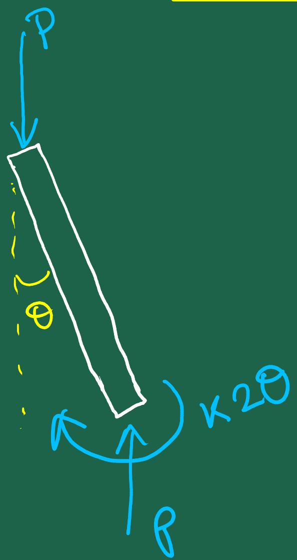
Buckling



$$P\frac{L}{2}\sin\theta - K2\theta = 0$$

$$\Rightarrow P\frac{L}{2}\sin\theta = K2\theta$$

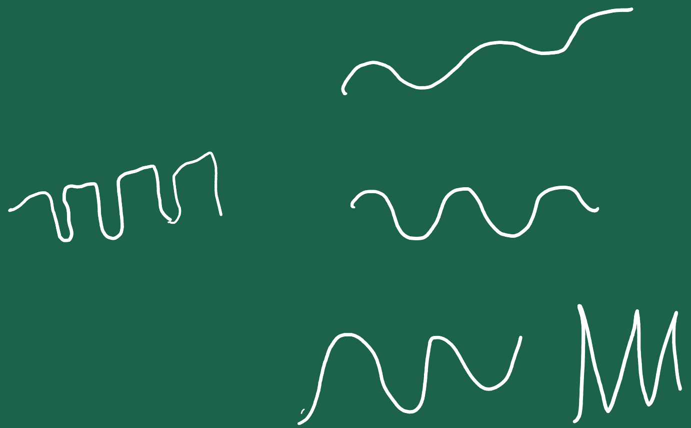
θ small , $\sin\theta \approx \theta$

$$P\frac{L}{2}\theta = K2\theta$$

$$\Rightarrow P = \frac{4K}{L}$$ → Neutral eqb

$P < \frac{4K}{L}$ → stable

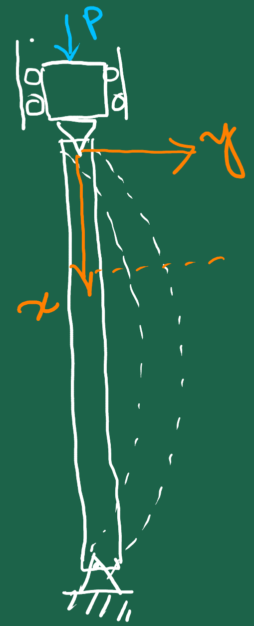
$P > \frac{4K}{L}$ → Unstable

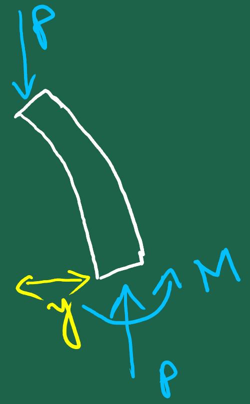


#1

Vertical beam

Column





$$M = -Py$$

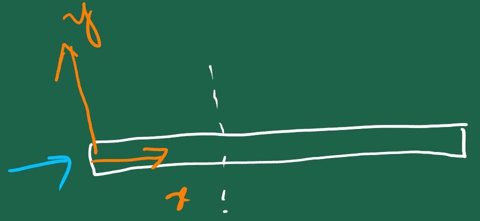
$$EI\frac{d^2y}{dx^2} = M = -Py$$

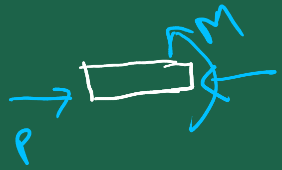
$$\Rightarrow EI\frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

Say $\frac{P}{EI} = k^2$

$$\frac{d^2y}{dx^2} + k^2y = 0$$





$$\frac{d^2y}{dx^2} + k^2 y = 0$$

Assume $y = e^{mx}$

$$m^2 e^{mx} + k^2 e^{mx} = 0$$

$$\Rightarrow \quad m^2 + k^2 = 0$$

$$\Rightarrow \quad m = \pm ki \,, \quad i = \sqrt{-1}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$= C_1 e^{kix} + C_2 e^{-kix}$$

$$= C_1 (\cos kx + i \sin kx) + C_2 (\cos kx - i \sin kx)$$

$$= \underbrace{(C_1 + C_2)}_{B} \cos kx + \underbrace{(iC_1 - iC_2)}_{A} \sin kx = A \sin kx + B \cos kx$$

$y = A \sin kx + B \cos kx$

@ $x = 0$, $y = 0$ $\Rightarrow$ $B = 0$

@ $x = L$, $y = 0$ $\Rightarrow$ $A \sin kL = 0$

If $A = 0$, $y = 0$ (trivial)

For non-trivial solutions (bent shape!)

$\sin kL = 0$ (with $A \neq 0$)

$\Rightarrow$ $kL = n\pi$, $n = 0, 1, 2, 3$

$\Rightarrow$ $k = \dfrac{n\pi}{L}$

$$K = \frac{n\pi}{L} \qquad (KL = \pi)$$

$$\Rightarrow K^2 = \frac{n^2\pi^2}{L^2}$$

$$\Rightarrow \frac{P}{EI} = \frac{n^2\pi^2}{L^2}$$

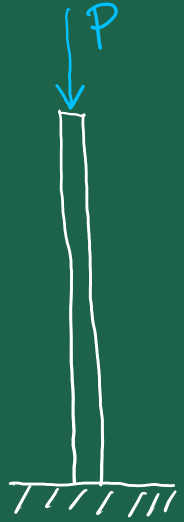
$$\Rightarrow P = \frac{n^2\pi^2 EI}{L^2} \rightarrow \text{Critical values}$$

$$n=1 \rightarrow \boxed{P_{cr} = \frac{\pi^2 EI}{L^2}} \rightarrow \text{Euler's load}$$

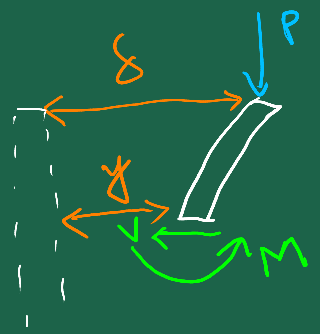
Euler's critical buckling formula

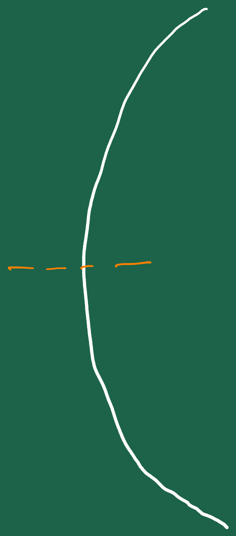
(Remember that this is only for pinned-pinned end)

2



$$M = P(\delta - y)$$





$$EI\frac{d^2y}{dx^2} = M$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = P(\delta - y)$$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}\delta$$

$$\Rightarrow \frac{d^2y}{dx^2} + k^2 y = k^2\delta$$

$$y = A\sin kx + B\cos kx + \delta$$

$$L_{eff} = 2L$$

BCs

$y = A\sin Kx + B\cos Kx + \delta$

$\frac{dy}{dx} = AK\cos Kx - BK\sin Kx$

@ $x = L$, $y = 0$

$A\sin KL + B\cos KL + \delta = 0$

@ $x = L$, $\frac{dy}{dx} = 0$

$AK\cos KL - BK\sin KL = 0$

$\Rightarrow \quad A = B\tan KL$

$\therefore \quad B\frac{\sin^2 KL}{\cos KL} + B\cos KL + \delta = 0$

$\Rightarrow \quad B + \delta\cos KL = 0 \Rightarrow B = -\delta\cos KL$

$\therefore A = -\delta\sin KL$

$y = -\delta\sin KL\sin Kx - \delta\cos KL\cos Kx + \delta$

$= -\delta\cos\{K(L-x)\} + \delta$

$$y = -\delta \cos\{k(L-x)\} + \delta$$

@ $x=0$, $y=\delta$

$$\cancel{\delta} = -\delta \cos(kL) + \cancel{\delta}$$

$$\Rightarrow \delta \cos kL = 0$$

$$\Rightarrow \cos kL = 0$$

$$\Rightarrow kL = (2n-1)\frac{\pi}{2}, \quad n = 1, 2, 3, \ldots$$

$$k = (2n-1)\frac{\pi}{2L}$$

$$K = (2n-1)\frac{\pi}{2L}$$

$$K^2 = \frac{P}{EI}$$

$n=1$

$$K = \frac{\pi}{2L}$$

$$KL = \frac{\pi}{2}$$

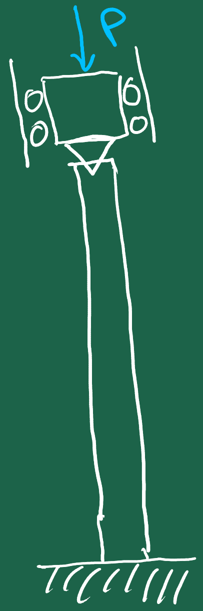
$$\Rightarrow \quad K^2 = \frac{\pi^2}{(2L)^2}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\Rightarrow \quad P = \frac{\pi^2 EI}{(2L)^2}$$

$\hookrightarrow L_{eff} = 2L$

#3



$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow EI\frac{d^3y}{dx^3} + P\frac{dy}{dx} = 0$$

$$\Rightarrow EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^4y}{dx^4} + k^2\frac{d^2y}{dx^2} = 0 \qquad \left[k^2 = \frac{P}{EI}\right]$$

$$y = e^{mx} \rightarrow m^4 + k^2m^2 = 0$$

$$m^4 + k^2 m^2 = 0$$

$$\Rightarrow m^2 (m^2 + k^2) = 0$$

$$\Rightarrow m = 0, 0, -ik, +ik$$

$0, -ik, ik$

$A \sin kx + B \cos kx$

$+ C$

$$y = A \sin kx + B \cos kx + Cx + D$$

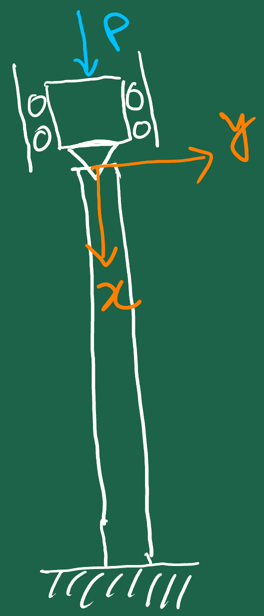
$@ x = 0, \quad y = 0 \Rightarrow B + D = 0$

$@ x = 0, \quad M = 0$

$$\frac{dy}{dx} = Ak \cos kx - Bk \sin kx + C$$

$$\frac{d^2y}{dx^2} = -Ak^2 \sin kx - Bk^2 \cos kx$$

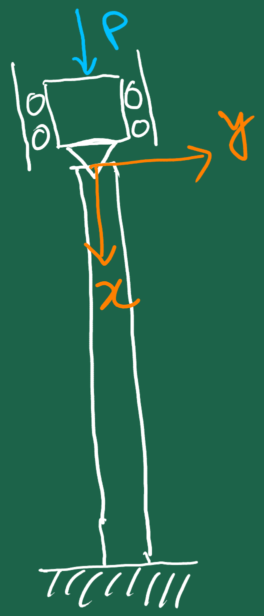
$$B = 0$$



$@\ x = L, \quad y = 0$

$A \sin KL + B \cos KL + CL + D = 0$

$@\ x = L, \quad \frac{dy}{dx} = 0$



$\frac{dy}{dx} = AK \cos Kx - BK \sin Kx + C$

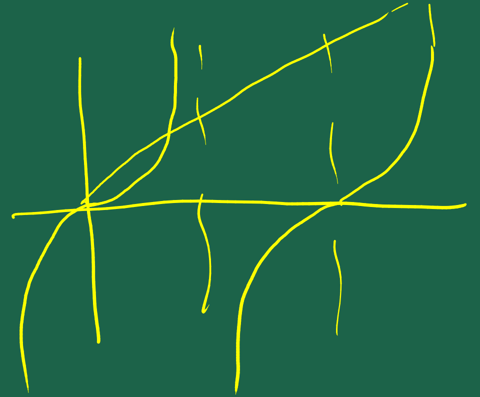
$AK \cos KL - BK \sin KL + C = 0$

$A \sin KL + CL = 0 \Rightarrow A \sin KL = -CL \quad - *_1$

$AK \cos KL + C = 0 \Rightarrow AK \cos KL = -C \quad - *_2$

$*_1 \div *_2$

$\frac{\tan KL}{K} = L \Rightarrow \boxed{\tan KL = KL}$



$$\tan kL = kL$$

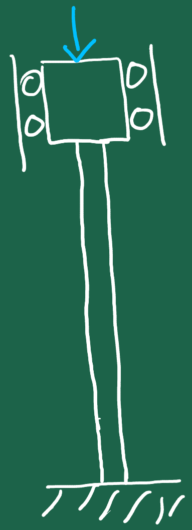
$$kL = 4.4934$$

$$\frac{P}{EI} = \left(\frac{4.4934}{L}\right)^2$$

$$\Rightarrow P_{cr} \approx \frac{4.5^2\, EI}{L^2} = \frac{\pi^2 EI}{\left(\pi L/4.5\right)^2}$$

$$L_{eff} = \frac{\pi L}{4.5}$$

4



Both ends clamped

(Part of TS 9)

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 EI}{(KL)^2}$$

#1

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\frac{P_{cr}}{A} = \sigma_{cr} = \frac{\pi^2 EI}{L^2 A}$$

$$I = A r^2$$

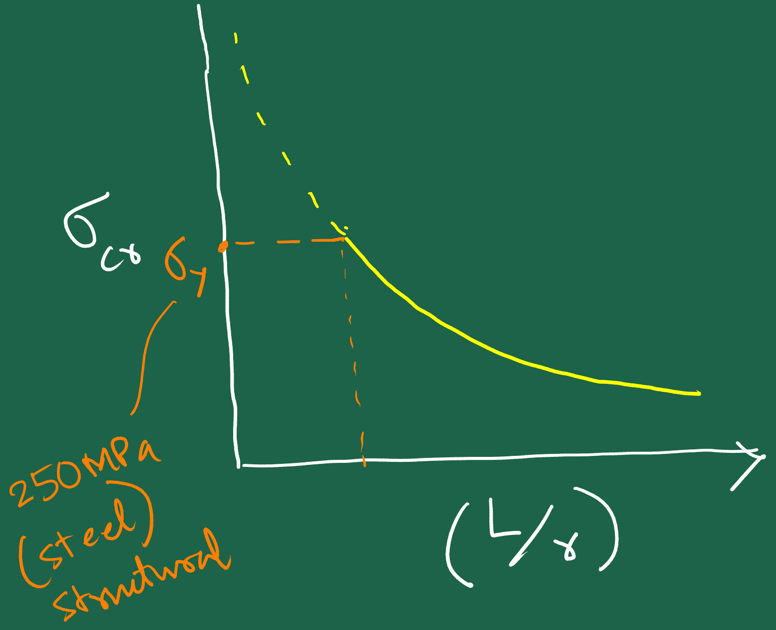
↳ radius of gyration

$$\sigma_{cr} = \frac{\pi^2 E \cancel{A} r^2}{L^2 \cancel{A}} = \frac{\pi^2 E r^2}{L^2} = \frac{\pi^2 E}{(L/r)^2}$$

$\frac{L}{r}$: slenderness ratio

$$I = \frac{1}{12} b h^3$$

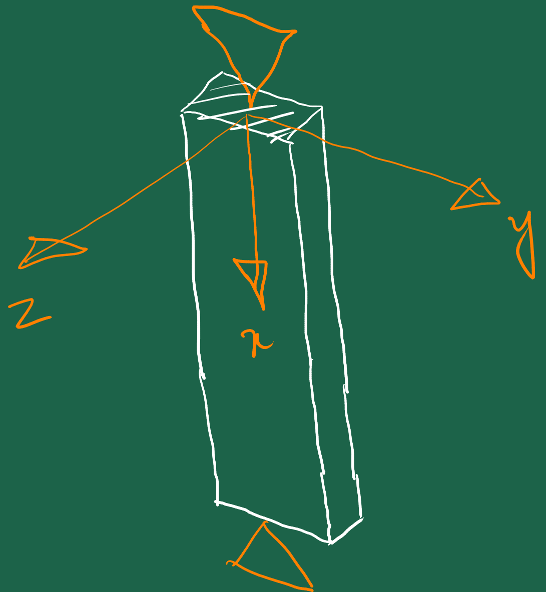
$$[I] = m^4$$

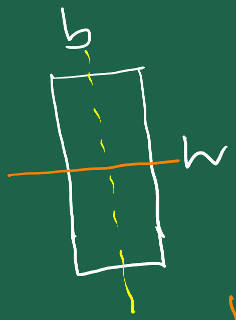


#1 $P_{cr} = \frac{\pi^2 EI}{L^2}$

For actual P_{cr},

consider the I_{min}





$I_z = \frac{1}{12} b h^3$, $I_y = \frac{1}{12} h b^3$

$I_z > I_y$