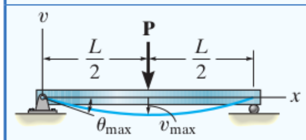
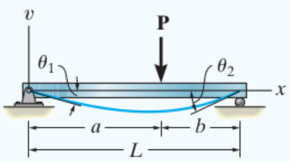
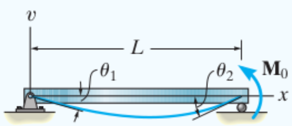
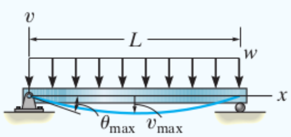
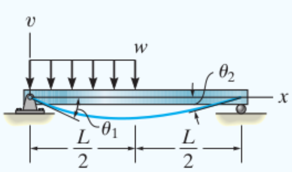
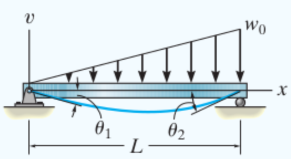
TUTORIAL SHEET 8: DEFLECTION OF BEAMS

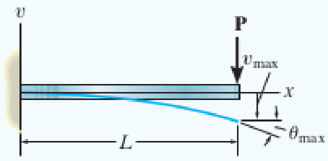
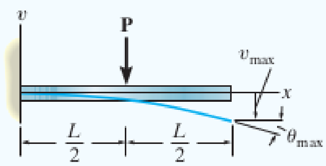
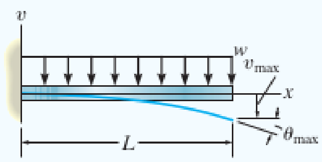
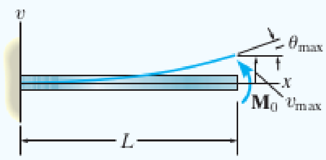
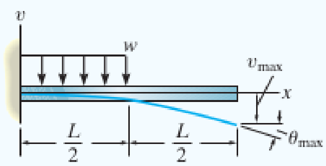
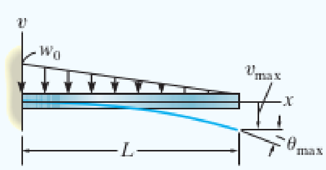
1. Determine the expressions as tabulated in the second, third, and fourth columns in the following:

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L+b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0 L}{6EI}$ $\theta_2 = \dfrac{M_0 L}{3EI}$	$v_{max} = \dfrac{-M_0 L^2}{9\sqrt{3}\,EI}$ at $x = 0.5774L$	$v = \dfrac{-M_0 x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0 L^3}{360EI}$ $\theta_2 = \dfrac{w_0 L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0 L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0 x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

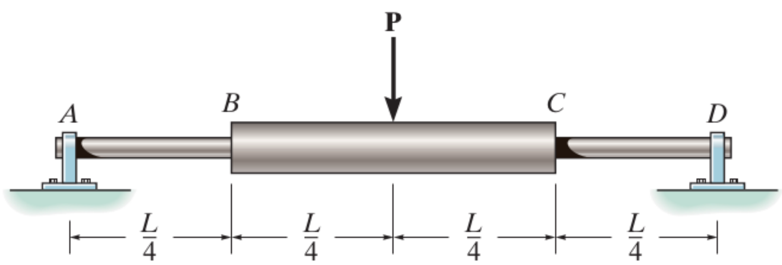
2. Determine the expressions as tabulated in the second, third, and fourth columns in the following:

Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0 L}{EI}$	$v_{max} = \dfrac{M_0 L^2}{2EI}$	$v = \dfrac{M_0 x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right) \quad 0 \le x \le L/2$ $v = \dfrac{-wL^3}{384EI}(8x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0 L^3}{24EI}$	$v_{max} = \dfrac{-w_0 L^4}{30EI}$	$v = \dfrac{-w_0 x^2}{120EIL}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$

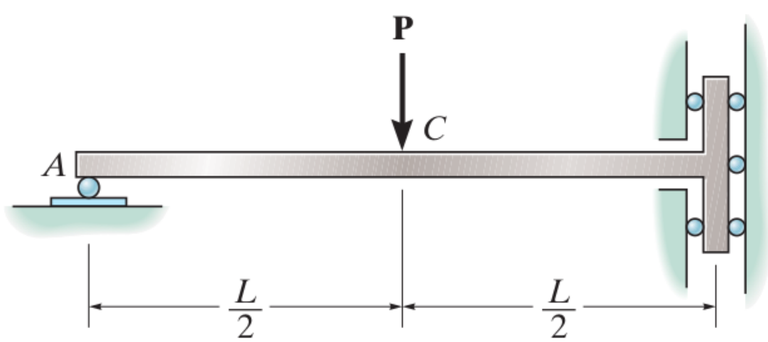
3. The simply supported shaft has a moment of inertia (or, second moment of area) of $2I$ for the region BC and a moment of inertia of I for the regions AB and CD. Determine the magnitude of the maximum deflection of the shaft and the location of this maximum deflection.

[$\dfrac{3PL^3}{256EI}$ at $L/2$.]



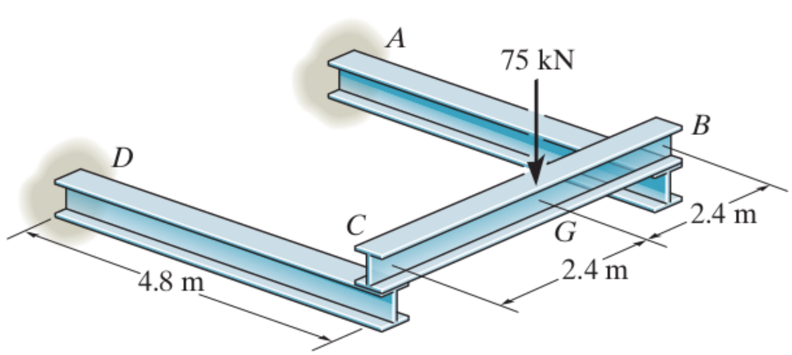
4. The bar is supported by a roller constraint at B which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A, and the deflections at B and C.

[$-\dfrac{3PL^2}{8EI}, -\dfrac{11PL^3}{48EI}, -\dfrac{PL^3}{6EI}$]

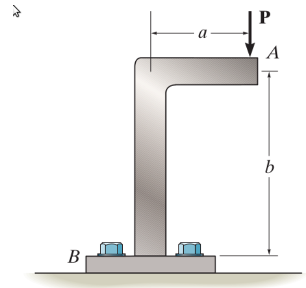


5. The framework consists of two steel cantilevered beams CD and BA and a simply supported beam CB. If each beam has a Young's modulus of 200 GPa and a moment of inertia about its neutral axis of 46×10^6 mm^4, determine the deflection at the centre G of beam CB.

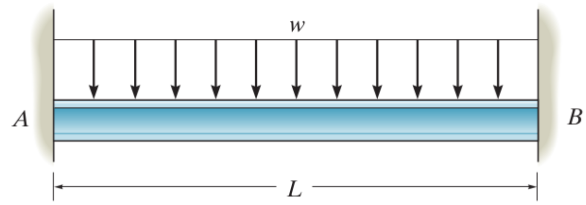
[-169 mm]



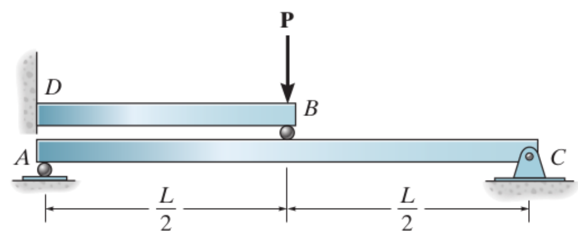
6. Determine the vertical deflection at the end A of the bracket. Assume that the bracket is fixed supported at its base B and neglect axial deflection. $[-\dfrac{Pa^2(3b+a)}{3EI}]$



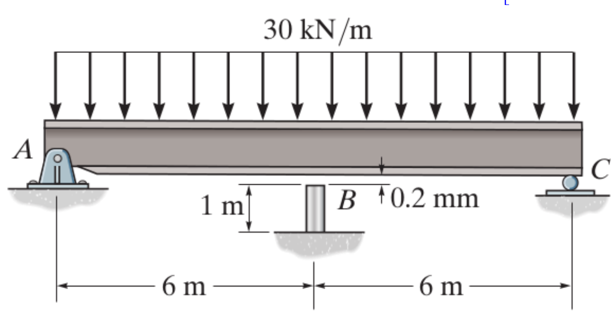
7. Determine the moment reactions at the supports A and B of the fixed-fixed beam. $[\dfrac{wL^2}{12}]$



8. Determine the vertical reaction at support C in the beam arrangement shown. $[P/3]$



9. Before the uniformly distributed load is applied on the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is 875×10^6 mm^4. Both the post and the beam are made of steel having modulus of elasticity 200 GPa. [70.11 kN, 219.78 kN, 70.11 kN]



10. Determine the force in the spring. $[\dfrac{3kwL^4}{24EI+8kL^3}]$