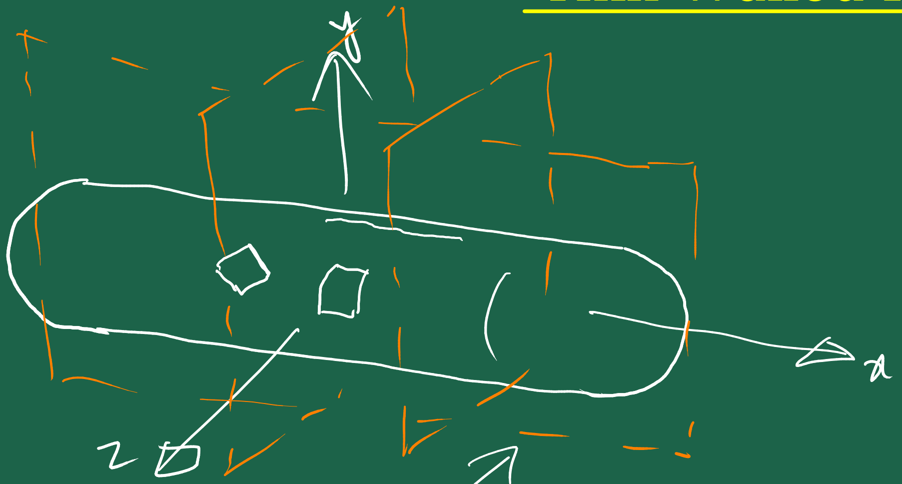
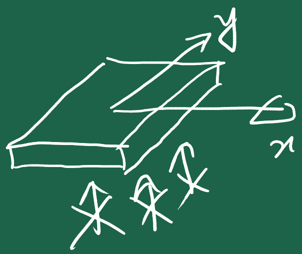
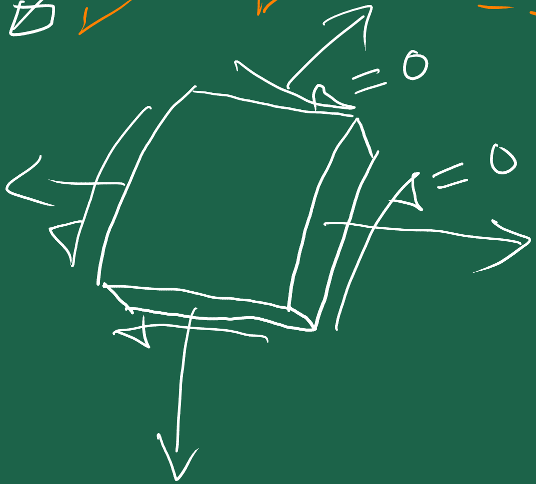
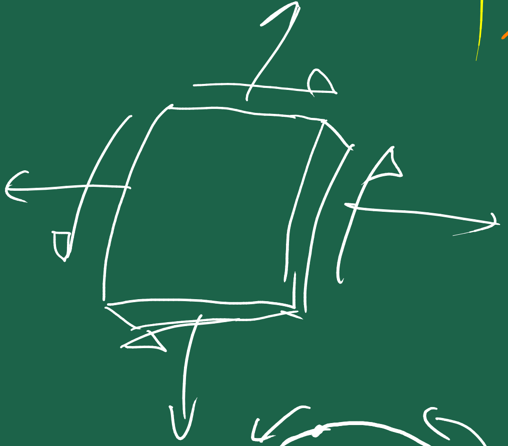
Thin-Walled Pressure Vessels

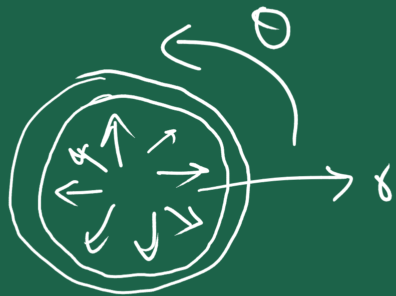




PLANE

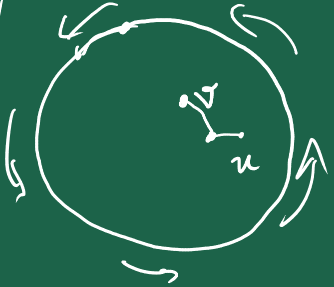






u : displ in r-dir^n

v : displ in θ-dir^n



$v \neq 0$

Axisymmetry

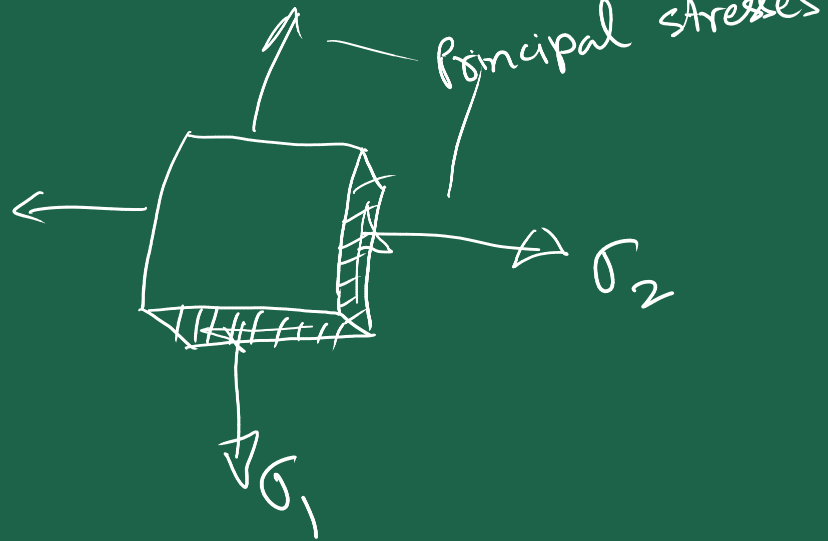
→ No variation in θ-dir^n

$\Rightarrow \frac{\partial}{\partial \theta}(\;) = 0$

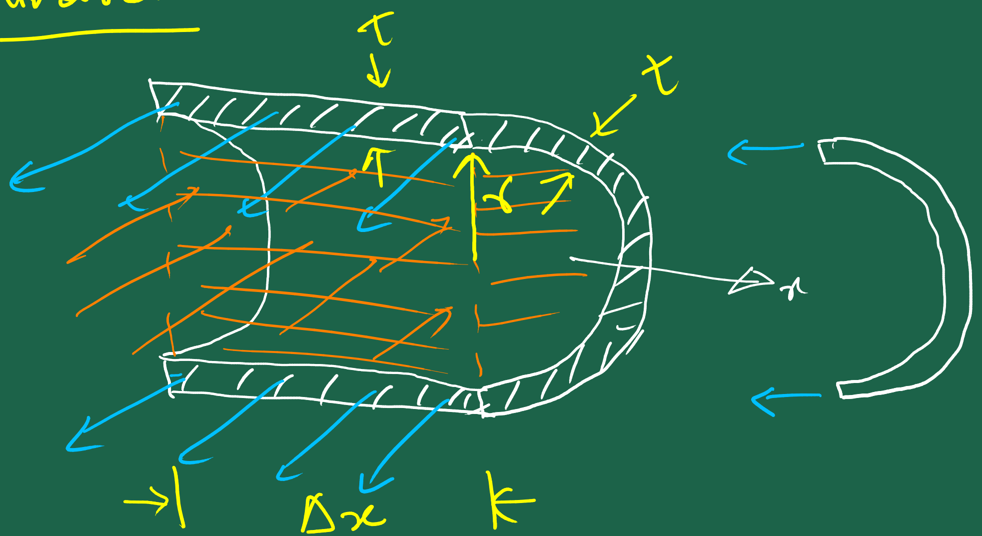
No loading in θ-dir^n $\Rightarrow v = 0$

$$\underbrace{\frac{\partial}{\partial \theta}(\ ) = 0 \qquad + \qquad v = 0}$$

$\hookrightarrow \varepsilon_{r\theta} = 0 \Rightarrow \tau_{r\theta} = 2G\,\varepsilon_{r\theta} = 0$



Cylindrical

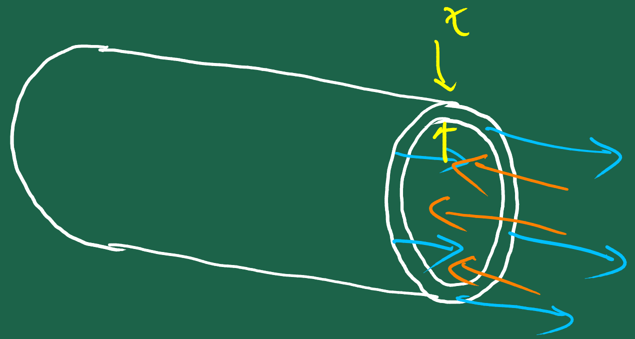


$\tau_{z\theta} = 0$

$\tau_{\theta z} = 0$

$2 \times (\sigma_1 t \Delta x) = p\, 2r\, \Delta x$

$\Rightarrow \boxed{\sigma_1 = \frac{pr}{t}}$ Circumferential OR Hoop Stress

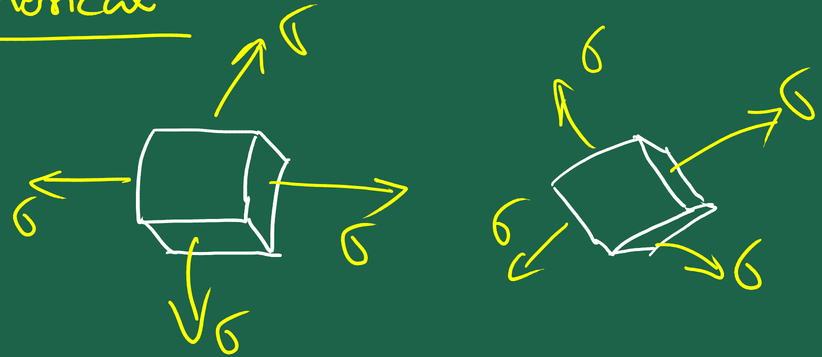


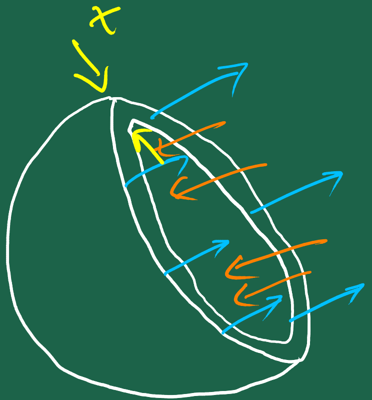
$$\sigma_2 \, 2\pi r t = p \pi r^2$$

$$\Rightarrow \boxed{\sigma_2 = \frac{pr}{2t}} \rightarrow$$ Longitudinal stress

$$\sigma_1 = 2\sigma_2$$

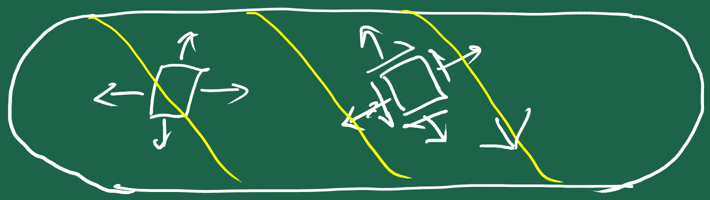
Spherical

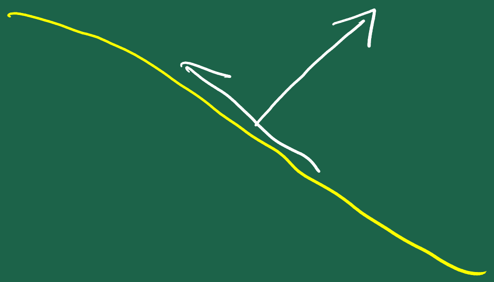




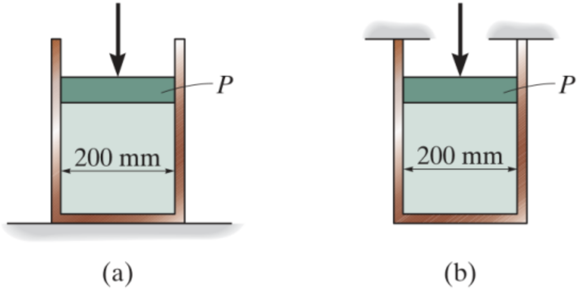
$$\sigma \, 2\pi r t = p \pi r^2$$

$$\Rightarrow \boxed{\sigma = \frac{pr}{2t}}$$





1. The thin-walled cylinder can be supported in one of two ways shown. If the piston causes the internal pressure to be 0.6 MPa, determine the state of stress in the wall of the cylinder for both cases. The inner diameter of the cylinder is 250 mm and the wall has a thickness of 6 mm. [(a) 12.5 MPa, 0 MPa (b) 12.5 MPa, 6.25 MPa]



$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\boxed{r \gg t}$$

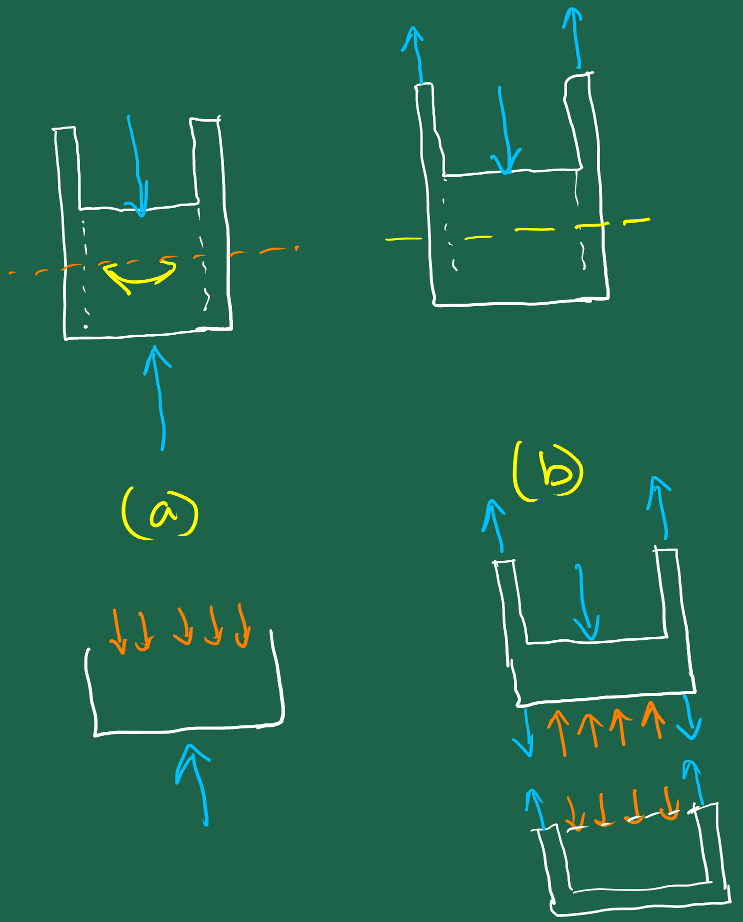
$$r > 10t$$

(a) $\sigma_1 = \frac{pr}{t}$ ✓
(hoop)

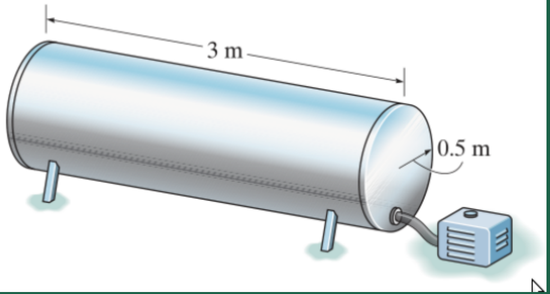
$\sigma_2 = 0$
(longitudinal)

(b) $\sigma_1 = \frac{pr}{t}$ ✓

$\sigma_2 = \frac{pr}{2t}$ ✓



5. The thin-walled cylindrical pressure vessel shown in the figure is subjected to an internal gauge pressure of 15 MPa. If the thickness is 10 mm, and the material constants are $E = 200$ GPa and $\nu = 0.3$, determine the increase in both the diameter and the length of the pressure vessel. [3.19 mm, 2.25 mm]



$\sigma_{rr} = p \ll p\frac{r}{t}$ $\because \frac{r}{t} \gg 1$

$\varepsilon = \frac{\Delta d}{d}$ ✗

$\sigma_1 = \frac{pr}{t}$ ✓, $\sigma_2 = \frac{pr}{2t}$ ✓, $\sigma_{rr} \approx 0$

✓ $\varepsilon_{\theta\theta} = \frac{1}{E}\left[\underbrace{\sigma_{\theta\theta}}_{\sigma_1} - \nu\left(\underbrace{\sigma_{rr}}_{\approx 0} + \underbrace{\sigma_{zz}}_{\sigma_2}\right)\right]$

$\varepsilon_{\theta\theta} = \frac{2\pi(r+\Delta r) - 2\pi r}{2\pi r} = \frac{2\Delta r}{2r} = \frac{\Delta d}{d} \Rightarrow \Delta d = \varepsilon_{\theta\theta}\, d$

✓ $\varepsilon_{zz} = \frac{1}{E}\left[\underbrace{\sigma_{zz}}_{\sigma_2} - \nu\left(\underbrace{\sigma_{rr}}_{\approx 0} + \underbrace{\sigma_{\theta\theta}}_{\sigma_1}\right)\right]$; $\varepsilon_{zz} = \frac{(L+\Delta L) - L}{L} = \frac{\Delta L}{L} \Rightarrow \Delta L = \varepsilon_{zz} L$

$\frac{r}{t} = \frac{0.5}{0.01} = 50$

4. A thin walled spherical pressure having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = \dfrac{2p\pi r^4}{Et}(1-\nu)$. Use a small-strain analysis.

θ ϕ

$$\sigma = \frac{pr}{2t}$$

$$\frac{pr}{t}$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left[\underset{\sigma}{\sigma_{\theta\theta}} - \nu(\underset{\simeq 0}{\sigma_{rr}} + \underset{\sigma}{\sigma_{\phi\phi}})\right]$$

$$= \frac{\sigma(1-\nu)}{E}$$

$$\varepsilon_{\theta\theta} = \frac{2\pi(r+\Delta r) - 2\pi r}{2\pi r}$$

$$= \frac{\Delta r}{r}$$

$$\therefore \frac{\Delta r}{r} = \frac{\sigma(1-\nu)}{E}$$

$$V = \frac{4}{3}\pi r^3$$

$$V + \Delta V = \frac{4}{3}\pi(r+\Delta r)^3 = \frac{4}{3}\pi r^3\left(1+\frac{\Delta r}{r}\right)^3$$

$$\frac{\Delta r}{r} \ll 1$$

$$\therefore V + \Delta V \simeq \frac{4}{3}\pi r^3\left(1 + 3\frac{\Delta r}{r}\right)$$

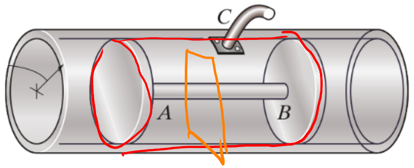
$$\therefore \Delta V = \frac{4}{3}\pi r^3\left(3\frac{\Delta r}{r}\right)$$

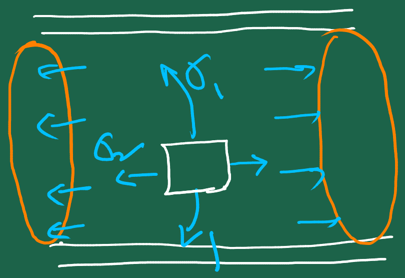
6. Air is pumped into the steel thin-walled presure vessel at C. The inner radius of the pressure vessel is 400 mm, and its thickness is 10 mm. For steel: $E = 200$ GPa and $\nu = 0.3$.

   (a) If the ends of the vessel are closed using two pistons connected by a rod AB, determine the increase in the diameter of the pressure vessel when the internal gauge pressure is 5 MPa. Also, what is the tensile stress in rod AB if it has a diameter of 100 mm?

   (b) If the pistons in part (a) are replaced by walls connected to the ends of the vessel, determine the increase in the diameter of the pressure vessel.

[(a) 0.800 mm, 315 MPa,
(b) 0.680 mm]





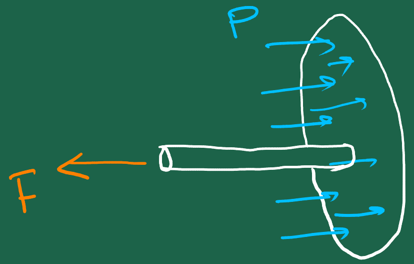
$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = 0$$

(a) $\sigma_1 = \frac{pr}{t}$, $\sigma_2 = 0$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\Big[\underbrace{\sigma_{\theta\theta}}_{\sigma_1} - \nu\big(\underbrace{\sigma_{rr}}_{\simeq 0} + \underbrace{\sigma_{zz}}_{\sigma_2 = 0}\big)\Big]$$

$$\varepsilon_{\theta\theta} = \frac{2\pi(r+\Delta r) - 2\pi r}{2\pi r} = \frac{\Delta r}{r} = \frac{\Delta d}{d}$$

$$\Delta d = d\,\varepsilon_{\theta\theta}$$

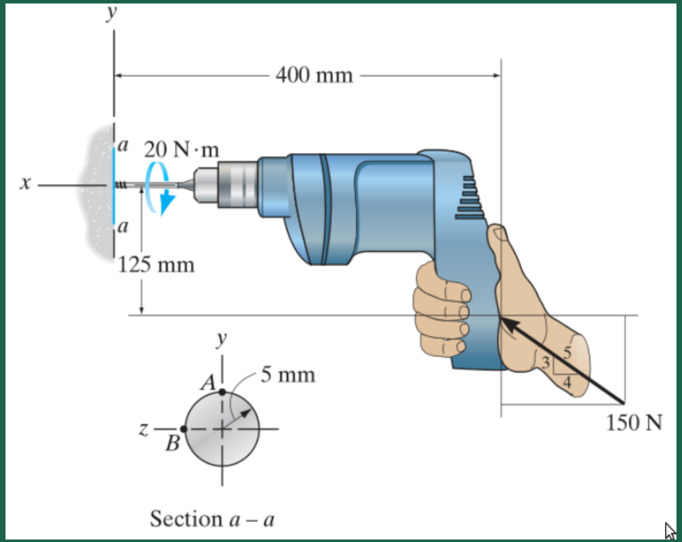


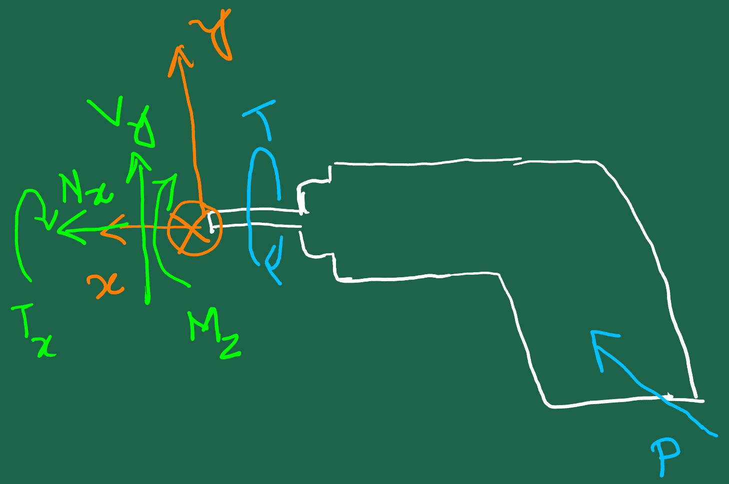
$$F = p\left(\pi r^2 - \pi r_{rod}^2\right)$$

$$\sigma_T = \frac{F}{\pi r_{rod}^2}$$

15. The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at points A and B on the cross section of drill bit at section *a*-*a*.

$[\sigma_A = 215 \text{ MPa (C)},$
$(\tau_{xy})_A = 0,\ (\tau_{xz})_A = 102 \text{ MPa},$
$\sigma_B = 1.53 \text{ MPa (C)},$
$(\tau_{xy})_B = \tau_{\text{torsion}} - \tau_{\text{transverse}} = 100.33 \text{ MPa}\ ]$





$$\sum F_x = 0 \Rightarrow N_x + 150\left(\frac{4}{5}\right) N = 0$$

$$\Rightarrow N_x = -120 \text{ N}$$

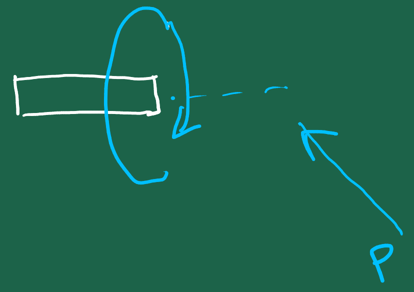
$$\sum F_y = 0 \Rightarrow V_y + 150\left(\frac{3}{5}\right) N = 0$$

$$\Rightarrow V_y = -90 \text{ N}$$

$$\sum M'_x = 0$$

$$\Rightarrow T_x + 20 \text{ Nm} = 0$$

$$\Rightarrow T_x = -20 \text{ Nm}$$



$$\sum M'_z = 0 \Rightarrow M_z + 150\left(\frac{4}{5}\right) N (0.125 \text{ m}) - 150 N\left(\frac{3}{5}\right) N (0.400 \text{ m}) = 0$$

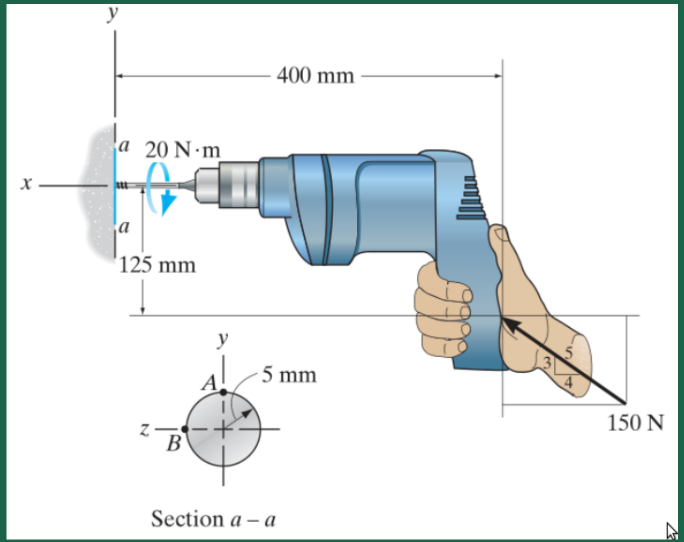
$$\Rightarrow M_z = 21 \text{ Nm}$$

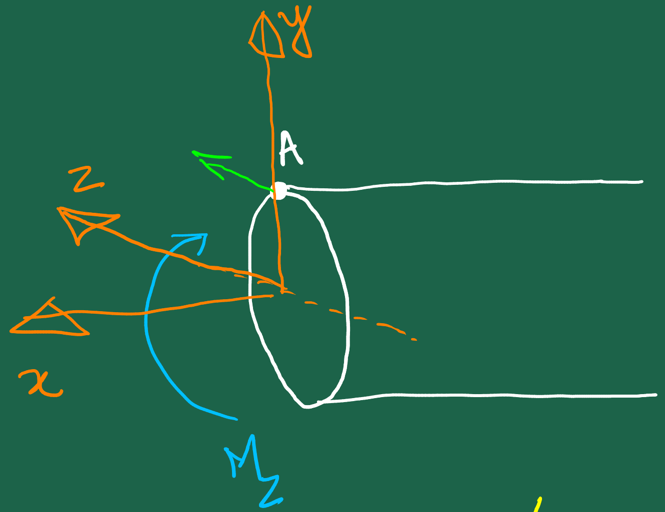
A

$$\sigma_A = -\frac{|N_x|}{A} - \frac{|M_z|r}{I} \quad (C)$$

$$\tau_{xy}\Big|_A = \left(\frac{V_y Q_z}{I_z t}\right) = 0$$

$$\tau_{xz}\Big|_A = -\frac{|T_x|r}{J}$$





$$I = \frac{\pi}{4}r^4$$

$$J = \frac{\pi}{2}r^4$$

B

$$\sigma_B = -\frac{|N_x|}{A} \qquad (c)$$

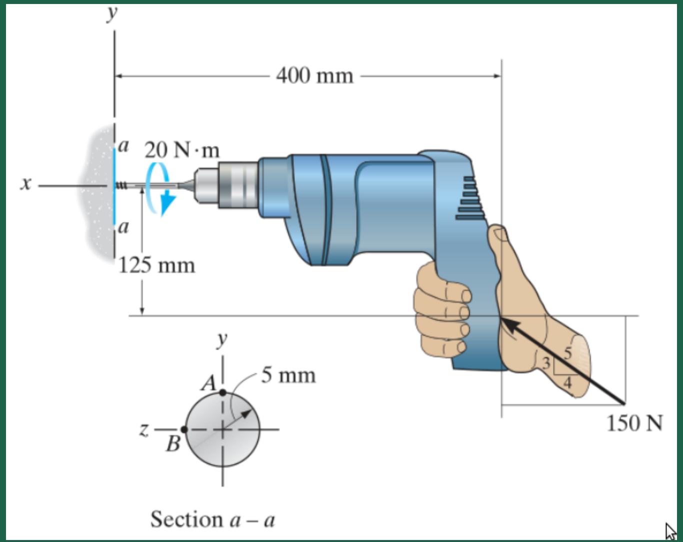
$$\tau_{xy}\Big|_B = \frac{|T_x|\, r}{J} - \frac{|V_y| Q_z}{I_z\, t}$$

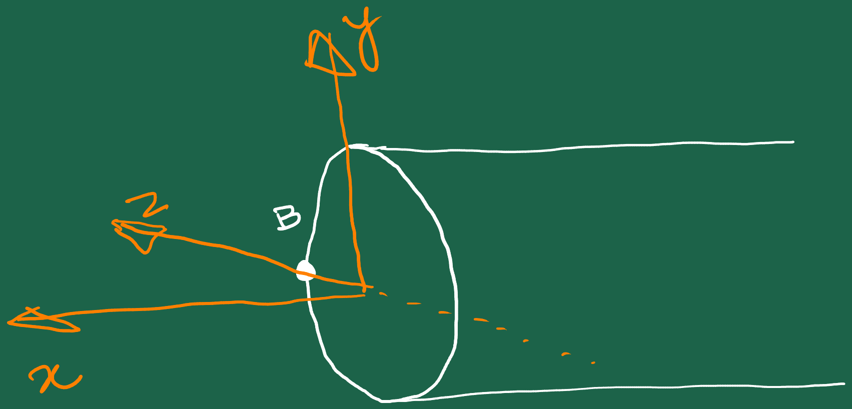
$$Q_z = \left(\frac{4r}{3\pi}\right)\left(\frac{1}{2}\pi r^2\right)$$

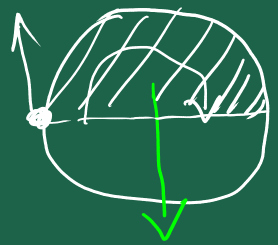
$$I_z = \frac{\pi}{4} r^4$$

$$t = 2r$$

$$\tau_{xz}\Big|_B = 0$$

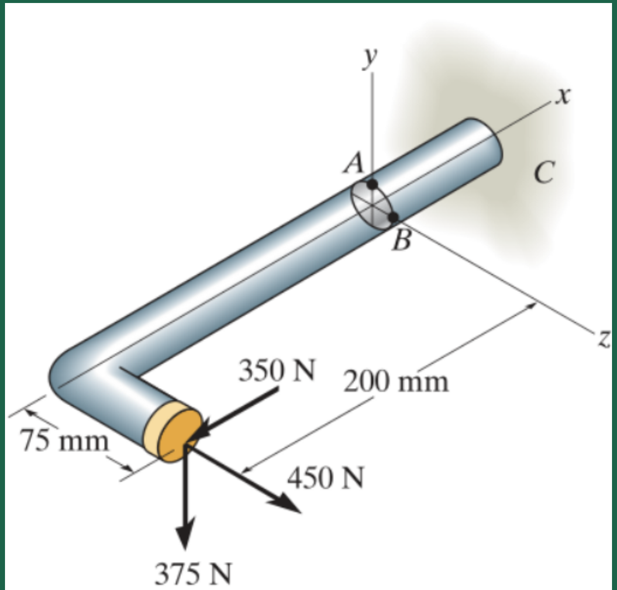


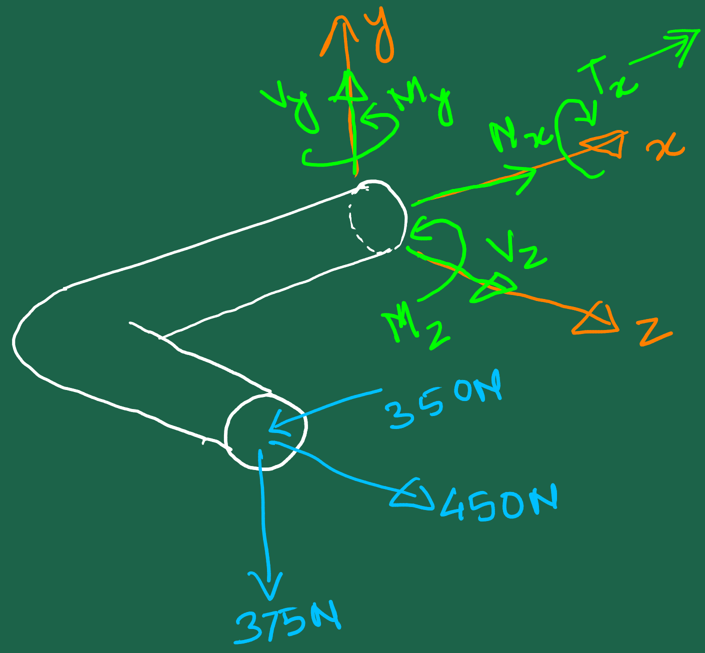




16. The 25 mm diameter rod is subjected to the loads shown. Determine the state of stress at points A and B.

[$\sigma_A = 49.61$ MPa (C), $\tau_A = 10.39$ MPa,
$\sigma_B = 40.84$ MPa (C), $\tau_B = 9.58$ MPa]





$\Sigma F_y = 0 \Rightarrow V_y - 375\text{N} = 0 \Rightarrow \boxed{V_y = 375\text{N}}$

$\Sigma M'_z = 0 \Rightarrow M_z + 375\text{N} \times 0.200\text{m} \Rightarrow \boxed{M_z = -75\text{N m}}$

$\Sigma M'_y = 0 \Rightarrow M_y + 450\text{N} \times 0.200\text{m} - 350\text{N} \times 0.075\text{m} = 0$

$\Rightarrow \boxed{M_y = -63.750\text{ N m}}$

$\Sigma M'_x = 0 \Rightarrow T_x + 375\text{N} \times 0.075\text{ m} \Rightarrow \boxed{T_x = -28.125\text{ Nm}}$

$\Sigma F_z = 0 \Rightarrow V_z + 450\text{N} = 0 \Rightarrow \boxed{V_z = -450\text{ N}}$

$\Sigma F_x = 0 \Rightarrow N_x - 350\text{N} = 0 \Rightarrow \boxed{N_x = 350\text{N}}$

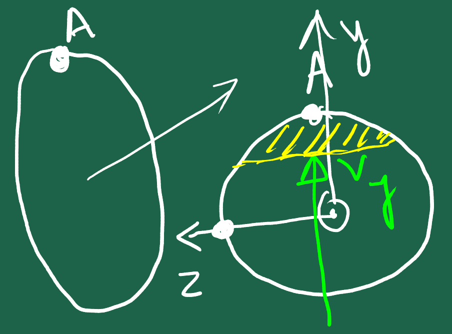
A

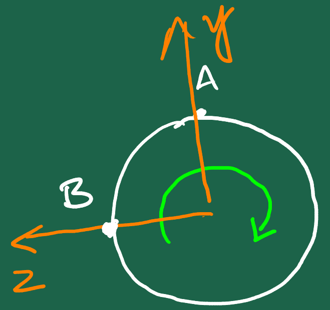
$$\sigma_A = \frac{|N_x|}{A} + \frac{|M_z|\, r}{I_z}$$

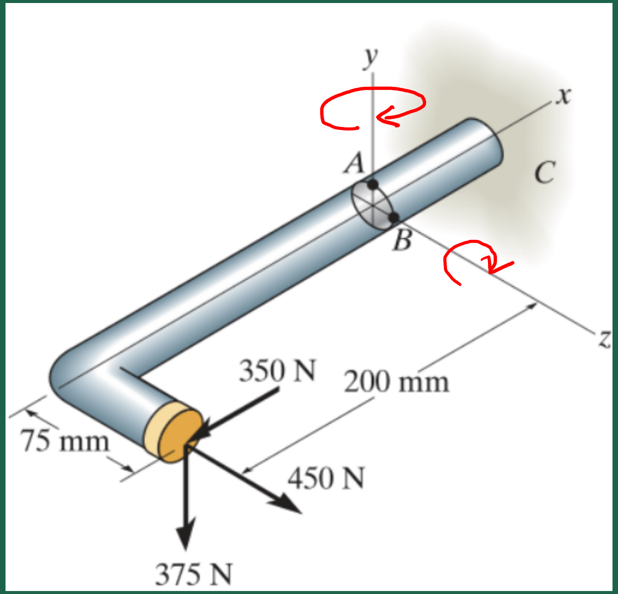
$$\tau_{xy}\Big|_A = \cancel{\text{Torsion}}^{\,0} + \cancel{\text{Transverse load}}^{\,0}$$

$$\tau_{xz}\Big|_A = \text{Torsion} + \text{Transverse load}$$

$$= -\frac{|T_x|\, r}{J} - \frac{|V_z|\, Q}{I\, t}$$







$$I_z = \frac{\pi}{4}\pi r^4$$

$$I_y = I_z$$

$$J = \frac{\pi}{2} r^4$$

$$Q = \frac{4r}{3\pi} \times \frac{1}{2}\pi r^2 = \frac{2}{3} r^3$$

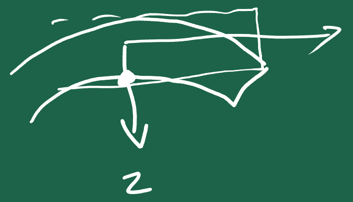
$$A = \pi r^2$$

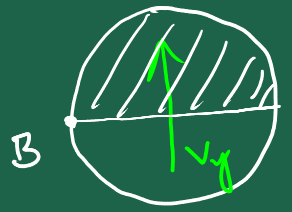
B

$$\sigma_B = \frac{|N_x|}{A} - \frac{|M_y|\, r}{I_y}$$

$$\tau_{xy}\Big|_B = \text{Torsion} + \text{Transverse loading}$$

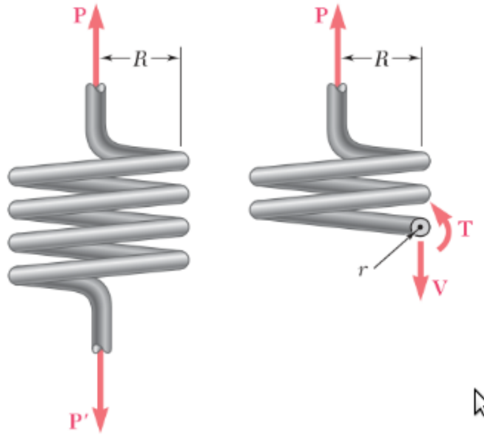
$$= \frac{|T_x|\, r}{J} + \frac{|V_y|\, Q}{I_z\, t}, \quad t \to 2r$$

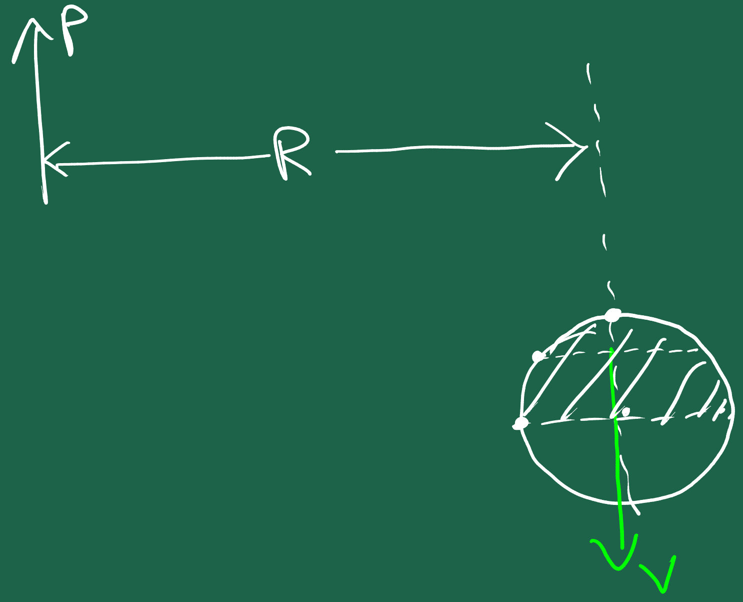




$$\tau_{xz}\Big|_B = 0 + 0 = 0$$

19. A close-coiled spring is made of a circular wire of radius r tht is formed into a helix of radius R. Determine the maximum shearing stress produced by the two equal and opposite forces P and P'. (First determine the shear and the torque in a transverse cross section of the wire.) $\left[\tau_{\max} = \frac{P}{\pi r^3}\left(2R + \frac{4r}{3}\right)\right]$





$$\boxed{V = P}$$

$$\boxed{T = PR}$$

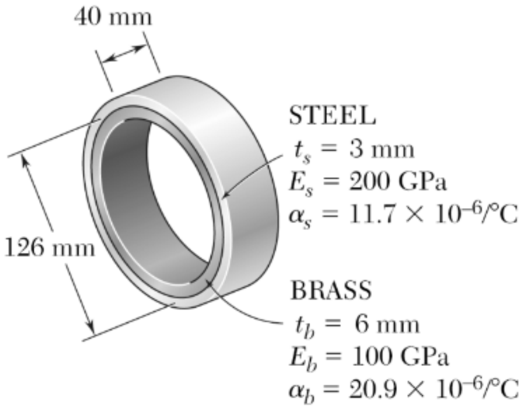
$$\tau_V = \frac{V \cdot Q}{It}$$

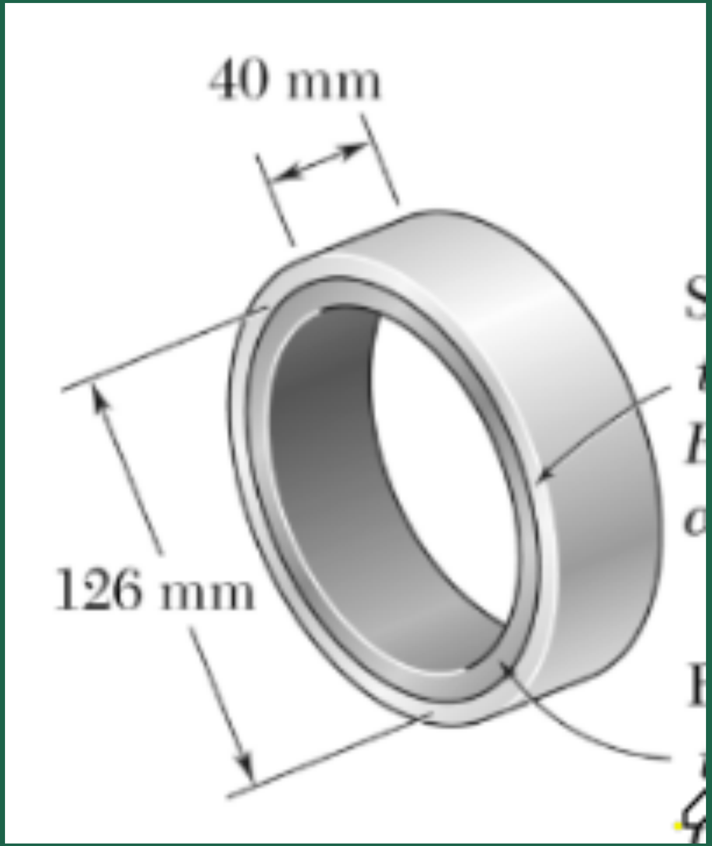
$$\tau_T = \frac{Tr}{J}$$

$$\tau_{max} = \tau_V + \tau_T$$

$$\frac{VQ}{It}$$

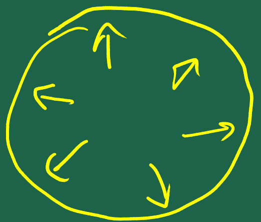
10. A brass ring of 126 mm outer diameter and 6 mm thickness fits exacts inside a steel ring of 126 mm inner diameter and 3 mm thickness when the temperature of boths rings is 10°. Knowing that the temperature of both rings is then raised to 52°, determine the pressure exerted by the brass ring on the steel ring and the tensile stress in the steel ring. Note that the strain due to temperature change is given by $\varepsilon_T = \alpha \Delta T$ and the strains due to pressure and temperature change add up to give the total strain. [1.67 MPa, 35.1 MPa]





steel

$$\sigma_{s,h} = \frac{p r_s}{t_s} \quad , \quad \sigma_{s,l} = 0$$



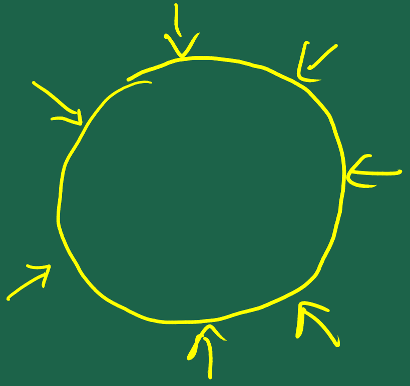
steel

$$\varepsilon_s = \frac{1}{E_s}\left[\sigma_{s,h} - \nu\left(\underbrace{\cancel{\sigma_{s,l}}}_{0} + \underbrace{\cancel{\sigma_{s,r}}}_{\approx 0}\right)\right] + \varepsilon_T$$

$$= \frac{p r_s}{E_s t_s} + \alpha_s \Delta T$$

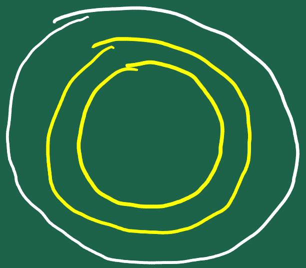
$$\sigma_{B,h} = \frac{-pr}{t}, \quad \sigma_{B,\ell} = 0$$

$$\varepsilon_B = \frac{1}{E_B}\left[\sigma_{B,h} - \nu\left(\underbrace{\cancel{\sigma_{B,\ell}}}_{=0} + \underbrace{\cancel{\sigma_{B,r}}}_{\simeq 0}\right)\right] + \varepsilon_T$$



Brass

$$= \frac{-p r_B}{E_B t_B} + \alpha_B \Delta T$$



$$(\Delta L_{circ})_s = (\Delta L_{circ})_B$$

$$\Rightarrow \quad 2\pi r_s \varepsilon_s = 2\pi r_B \varepsilon_B \rightarrow \text{Eqn in the unknown } p \rightarrow p \checkmark$$

2. A boiler is constructed of 8 mm thick steel plates that are fastened together at their ends using a butt joint consisting of two 8 mm thick cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the hoop stress in the boiler plate apart from the seam, (b) the hoop stress in the outer cover plate along the rivet line *a*-*a*, and (c) the shear stress in the rivets. [(a) 127 MPa, (b) 79.1 MPa, (c) 322 MPa]