + e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}} \tag{2a}
\]

\[
P_{E=0} = \frac{1}{1 + e^{\beta_0 + \sum_{k=1}^{K} \beta_k X_k}} \tag{2b}
\]

5.2. Discussion of binomial logit analysis

The independent quantitative design parameters that were considered to affect the performance of the connection were taken as follows: (1) the ratio of provided embedment depth to required development length as per ACI 352 ([4], labeled as \( A \)); (2) the ratio of head thickness to bar diameter (\( \text{head depth} / \text{bar dia.} \); labeled as \( B \)); (3) the ratio of net bearing area to bar area (\( A_{\text{net}} / A_b \); labeled as \( C \)); (4) the ratio of joint shear at probable beam moment to joint shear capacity (\( V_{c} / A_b d_f \); labeled as \( D \)); (5) the specified yield strength of the headed bar (\( f_y \); labeled as \( E \)); (6) the joint transverse reinforcing ratio in the direction of lateral loading (\( A_{x, \text{joint}} / S_b h' \); labeled as \( F \)); (7) the side cover for the headed bar to bar diameter (\( c_s / d_b \); labeled as \( G \)); and (8) the ratio of column axial load to compressive strength of the concrete (\( P / A_{f, c} \); labeled as \( H \)). Here, \( A_{x, \text{joint}} \) is the cross-sectional area of joint transverse reinforcement placed in the direction of lateral loading within the joint in a layer, \( S_b \) is the joint hoop spacing, \( h' \) is the joint core width, \( A_{f, c} \) is the gross column section area, and \( f_y \) is the specified concrete compressive strength.

The method of maximum likelihood [5], which provides a means of choosing an asymptotically efficient estimator for a set of parameters, was used to compute logistic regression parameters (\( \beta_i \)) in Eq. (1). For each of the independent variables in the models, Table 2 shows the computed regression parameters (\( \beta_i \)). It should be noted that by obtaining the logistic regression parameters (\( \beta_i \)), one can use Eqs. (2a) and (2b) to evaluate the probability of occurrence of Events 1 and 0, respectively. Given the definition of \( Y \) in Eq. (1), the sign of a regression parameter indicates whether an increase in the associated design parameter increases or decreases the likelihood of the satisfactory performance of exterior connections with headed bars. A positive regression parameter indicates that increasing the associated design parameter increases the likelihood of a satisfactory performance. Similarly, a negative regression parameter indicates that increasing the associated design parameter decreases the likelihood of unsatisfactory performance. Based on the signs of the parameters in Table 2, an increase in \( A, B, C \) and \( D \) would result in the likelihood of satisfactory performance of the connection response. An increase in other parameters such as \( E, F, G \) or \( H \) would result in increased likelihood of unsatisfactory performance of the connection response.

An increase in the ratio of provided embedment depth to required development length (\( A \)) typically means an improved bond condition of the specimen, and improved bond capacity would qualitatively mean better performance of the connection. Fig. 4 graphically supports the idea that the provided embedment depth directly affects the connection performance. An increase in ratio of the thickness of the anchored head to the diameter of the reinforcing bar (\( B \)) qualitatively means an increase in resistance of the head against deformation which would also equate to better performance of the connection. No comprehensive research on head thickness has been carried out, nor are standards on head thickness available in ACI 318 codes and ASTM specifications. Based on the new experimental work by Kang et al. [13], the head thickness of at least 1\( d_b \) is considered reasonable.

If the head size (\( C \)) is increased, a better bearing is achieved which qualitatively also results in better performance of the connection region. A comprehensive review of the database [10–12] showed that both the development length and head size determine the anchorage capacity of a headed bar, and that the head size should be large enough to ensure no anchorage failure (at least \( A_{\text{avg}} \geq 3 A_b \) according to the new experimental work by Kang et al. [13]). Better performance can also be achieved if the joint shear demand (\( D \)) is decreased. The results obtained from the statistical methodology are in direct agreement with what has been observed experimentally (e.g., [6]). Less joint shear deformation was monitored for the connection subject to a smaller joint shear demand, but with the same or comparable other conditions.

On the other hand, an increase in yield strength of the longitudinal bar (\( E \)) results in an increase in the elastic stiffness of the connection along with exhibition of a rather brittle response and reduced ductility. The reduced ductility due to the use of high strength bars (\( f_y > 60 \text{ ksi; } 430 \text{ MPa} \)) results in unsatisfactory performance of the connection. All of the specimens with very high strength steel (\( f_y \geq 120 \text{ ksi; } 830 \text{ MPa} \)) in the database [12] did not exhibit satisfactory seismic connection performance. Furthermore, only 3 of 16 specimens with \( f_y \geq 100 \text{ ksi \ (690 MPa)} \) [No. 3, [17]; Nos. 4 and 12, [16]] showed satisfactory seismic performance. Therefore, the feasibility of the applications of very high strength steel in beam-column connections is questionable, and further scientific research on this topic is required.

Contrary to the common belief, it was observed that increasing the area of the transverse reinforcement within the joint (\( F \)) results in unsatisfactory performance. On a related note, it has been pointed out by many researchers (e.g., [7,23]) that for exterior connections with conventional longitudinal reinforcement subject to
load reversal, transverse reinforcement within the joint does not have any significant influence on the failure mechanism of beam-column connections under low axial loads (e.g., \( P \leq 0.12A_g f')\). All of the specimens in the database were subjected to axial loads less than 0.12A_g f' \[12\]. Based on a review of the database, a ratio of \( \left( \rho_i / \rho_h \right) \) even at the level of about 0.3 appeared not to pose a serious joint shear distress problem under low axial loads, where \( \rho_h = (A_{sh} / h^2) \), \( A_{sh} \) is the area of joint transverse reinforcement in the principal direction within hoop spacing \( (s_h) \), \( s_h \) is the joint hoop spacing, and \( h^* \) is the joint core width.

It was observed that increasing the side cover \( (C) \) to the headed bar beyond the minimum value results in an increased probability of unsatisfactory performance. Typically, the larger side cover is, the less vulnerable the connection is to side-face blowout failure. As noted earlier, side-face blowout is not a concern for headed bars anchored in interstory exterior connections because of its sufficient side cover. Only two specimens in the database had the side cover to the headed bar less than \( 2d_p \). Therefore, the adverse effect of increased side cover, shown in Table 2, appears to have no significant physical meaning. Increasing the column axial ratio \( (H) \) also results in an increase in the probability of unsatisfactory performance; however, as mentioned earlier, the level of the applied axial load was quite low for all specimens \( (P \leq 0.12A_g f') \). Thus, no information regarding the effect of high axial loads is available from the database.

The magnitude of a regression parameter multiplied with the mean of its corresponding design variables (referred to as the “influence factor” column in Table 2) indicates the relative importance of the design variables in determining connection failure initiation response. It should be noted that the sign of the influence factor is similar to the sign of the regression parameter. Based on results obtained in Table 2, the yield strength of the headed reinforcing bar is the most influential parameter, a decrease of which would result in higher satisfactory performance, whereas the least influential parameters are the side cover and head size. Again, this indicates that there is a great and urgent need to carry out research regarding high-strength headed bars in beam-column connections subject to load reversal.

6. Statistical evaluation of the applied statistical model

6.1. Goodness of fit of the developed logit model

To further evaluate the model, log-likelihood ratio test was performed which estimates the overall explanatory power of the model and also determines whether the inclusion of the independent parameters chosen for the model are statistically significant and improves the overall model prediction. The likelihood ratio test statistic is as follows:

\[
X^2 = 2[LL(\beta_0) - LL(\beta_i)]
\]

(3)

where \( LL(\beta_0) \) is the log likelihood at convergence of the “restricted” model (i.e., model in which all the parameters except the constant term are equal to 0), and \( LL(\beta_i) \) is the log likelihood at convergence of the “full” or “unrestricted” model (i.e., model under investigation with all parameters). The \( X^2 \) statistic is chi-square distributed with the degrees of freedom equal to the difference in the number of parameters in the “restricted” and “unrestricted” model. For the proposed logit model for exterior connections, the value of the obtained log-likelihood for the “unrestricted” or “full” model is \(-11.43\) whereas for the “restricted” model it is \(-42.08\). The chi-squared value obtained is 30.65 with 8 of freedom resulting in a \( p \)-value of less than 0.001, suggesting that inclusion of the independent parameters in the predicted model significantly improves the goodness-of-fit measure.

6.2. Predictive efficiency assessment

In order to assess the predictive efficiency of the statistical model, the likelihood of satisfactory performance \( (Event~1) \), computed using Eq. (2a) with \( \beta_i \) from Table 2, was plotted versus the observed event in Fig. 6. Connections from the dataset exhibiting unsatisfactory performance \( (Event~0) \) are plotted as circles and connections exhibiting satisfactory performance \( (Event~1) \) are plotted as squares. If the model were perfect, all connections exhibiting Event 0 would have a computed probability of occurrence of Event 1 of 0.0, while all connections exhibiting Event 1 would have a computed probability of occurrence of 1.0. The data in Fig. 6 indicate that although the model is not perfect for exterior connections with headed bars, the model is able to predict satisfactory performance for 94% of the connections and unsatisfactory performance for 89% of the connections. Overall, the correct prediction for the model is 92%.

7. Validation of the developed logit model using new data

In the preceding section, the logit model performance was evaluated using statistical methods. In this section, the developed model is validated with new experimental investigation data. The recommended procedure of using the developed model for predicting satisfactory or unsatisfactory performance is demonstrated with the help of three recently tested connections (see Section 3).

1. Calculate the design parameters considered in the logit model for the new data.
2. Obtain the probabilities of satisfactory and unsatisfactory performance using Eqs. (2a) and (2b), along with the estimated \( \beta_i \) values listed in Table 2.

The aforementioned procedure was applied when predicting the performance of three connections tested by Kang et al. \[13,14\], which were not included in the database used for the logit model development. Table 3 lists the design parameters calculated for the connection with small-headed bars \[13\], the connection with horizontally closely-spaced headed bars \[14\] and the connection with vertically closely-spaced headed bars \[14\]. The computed probabilities of satisfactory and unsatisfactory performance for the first connection are 84% and 16%, respectively. The probabilities of satisfactory performance of the second and third connections are approximately 99%. Therefore, it can be concluded that the statistical model, expressed by Eqs. (2a) and (2b) along with the \( \beta_i \) values in Table 2, predicts the observed three connection responses quite accurately, given the material and geometric properties as well as the reinforcing details.
8. Conclusions

The database spanning a wide range of design parameters for reinforced concrete exterior beam-column connections with headed bars has been statistically assessed to obtain forensic evidence of the observed behavior. As part of this study, binomial logistic regression methodology has been developed to quantify the effect of each design parameter in determining the performance of the beam-column connection with headed bars subject to load reversal. In the end, the statistical methodology was evaluated by using two robust goodness-of-fit tests and actual experimental data.

The following conclusions were drawn: (1) an increase in development length, head thickness and head size and a decrease in joint shear demand result in a qualitatively better performance of the connection; (2) an increase in bar yield strength, joint transverse reinforcement, and column axial force correlates to increased probability of unsatisfactory performance; (3) the joint shear demand and bar yield stress are two of the most influential design parameters on the connection performance; and (4) the feasibility of the applications of very high strength headed bars in beam-column connections is highly questionable. These conclusions were verified by applying the log-likelihood ratio test. Finally, the statistical methodology was validated using the actual data, revealing that the model is capable to reasonably predict the connection performance.

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