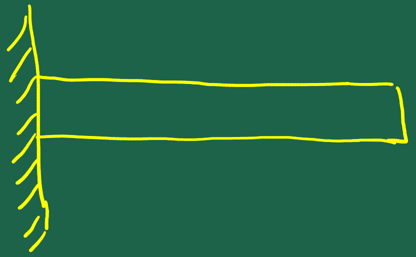
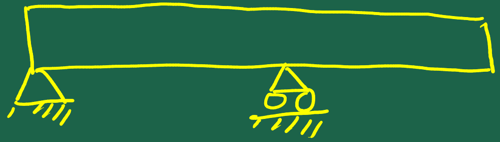
Bending of Beams

Simply-supported

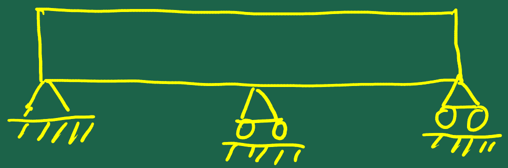


Cantilever

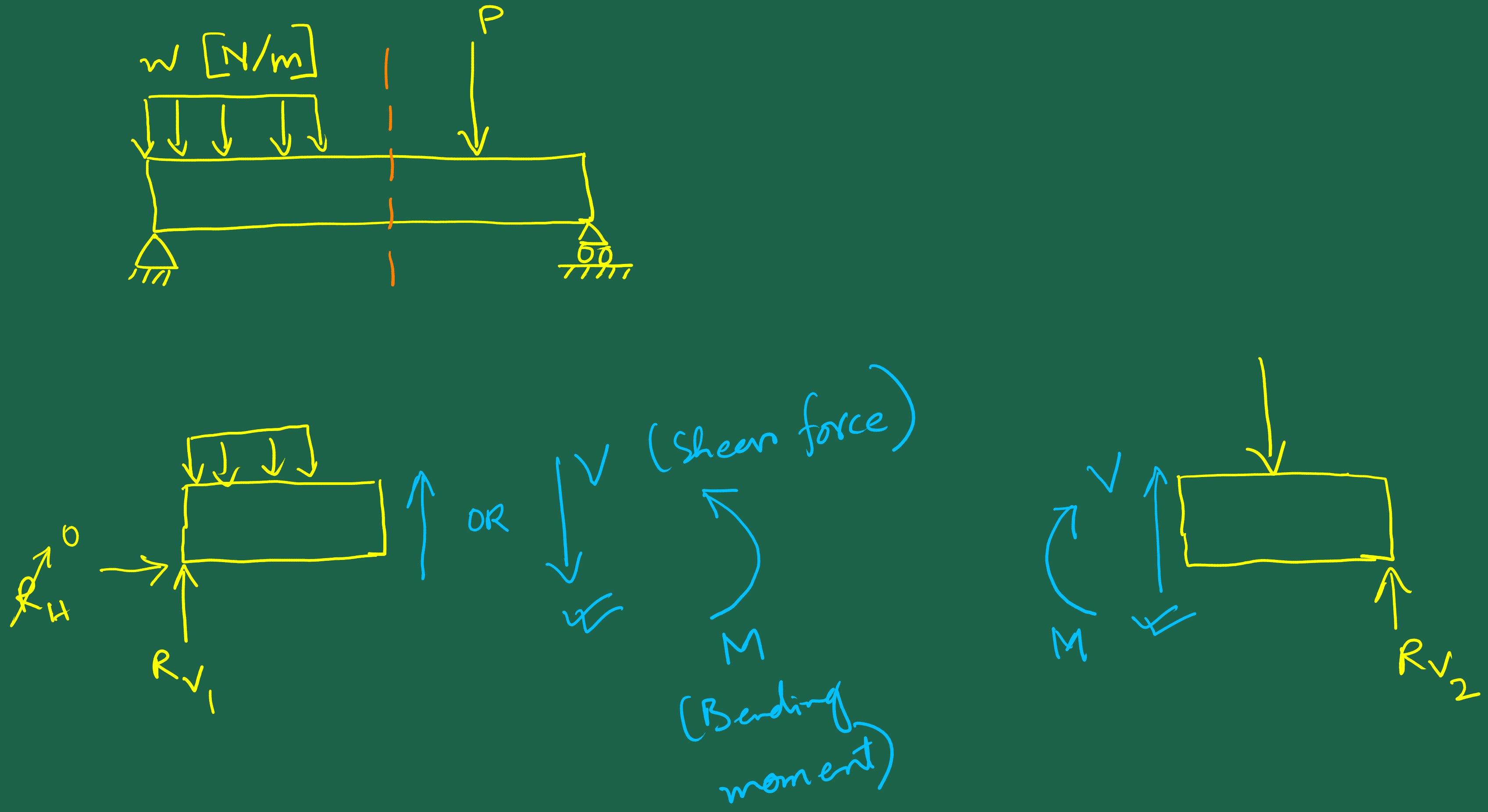


Overhanging

Statically Determinate

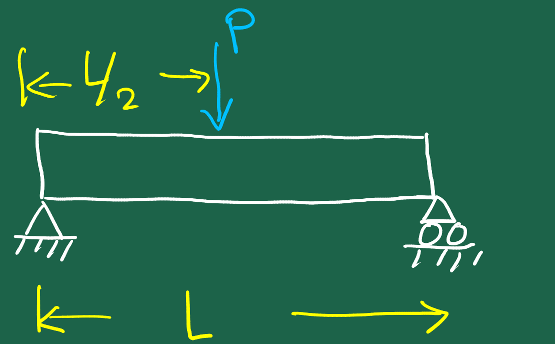


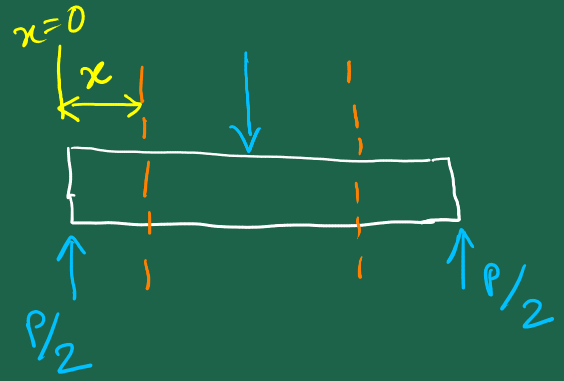
statically Indeterminate

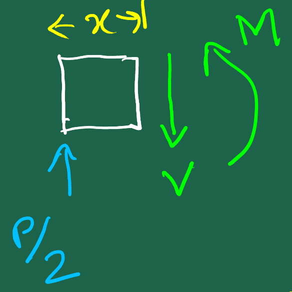


w [N/m]
P
R_H
0
R_V1
OR
V
(Sheer force)
M
(Bending moment)
V
M
R_V2

1





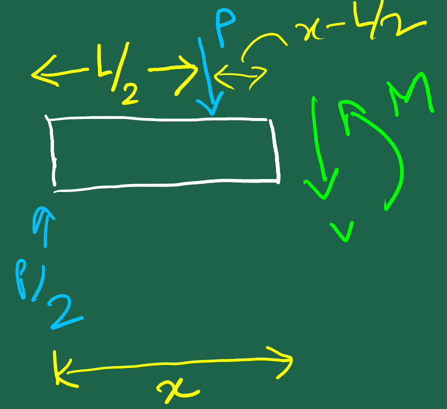


$0 < x < L/2$

$V - P/2 = 0 \Rightarrow V = P/2$

$\sum M = 0$ (+)

$M - \frac{P}{2}x = 0 \Rightarrow M = \frac{P}{2}x$



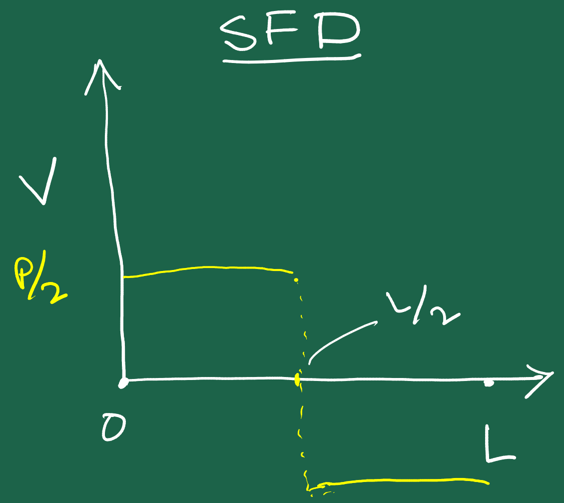
$L/2 < x < L$

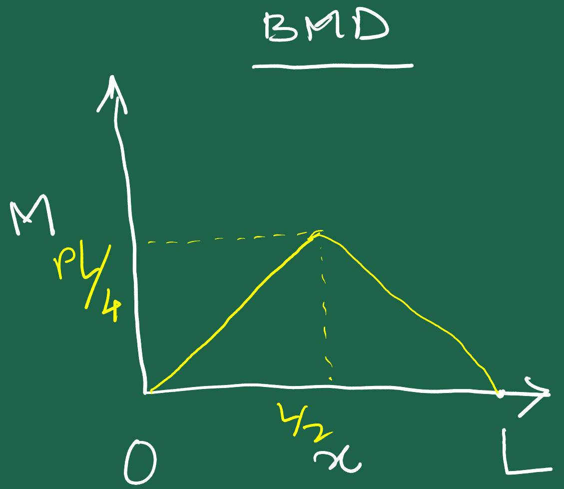
$V + P - P/2 = 0$

$\Rightarrow V = -P/2$

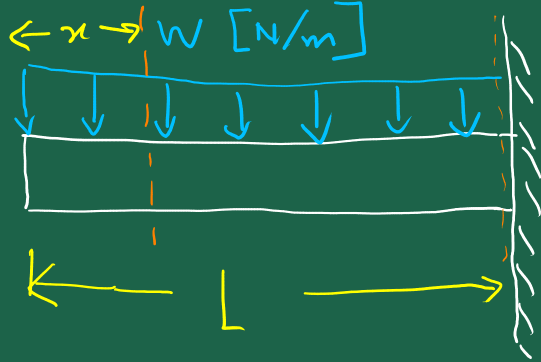
$M - \frac{P}{2}x + P\left(x - \frac{L}{2}\right) = 0$

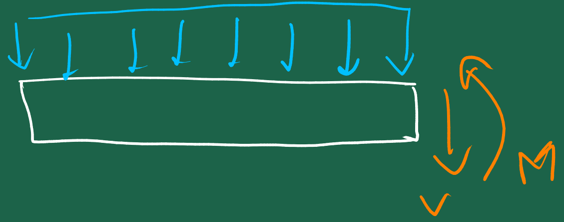
$\Rightarrow M = \frac{P}{2}(L - x)$



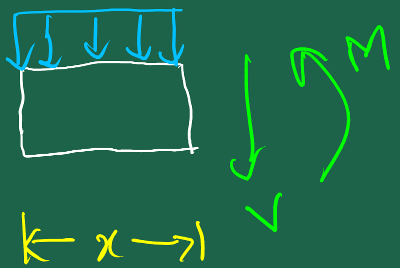


2





$wx \cdot \frac{x}{2}$

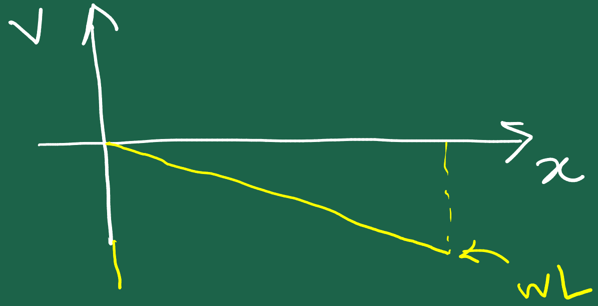


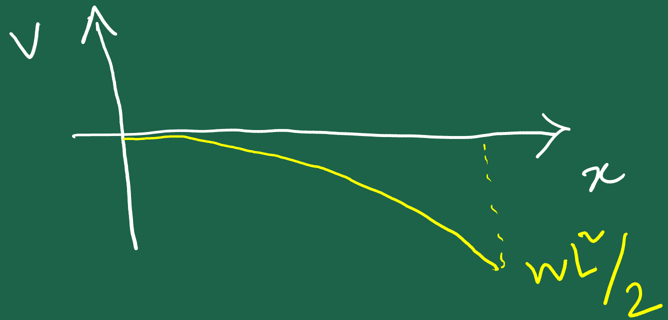
$$\sum M = 0 \quad (+)$$

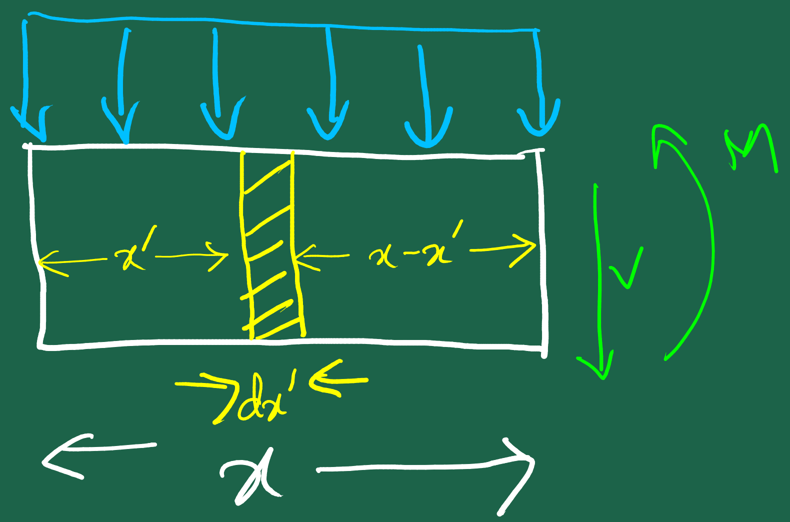
$$\Rightarrow \frac{wx^2}{2} + M = 0 \Rightarrow M = -\frac{wx^2}{2}$$

$$V + wx = 0$$

$$\Rightarrow V = -wx$$





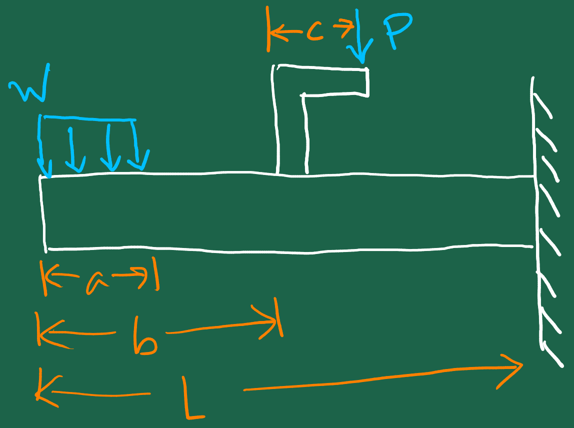


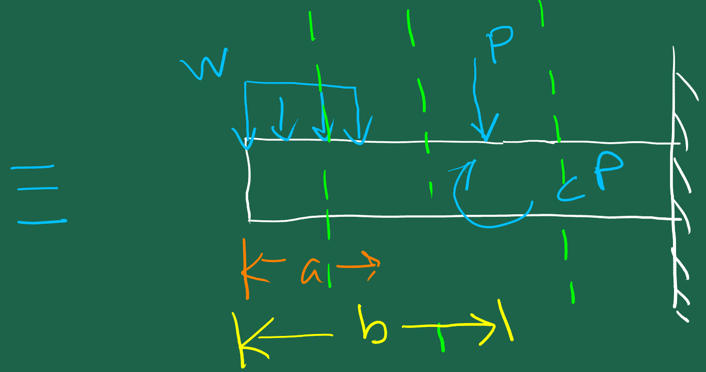
$$+\int_0^x (x-x')\, w\, dx' + M = 0$$

$$\Rightarrow -\left[\frac{w(x-x')^2}{2}\right]_0^x + M = 0$$

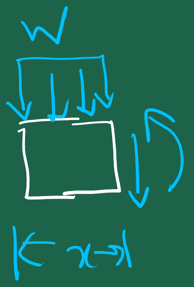
$$\Rightarrow +\frac{wx^2}{2} + M \Rightarrow M = -\frac{wx^2}{2}$$

3





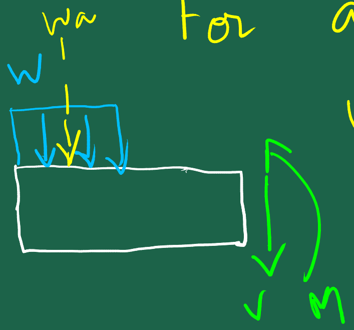
For $0 < x < a$



$$V = -wx$$

$$M = -wx^2/2$$

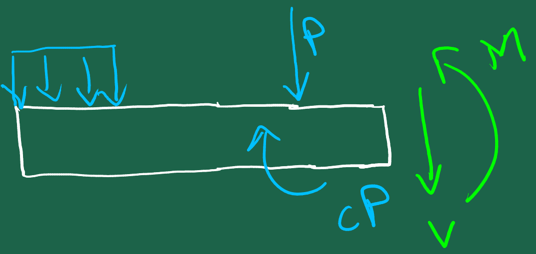
For $a < x < b$



$$V = -wa$$

$$M = -wa\left(x - \frac{a}{2}\right)$$

For $b < x < L$

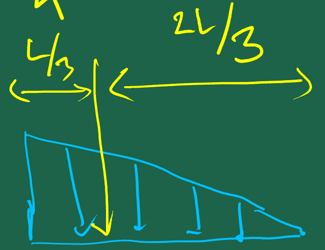


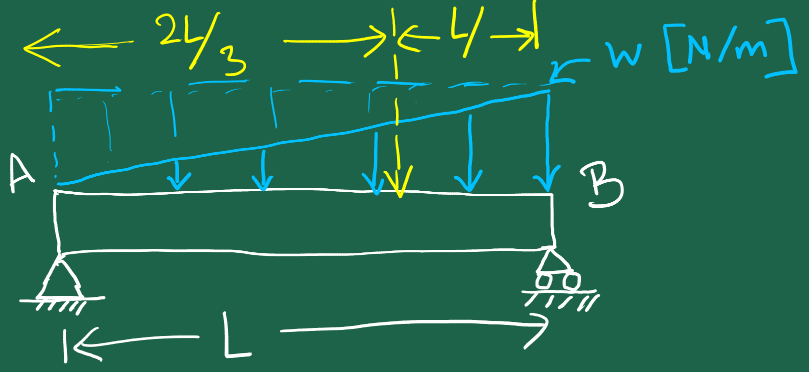
$$V + P + wa = 0 \Rightarrow V = -wa - P$$

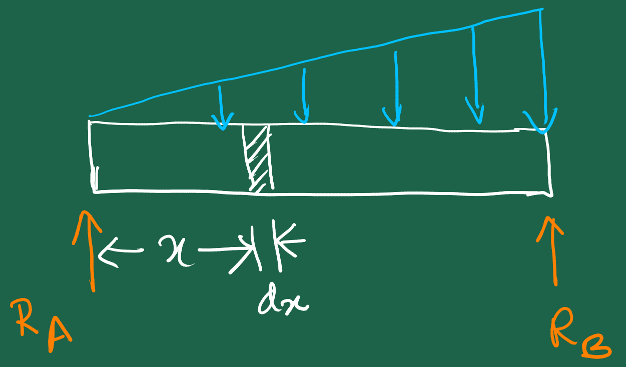
$$M - cP + P(x-b) + wa\left(x - \frac{a}{2}\right) = 0$$

$$\Rightarrow M = cP - P(x-b) - wa\left(x - \frac{a}{2}\right)$$

#4







at any pos^n x : $\frac{w}{L}x$

$$\sum M_A = 0 \quad (+)$$

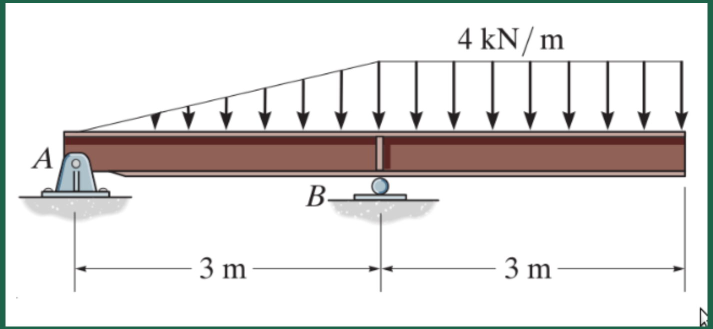
$$-\int_0^L x\left(\frac{w}{L}x\right)dx + R_B L = 0$$

$$\Rightarrow R_B L = \frac{w}{L}\frac{L^3}{3} = \frac{wL^2}{3}$$

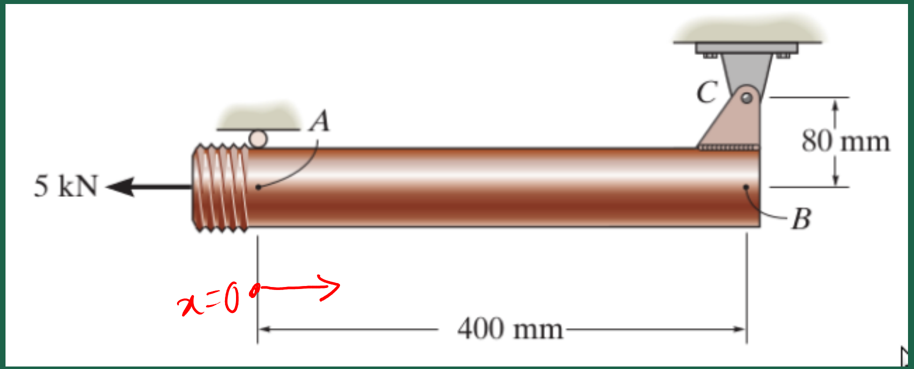
$$\Rightarrow R_B L = \left(\frac{wL}{2}\right)\left(\frac{2L}{3}\right)$$

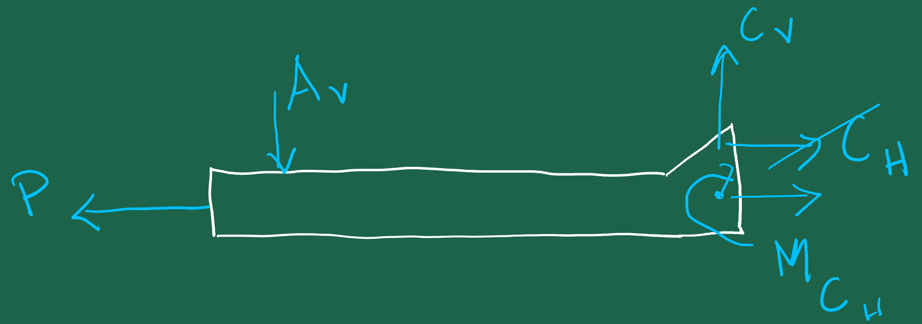
↳ wrt A (lhs)

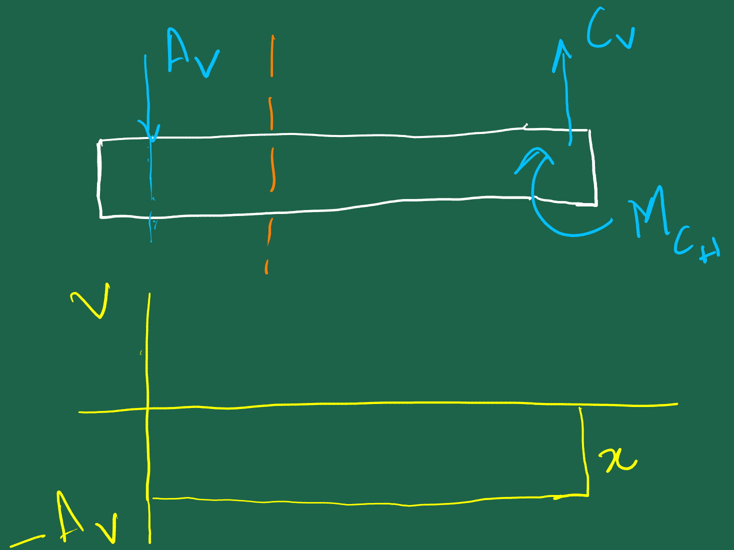
3. Draw the shear and moment diagrams for the overhanging beam.

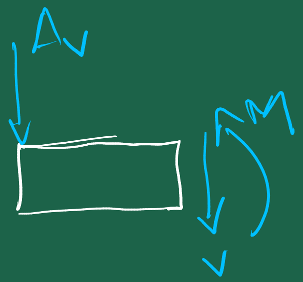


1. Draw the shear force and bending moment diagrams for the pipe. The end screw is subjected to a horizontal force of 5 kN.



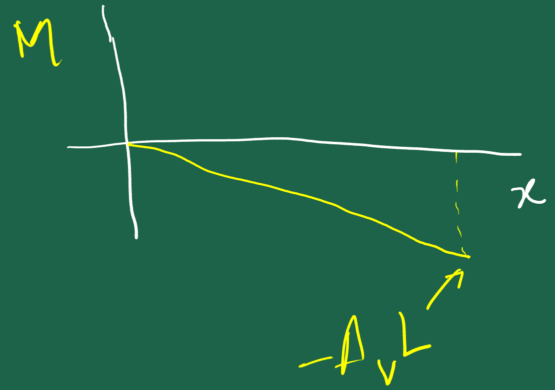






$V + A_v = 0$

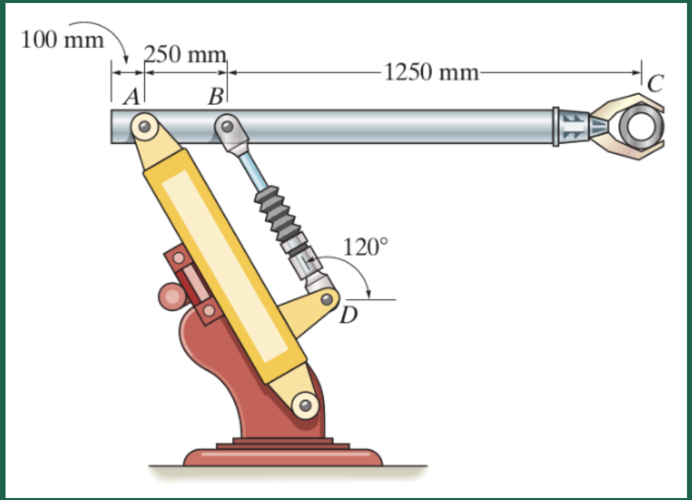
$\Rightarrow V = -A_v$

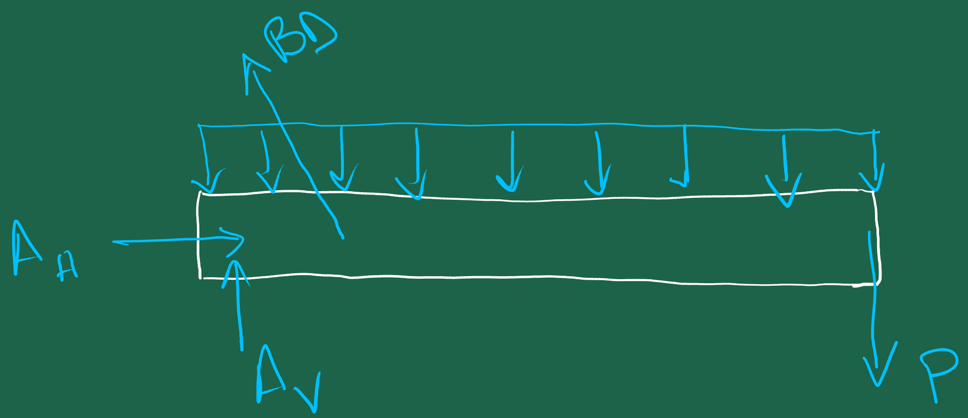


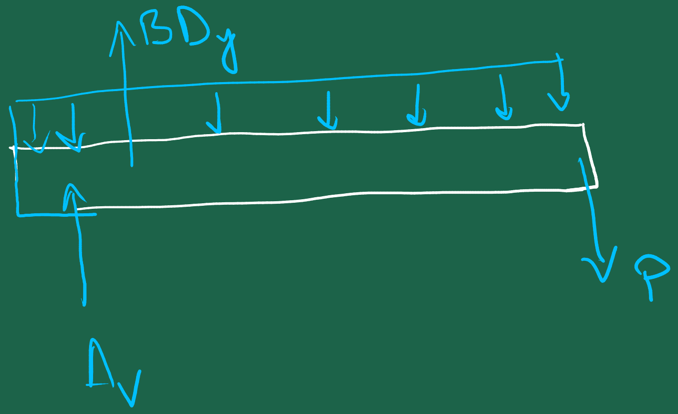
$\sum F_y = 0 \ (\uparrow +)$

$-V - A_v = 0 \Rightarrow V = -A_v$

2. The industrial robot is held in the stationary position shown. Draw the shear force and bending moment diagrams of the arm ABC if it is pin connected at A and connected to a hydraulic cylinder (two-force member) BD. Assume the arm and grip have a uniform weight of 0.3 N/mm and support the load of 200 N at C.





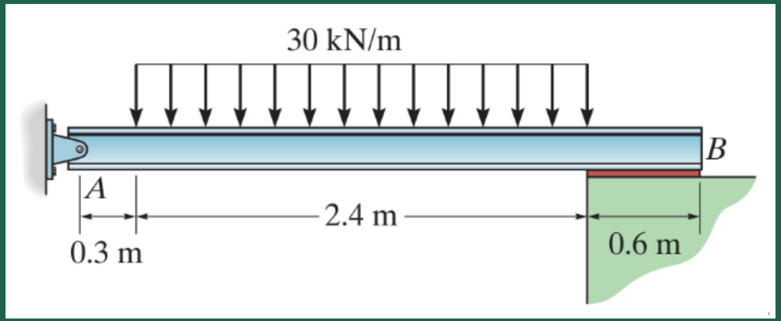


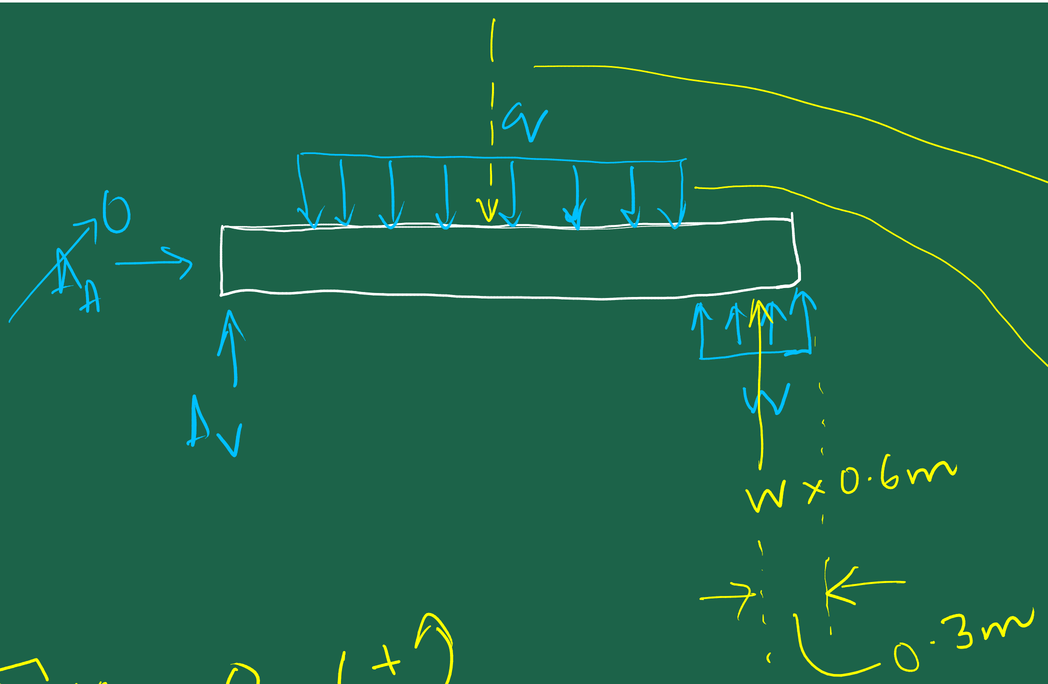
$\sum F_x = 0 \Rightarrow A_H = BD_x$

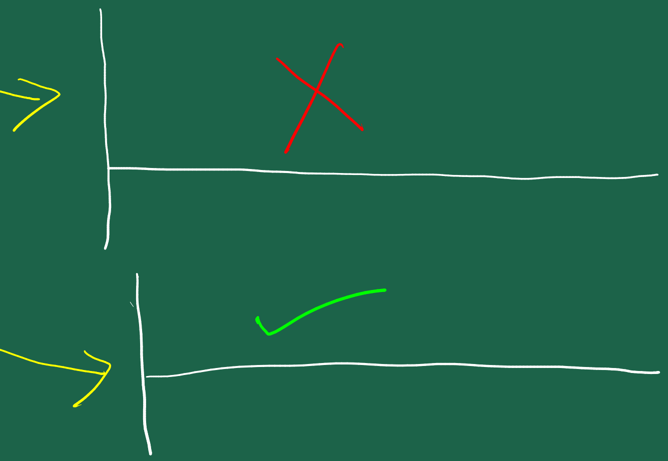
$\sum M_A = 0 \Rightarrow BD$ ✓

$\sum F_y = 0 \Rightarrow A_V$ ✓

4. The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its length of 0.6 m. If the beam supports a uniform loading of 30 kN/m, draw the shear and moment diagrams.



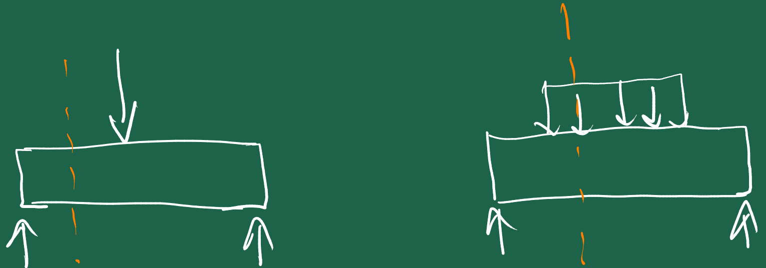




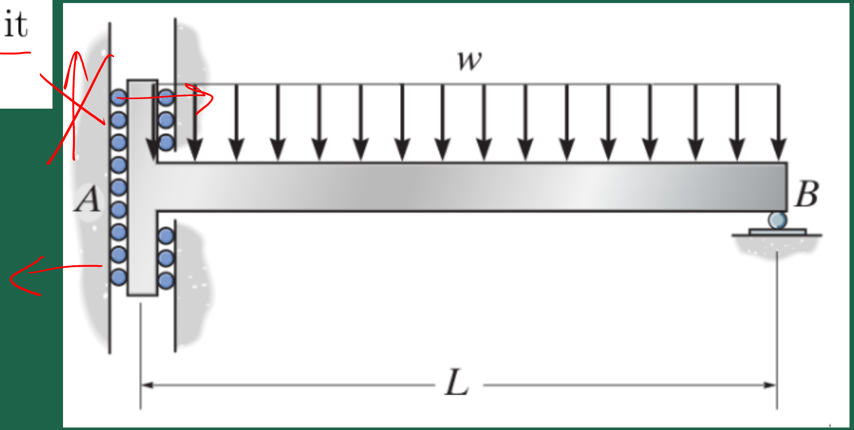
$$\sum M_A = 0 \quad (+\circlearrowleft)$$

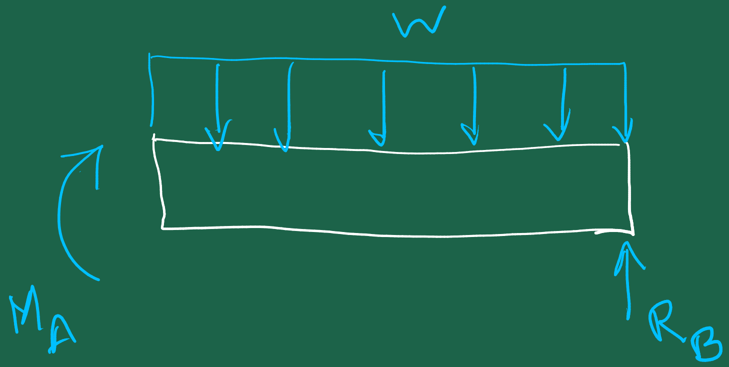
$$\Rightarrow -30\,\text{kN/m} \times 2.4\,\text{m} \times (1.2 + 0.3)\,\text{m} + (w \times 0.6\,\text{m}) \times 3\,\text{m} = 0 \Rightarrow w \checkmark$$

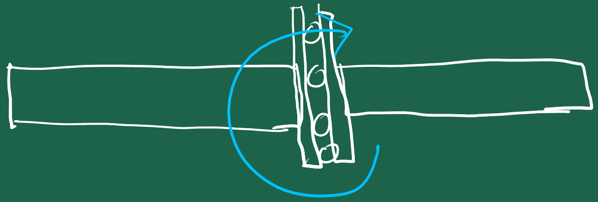
$$\sum F_y = 0 \Rightarrow A_v \checkmark$$



5. Draw the shear and moment diagrams for the beam when the support at A allows it to slide freely along the vertical guide and hence it cannot support a vertical force.



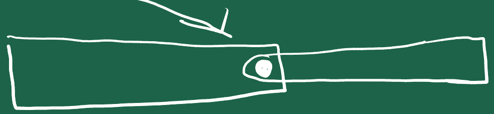




"Shear release"

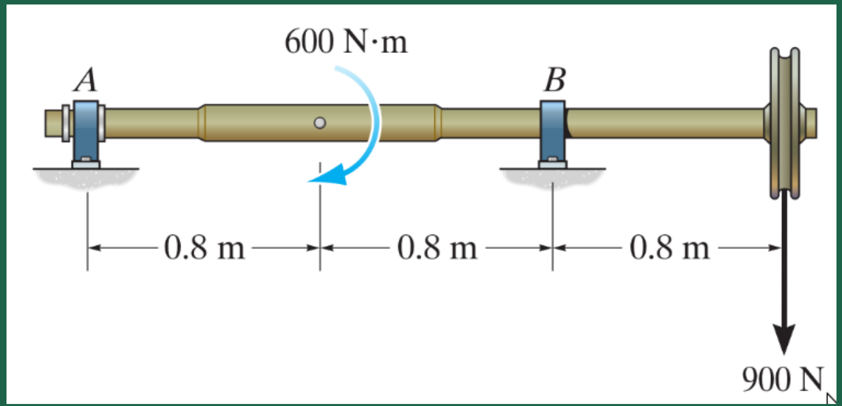
$V = 0$

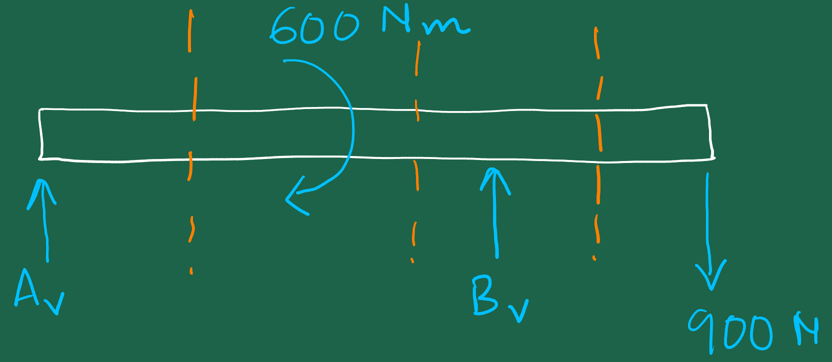
$M = 0$



"Moment release"

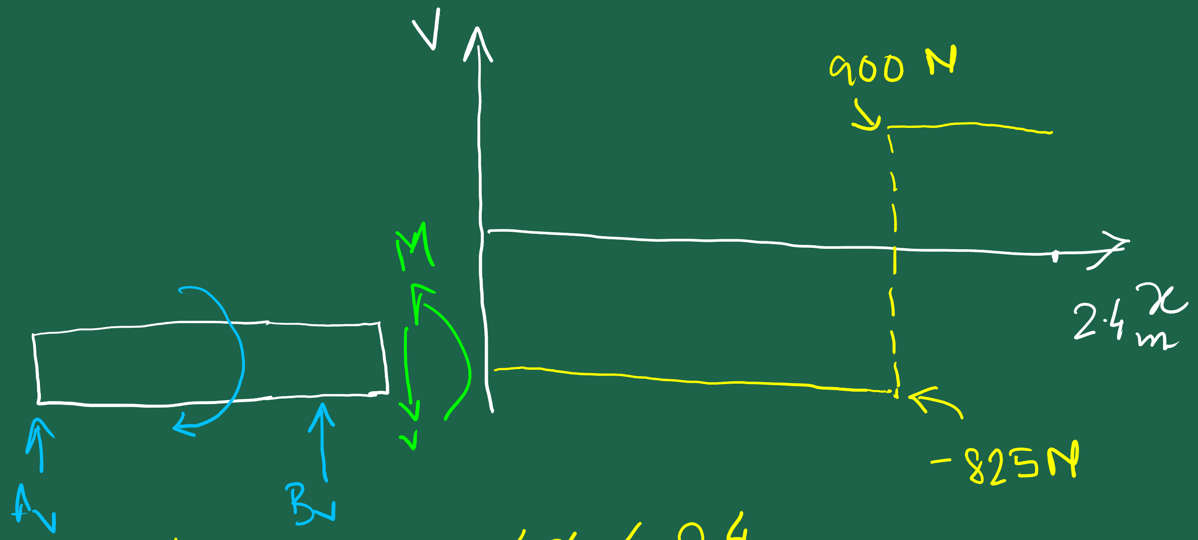
6. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.

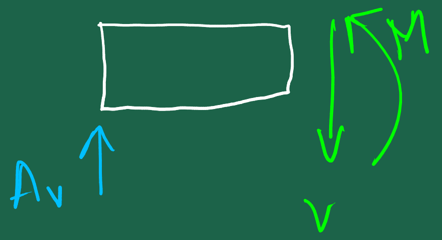




$\sum M_A = 0 \Rightarrow B_v = 1725\,N$

$\sum F_y = 0 \Rightarrow A_v = -825\,N$





For $0 < x < 1.6\,m$

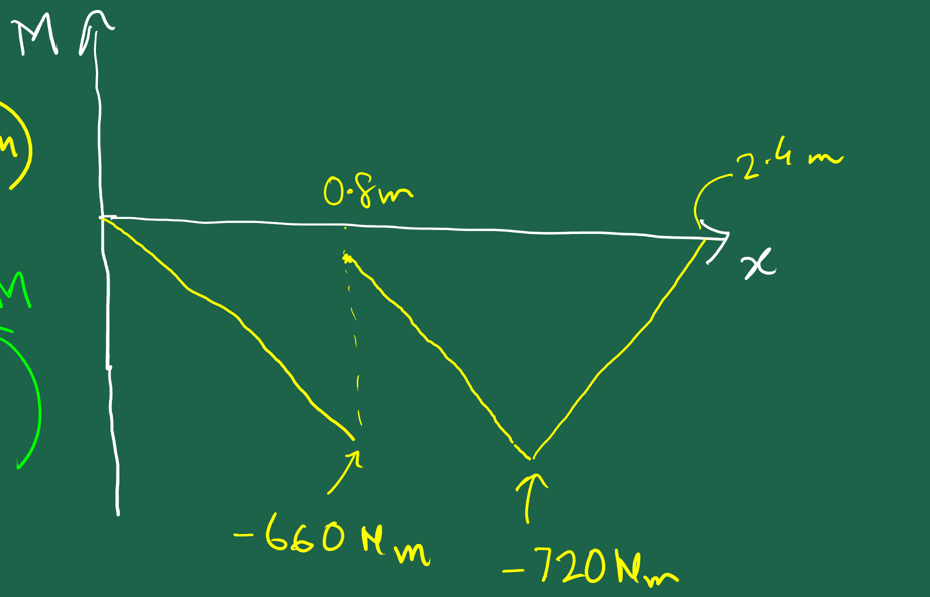
$\sum F_y = 0 \Rightarrow V - A_v = 0 \Rightarrow V = A_v = -825\,N$

For $1.6\,m < x < 2.4\,m$

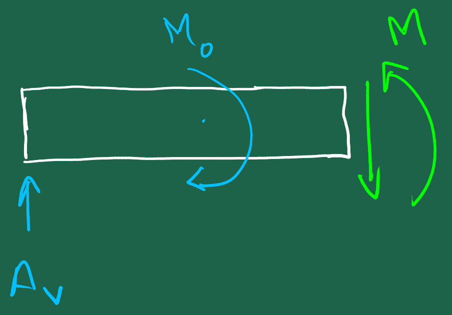
$\sum F_y = 0 \Rightarrow V = A_v + B_v = 900\,N$

For $0 < x < 0.8\,m$

$(+\circlearrowleft)\ \sum M = 0 \Rightarrow M = A_v x$ $\quad (@\ x = 0.8\,m,\ M = -660\,Nm)$



For $0.8\,m < x < 1.6\,m$



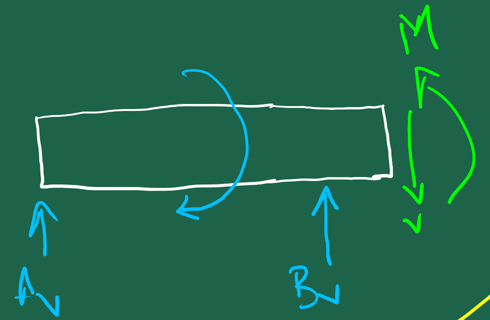
$(+\circlearrowleft)\ \sum M = 0 \Rightarrow -A_v x - 600 + M = 0$

$\Rightarrow M = 600 + A_v x$

$(@\ x = 0.8\,m,\ M = -60\,Nm)$

$(@\ x = 1.6\,m,\ M = -720\,Nm)$

For $1.6\,m < x < 2.4\,m$

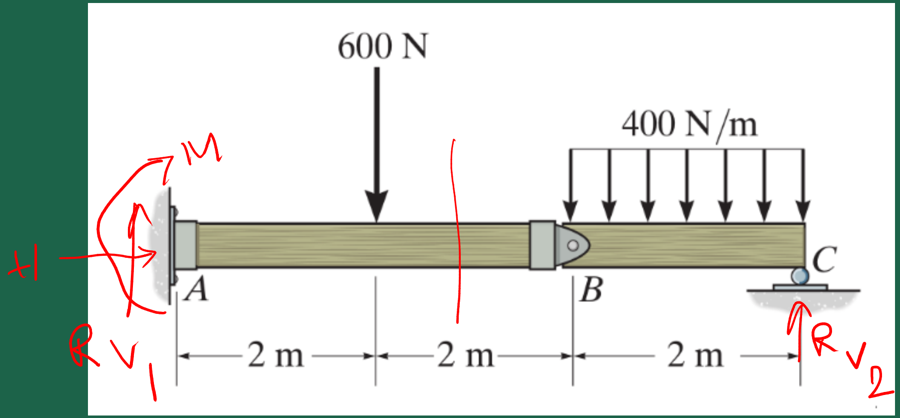


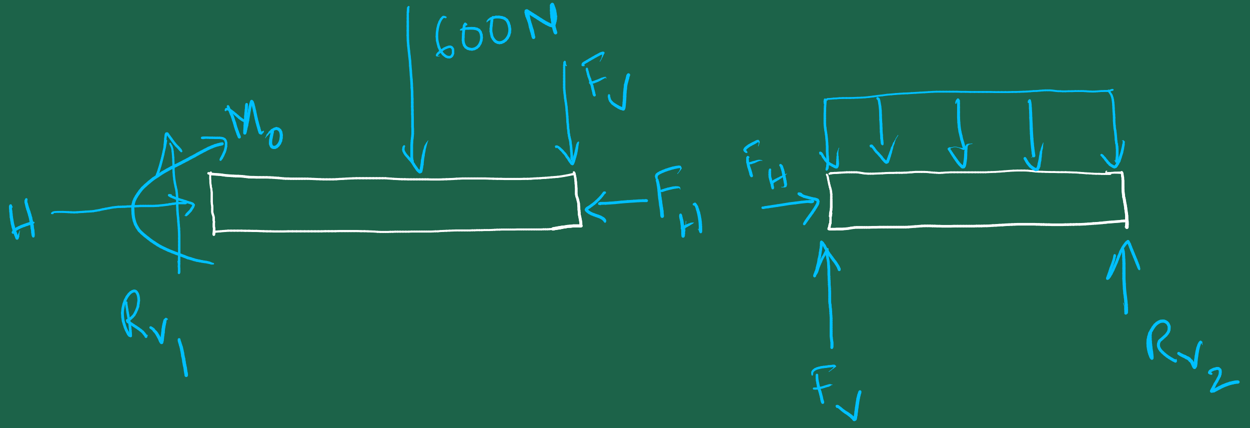
$(+\circlearrowleft)\ \sum M = 0 \Rightarrow M - A_v x - B_v(x - 1.6\,m) - 600\,Nm = 0$

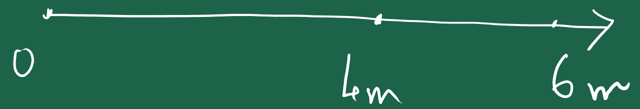
$@\ x = 1.6\,m,\ M = -720\,Nm$

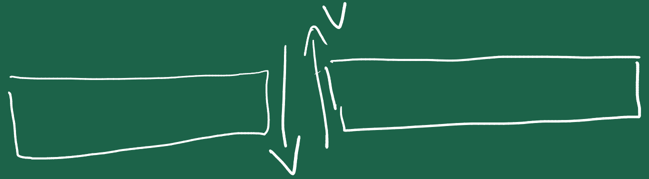
$@\ x = 2.4\,m,\ M = 0$

8. The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.

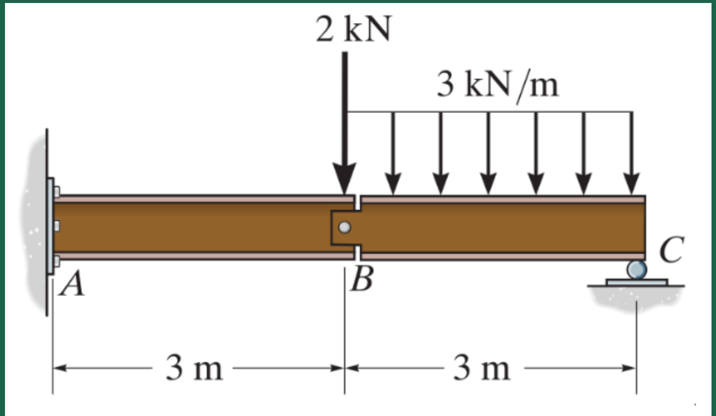


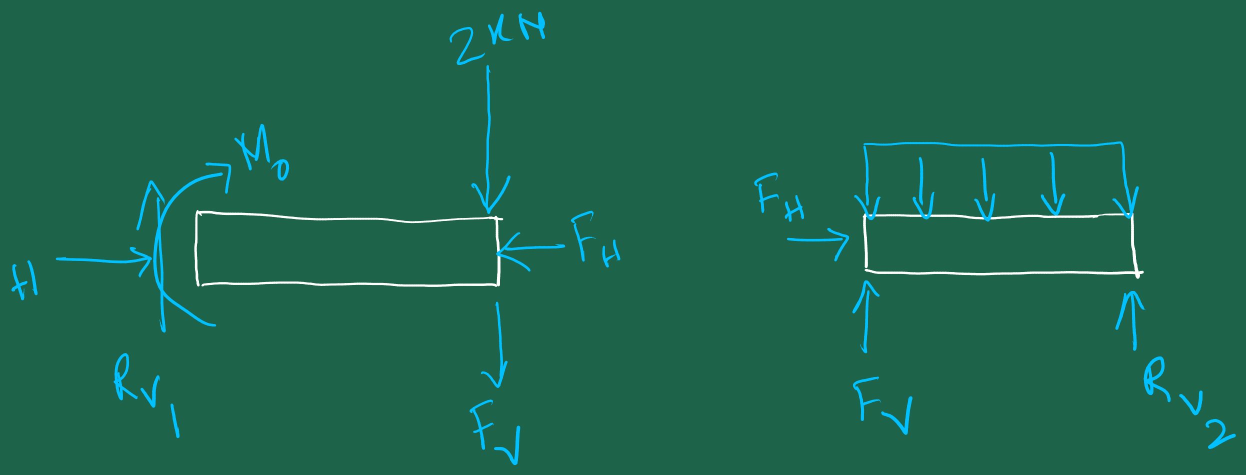


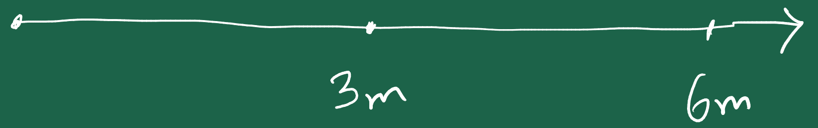




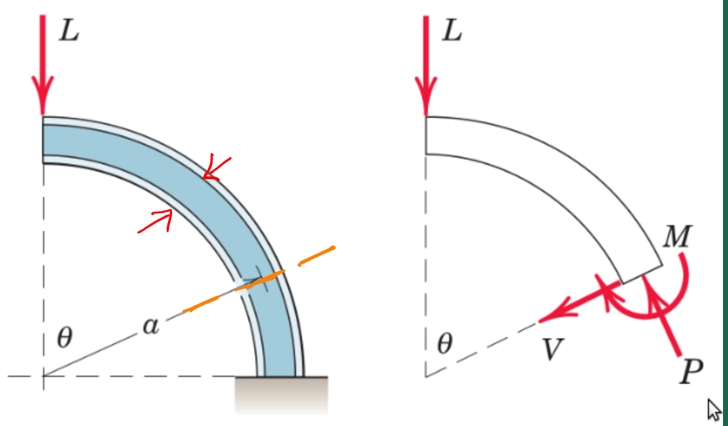
9. The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.







11. A curved cantilever beam has the form of a quarter circular arc. Determine the expressions of the shear force V and the bending moment M as functions of θ. The depth of the beam is much smaller than the arc radius.



Relations between load, shear, and bending moment