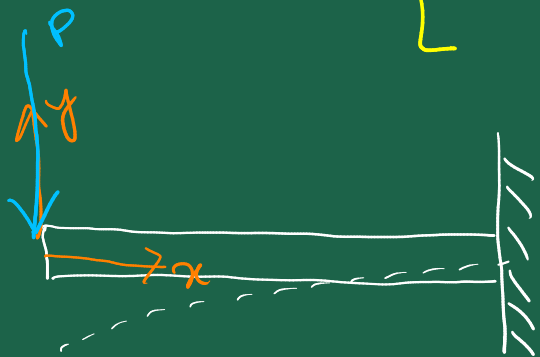


Deflection of Beams

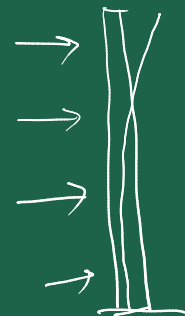
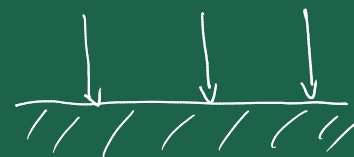
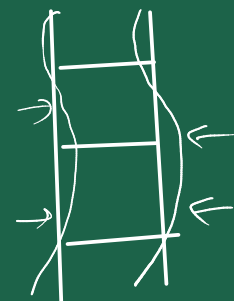
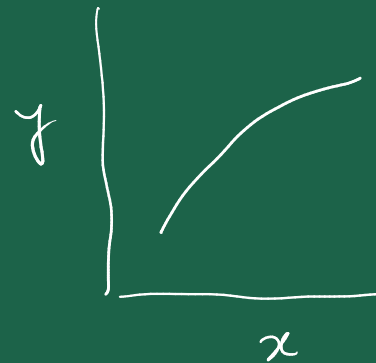
$$M = \frac{EI}{\rho}$$

$$\frac{1}{\rho} = \kappa = \frac{\frac{d^2 y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$



$$\left| \frac{dy}{dx} \right| \ll 1$$

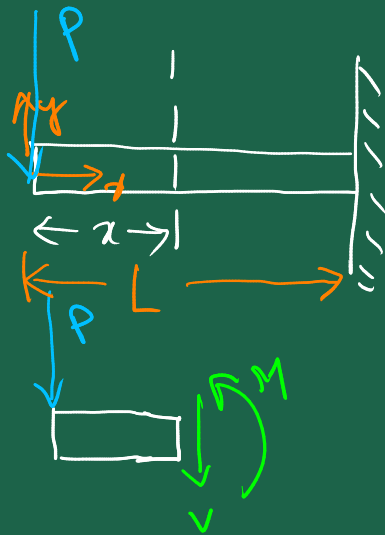
$$\frac{1}{\rho} = \kappa \approx \frac{d^2 y}{dx^2}$$



St. Venant's principle

$$M = \frac{EI}{\rho}$$

$$\Rightarrow EI \frac{d^2 y}{dx^2} = M$$

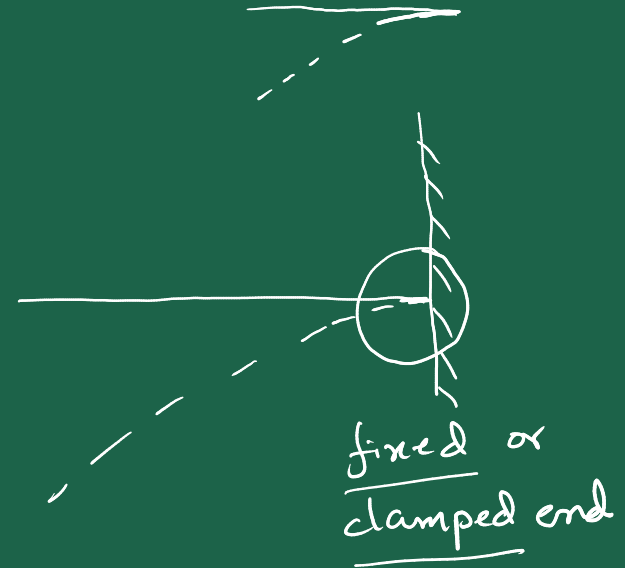


$$M = -Px$$

$$\therefore EI \frac{d^2 y}{dx^2} = -Px$$

$$\Rightarrow EI \frac{dy}{dx} = -P \frac{x^2}{2} + C_1$$

$$\Rightarrow EI y = -P \frac{x^3}{6} + C_1 x + C_2$$



fixed or clamped end

EI : flexural rigidity

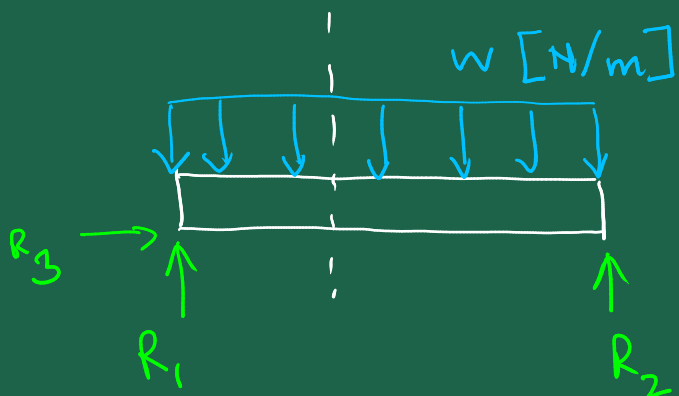
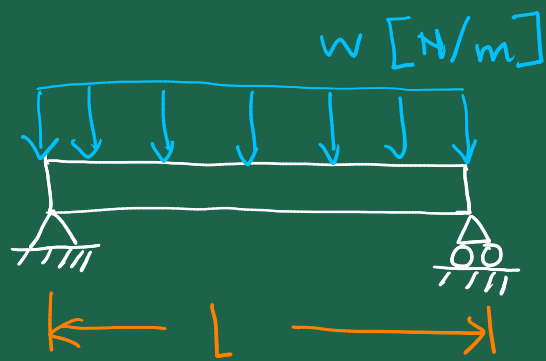
B.C.S

$$\text{At } x=L, y=0$$

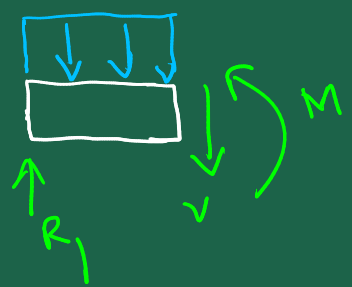
$$\text{At } x=L, \frac{dy}{dx} = 0$$

$$\text{After solving } y|_{x=0} = \frac{-PL^3}{3EI}, \quad \left. \frac{dy}{dx} \right|_{x=0} = ??$$

ex 2



$$R_1 = R_2 = \frac{wL}{2}, \quad R_3 = 0$$



$$EI \frac{d^2 y}{dx^2} = M \leftarrow$$

$$\Rightarrow EI \frac{d^3 y}{dx^3} = \frac{dM}{dx} = V$$

$$\Rightarrow EI \frac{d^4 y}{dx^4} = \frac{dV}{dx} = -w$$

$$EI \frac{d^4 y}{dx^4} = -w$$

$$y = \frac{w}{24EI} \times (-x^4 + 2Lx^3 - L^3x)$$

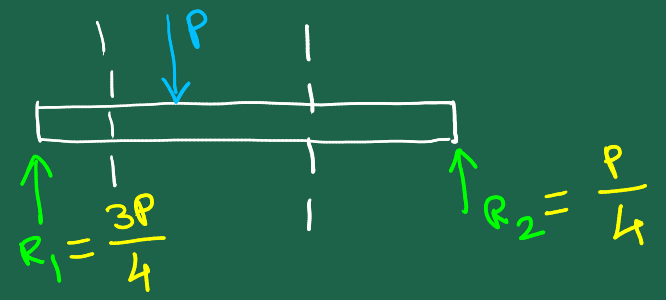
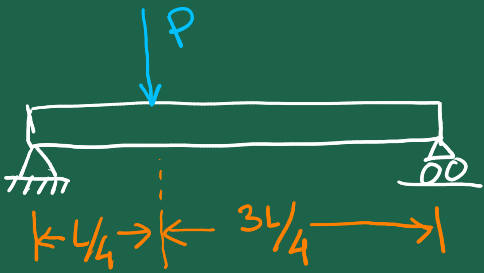
$$\text{At } x=0, \quad y=0$$

$$\text{At } x=L, \quad y=0$$

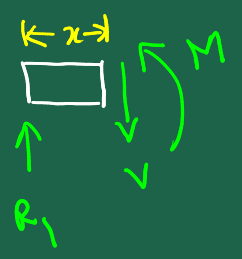
$$\text{At } x=0, \quad M=0 \Rightarrow EI \frac{d^2 y}{dx^2} \Big|_{x=0} = 0$$

$$\text{At } x=L, \quad M=0 \Rightarrow EI \frac{d^2 y}{dx^2} \Big|_{x=L} = 0$$

Ex 3



Für $0 < x < L/4$

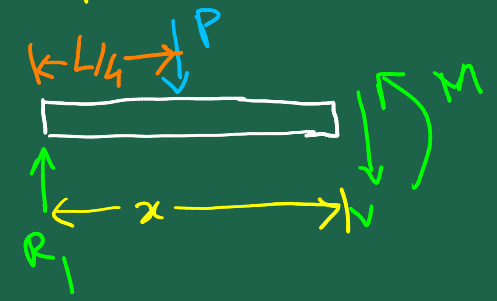


$$M - R_1 x = 0 \Rightarrow M = \frac{3P}{4} x$$

$$EI \frac{d^2 y}{dx^2} = \frac{3Px}{4}$$

→ C_1, C_2

Für $\frac{L}{4} < x < L$



$$M - R_1 x + P(x - L/4) = 0$$

$$\Rightarrow M = -\frac{P}{4}(x - L)$$

$$EI \frac{d^2 y}{dx^2} = -\frac{P}{4}(x - L)$$

→ C_3, C_4

To find C_1, C_2, C_3, C_4 :

$$@ x=0, y=0$$

$$@ x=L, y=0$$

$\left. \begin{array}{l} @ x=0, M=0 \\ @ x=L, M=0 \end{array} \right\}$ They do not help us in solving for C_1, C_2, C_3, C_4

Continuity of deflection and continuity of slope must be maintained between the 2 regions

$$y \Big|_{x=L/4^-} = y \Big|_{x=L/4^+}$$

$$\frac{dy}{dx} \Big|_{x=L/4^-} = \frac{dy}{dx} \Big|_{x=L/4^+}$$

Extremely important!

$$\frac{d^2 f}{dx^2} = g(x)$$

$$\frac{d^n f}{dx^n}$$

$$B.C.s \leq (n-1)$$

$$\frac{df}{dx} = 0$$

$$f = C_1 + C_2 x$$

$$x=0, f = \infty$$

$$\frac{df}{dx} \Big|_{x=L} = 0 \quad \checkmark$$

$$\frac{df}{dx} \Big|_{x=L} = \checkmark$$

Various kinds of BCs:

Clamped or Fixed End



$$y = 0$$
$$\frac{dy}{dx} = 0$$



Pinned End



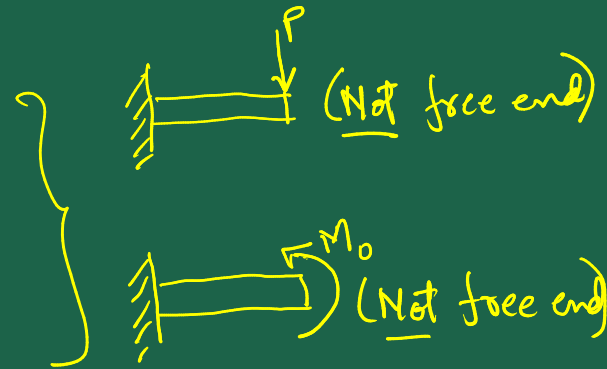
$$y = 0$$
$$M = 0 \Rightarrow EI \frac{d^2 y}{dx^2} = 0$$

Free End

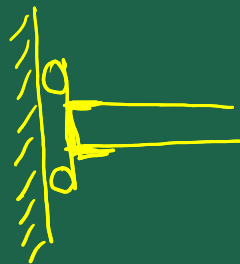


$$M = 0 \Rightarrow EI \frac{d^2 y}{dx^2} = 0$$

$$V = 0 \Rightarrow EI \frac{d^3 y}{dx^3} = 0$$



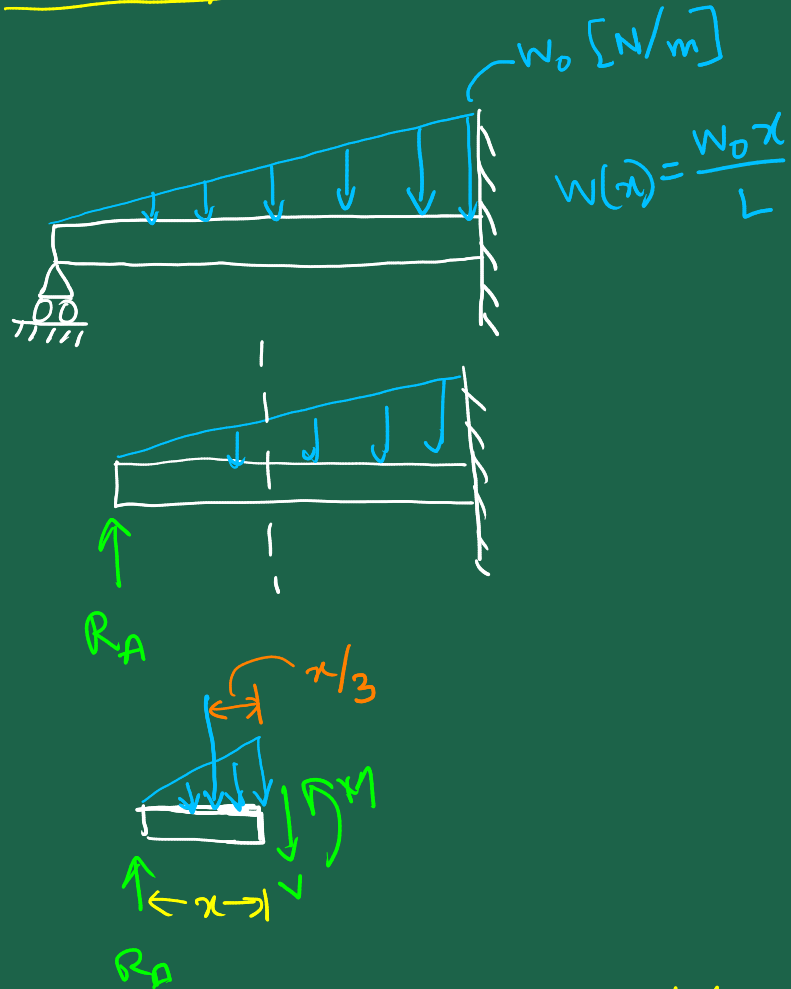
Guided support



$$V = 0 \Rightarrow EI \frac{d^3 y}{dx^3} = 0$$

$$\frac{dy}{dx} = 0$$

Statically Indeterminate Cases



$$M - R_A x + \frac{x}{3} \times \frac{w_0 x}{2} = 0$$

$$\Rightarrow M = R_A x - \frac{w_0 x^2}{6}$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x - \frac{w_0 x^2}{6}$$

$$y \rightarrow C_1, C_2 \text{ \& } R_A$$

$$\text{@ } x=0, y=0, \text{ (} M \neq 0 \text{)}$$

$$\text{@ } x=L, y=0$$

$$\text{@ } x=L, \frac{dy}{dx} = 0$$

$$EI \frac{d^4 y}{dx^4} = -w(x) = -\frac{w_0 x}{L}$$

$$EI \frac{d^4 y}{dx^4} = - \frac{w_0 x}{L}$$

$$\Rightarrow EI \frac{d^3 y}{dx^3} = - \frac{w_0 x^2}{2L} + C_1$$

$$\Rightarrow EI \frac{d^2 y}{dx^2} = - \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$$\Rightarrow EI \frac{dy}{dx} = - \frac{w_0 x^4}{24L} + C_1 \frac{x^2}{2} + C_2 x + C_3$$

$$\Rightarrow EI y = - \frac{w_0 x^5}{120L} + C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

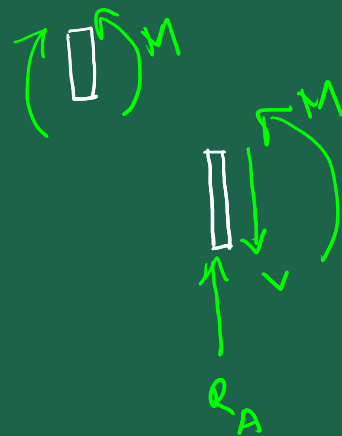
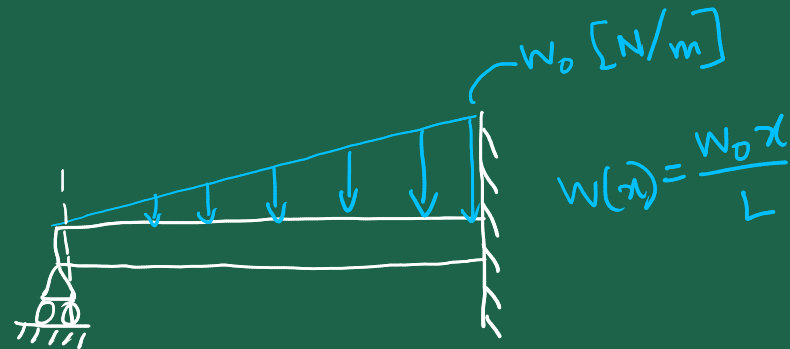
$$\textcircled{a} x=0, y=0$$

$$\textcircled{a} x=0, M=0 \Rightarrow EI \frac{d^2 y}{dx^2} = 0$$

$$\textcircled{a} x=L, y=0$$

$$\textcircled{a} x=L, \frac{dy}{dx} = 0$$

C_1, C_2, C_3, C_4 ✓

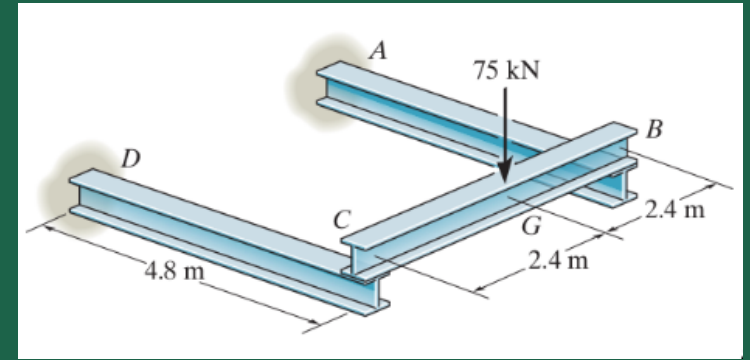


$$\sum F_y = 0$$

$$\Rightarrow V = R_A$$

$$\Rightarrow R_A = EI \left. \frac{d^2 y}{dx^2} \right|_{x=0}$$

5. The framework consists of two steel cantilevered beams CD and BA and a simply supported beam CB. If each beam has a Young's modulus of 200 GPa and a moment of inertia about its neutral axis of $46 \times 10^6 \text{ mm}^4$, determine the deflection at the centre G of beam CB. [-169 mm]

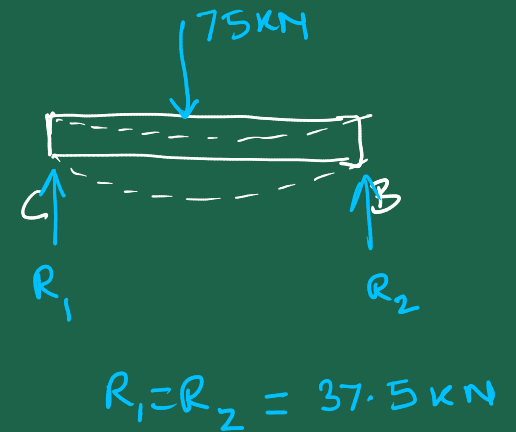


Method of superposition

$$\Delta'_C = \frac{PL^3}{3EI} = \frac{R_1 (4.8\text{m})^3}{3EI} \quad \checkmark$$

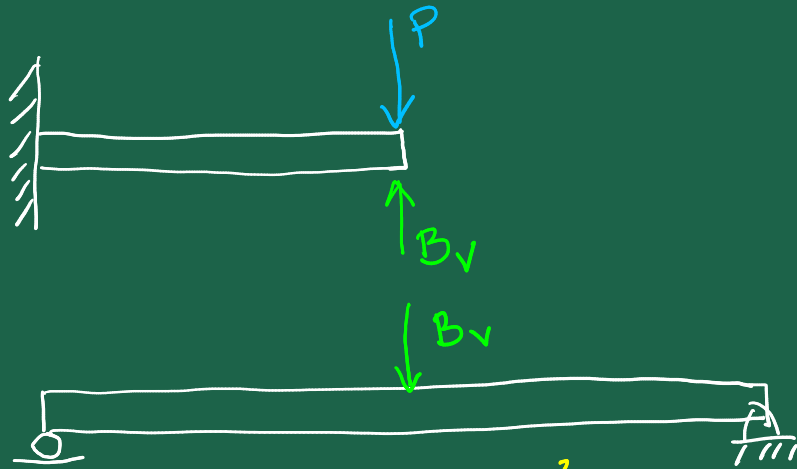
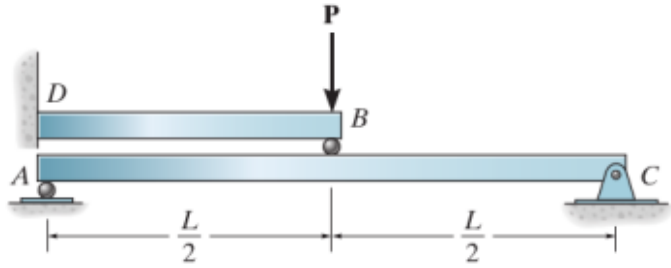
$$\Delta'_G = \frac{PL^3}{48EI} = \frac{(75\text{kN})(4.8\text{m})^3}{48EI}$$

$$\Delta_G = \Delta'_G + \Delta'_C \quad \checkmark$$



8. Determine the vertical reaction at support C in the beam arrangement shown.

[P/3]

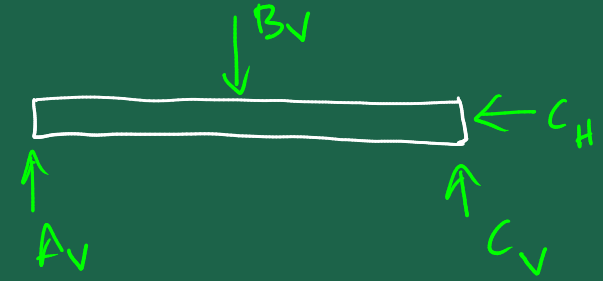


$$\Delta_{BD}^B = \frac{(P - B_V) \left(\frac{L}{2}\right)^3}{3EI}$$

$$\Delta_{AC}^B = \frac{B_V L^3}{48EI}$$

$$\Delta_{BD}^B = \Delta_{AC}^B$$

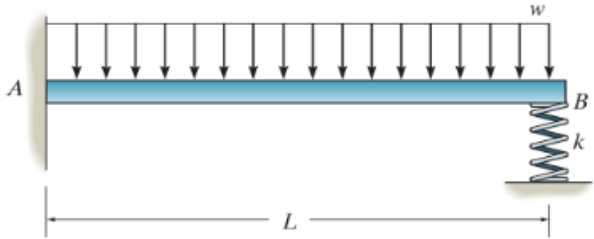
$$\Rightarrow B_V \checkmark$$



$$\sum M_A = 0$$

$$\Rightarrow C_V \checkmark$$

10. Determine the force in the spring.



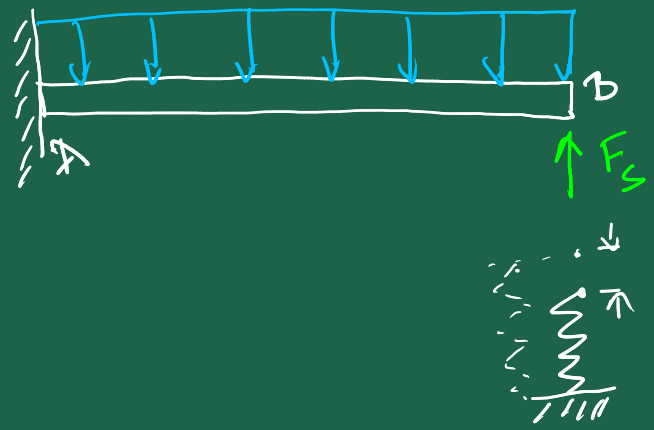
$$\left[\frac{3kwL^4}{24EI + 8kL^3} \right]$$

$$\Delta_{BA}^B = \frac{wL^4}{8EI} - \frac{F_s L^3}{3EI}$$

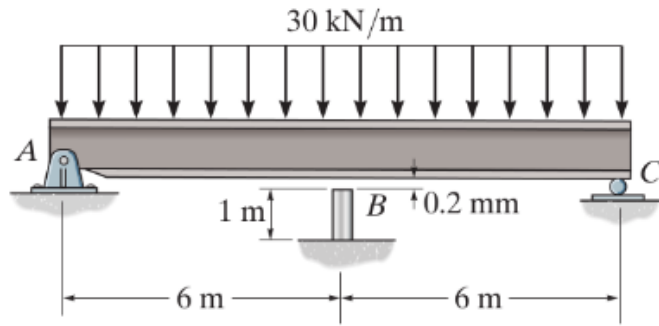
$$\Delta_{\text{Spring}}^B = \frac{F_s}{k}$$

$$\Delta_{BA}^B = \Delta_{\text{Spring}}^B$$

$$\Rightarrow F_s = \frac{3kwL^4}{24EI + 8kL^3}$$

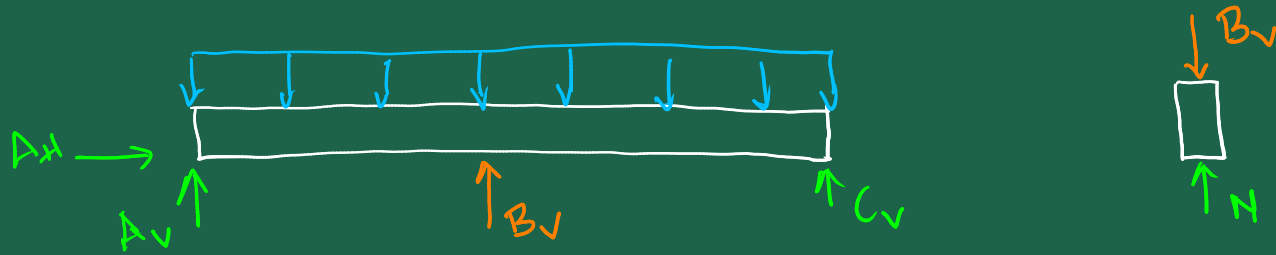


9. Before the uniformly distributed load is applied on the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is $875 \times 10^6 \text{ mm}^4$. Both the post and the beam are made of steel having modulus of elasticity 200 GPa. [70.11 kN, 219.78 kN, 70.11 kN]



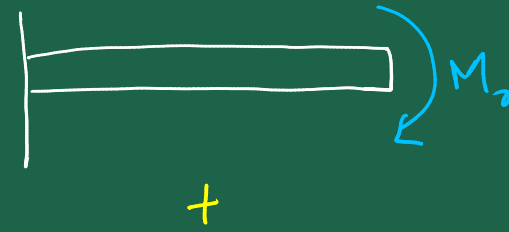
Check $\frac{5wL^4}{384EI} > 0.2 \text{ mm}$
 ?
 $\hookrightarrow 2.89 \text{ mm}$

$$\frac{PL}{AE}$$



$$(\downarrow) \Delta_{AC}^{\text{midpoint}} = \frac{5wL^4}{384EI} - \frac{B_v L^3}{48EI} = (\text{Gap-height}) = 0.2 \text{ mm} + \frac{B_v (1 \text{ m})}{\left(\frac{\pi \times 40^3}{4}\right) (200 \text{ GPa})}$$

$$\Rightarrow B_v \checkmark \rightarrow A_v, C_v \checkmark$$

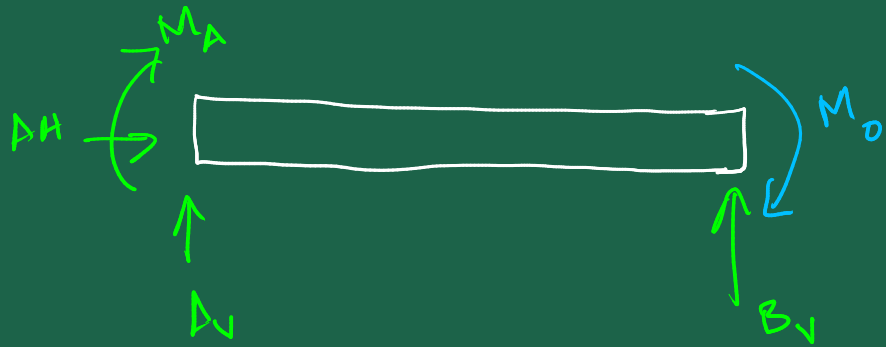


$$\frac{M_0 L^2}{2EI}$$

+



$$\frac{B_V L^3}{3EI}$$



$$EI \frac{d^2 y}{dx^2} = M = f(x)$$

Linear eqn → Add solutions

$$\frac{dy}{dx} = 3$$

$$\frac{d^2 y}{dx^2} = 4x$$

$$\frac{d^3 y}{dx^3} = 3 + 4x$$

$$\frac{d^4 y}{dx^4}$$

$$\frac{d^5 y}{dx^5}$$