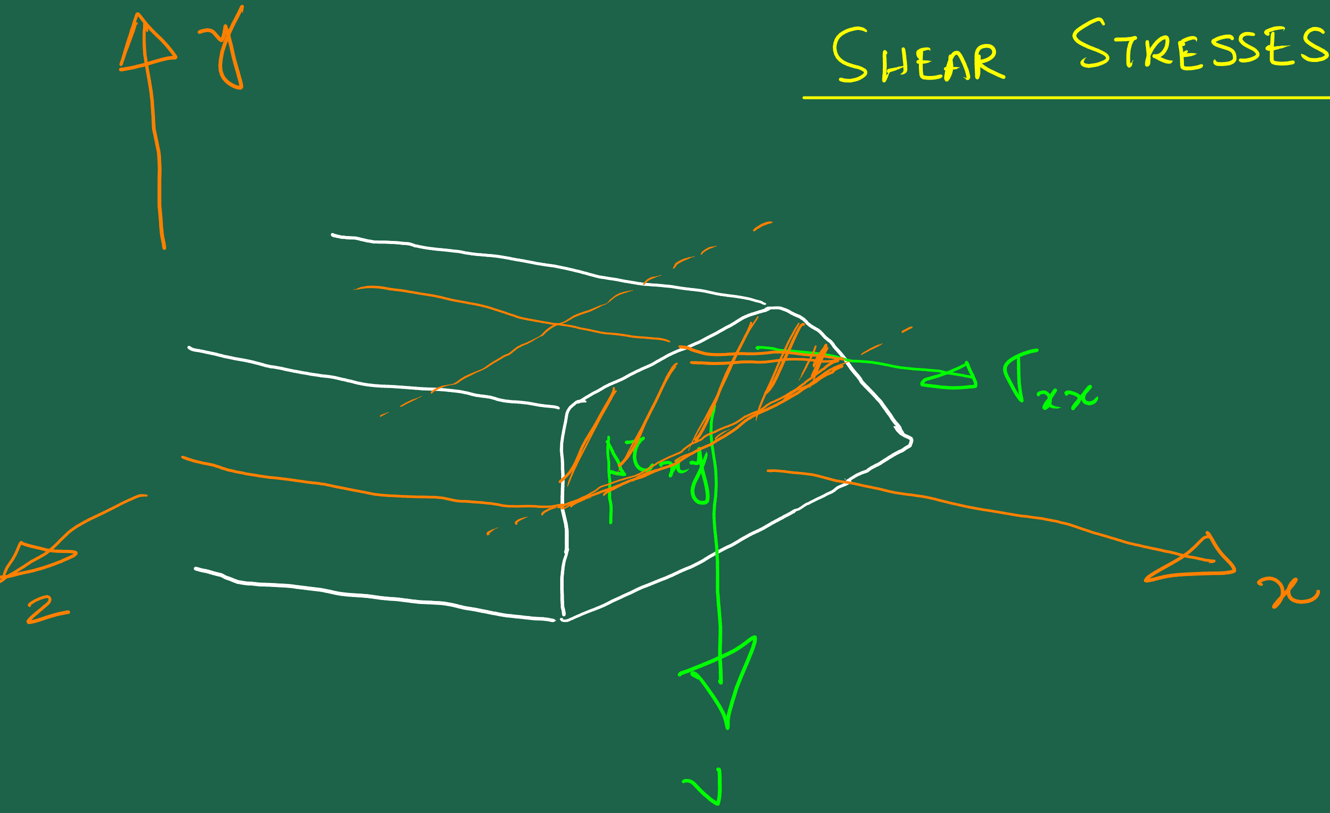
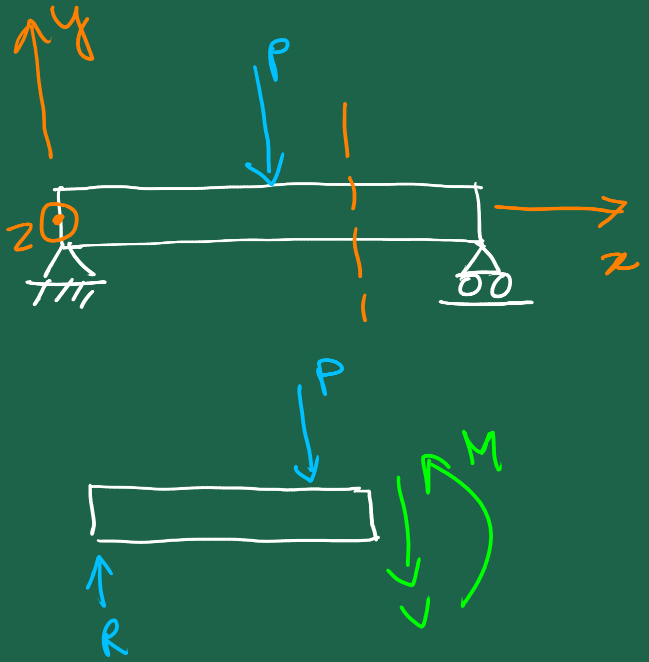
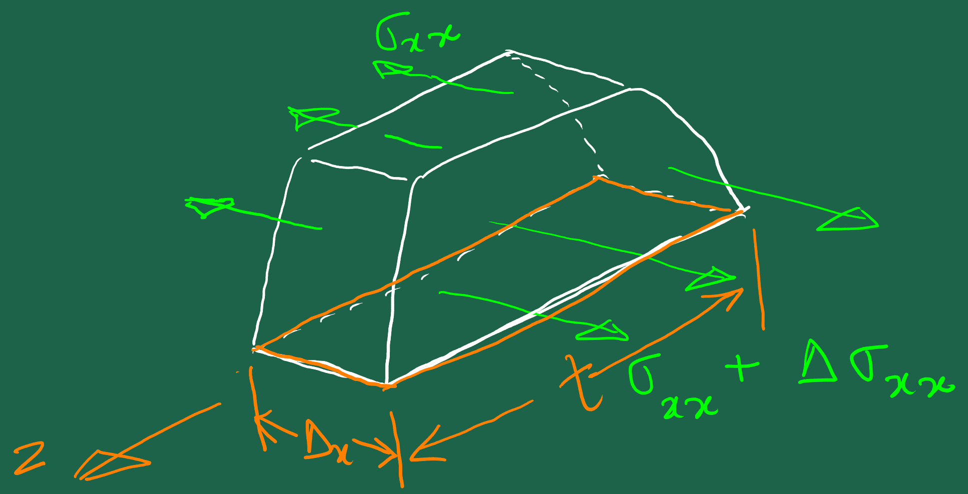
SHEAR STRESSES

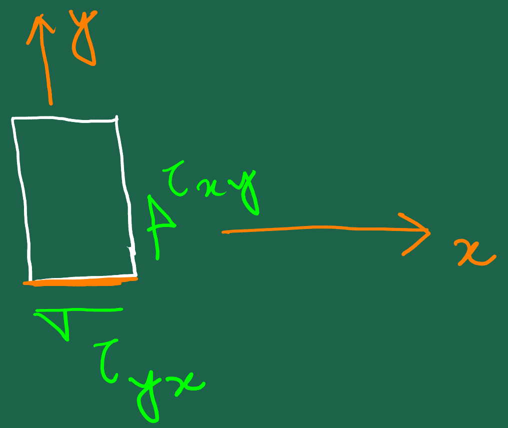


$$\int_A \tau_{xy}\, dA = -V$$



$$\int_A y\, \sigma_{xx}\, dy = -M$$





$$\int_{A'} (\sigma_{xx} + \Delta\sigma_{xx})\, dA' - \int_{A'} \sigma_{xx}\, dA' = \tau_{yx}\, t\, \Delta x$$

$$\Rightarrow \int_{A'} \Delta\sigma_{xx}\, dA' = \tau_{yx}\, t\, \Delta x$$

$$\Rightarrow \tau_{yx}\, t = \int_{A'} \frac{\Delta\sigma_{xx}}{\Delta x}\, dA' \xrightarrow[\Delta x \to 0]{} \int_{A'} \frac{d\sigma_{xx}}{dx}\, dA'$$

$$\tau_{yx}\, t = \int_{A'} \frac{d}{dx}\left(-\frac{My}{I}\right) dA' = -\int_{A'} \frac{y}{I}\frac{dM}{dx}\, dA'$$

$$= -\int \frac{Vy}{I}\, dA'$$

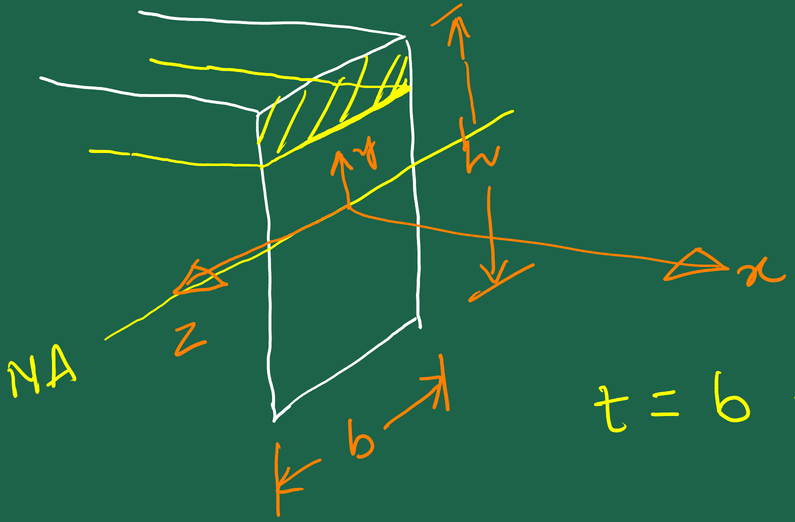
$$= -\frac{V}{I}\int y\, dA'$$

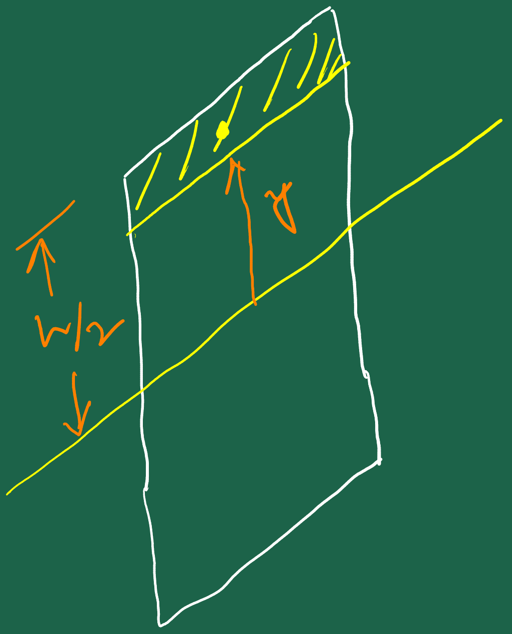
$$\therefore \boxed{\tau_{yx} = -\frac{VQ}{It}} \qquad \Big[\underbrace{Q = \int y\, dA'}\Big]$$

↳ 1st moment of area of the part of the c/s under consideration

Shear stress formula

$$\tau_{xy} = \tau_{yx} = -\frac{VQ}{It}$$





$$t = b \; ; \quad I = \frac{1}{12} b h^3$$

$$Q = b\left(\frac{h}{2} - y\right)\left(y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right)$$

$$= \frac{1}{2} b \left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{xy} = -\frac{VQ}{It} = \frac{-V}{\frac{1}{12} b h^3 b} \frac{1}{2} b \left(\frac{h^2}{4} - y^2\right)$$

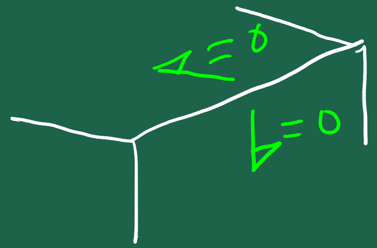
$$= -\frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

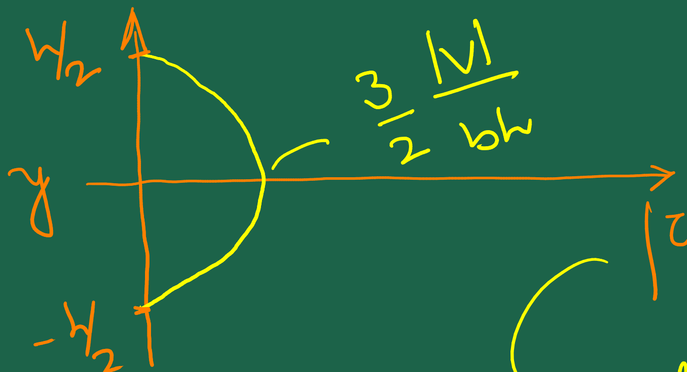
max → $\tau_{xy}\Big|_{y=0} = -\frac{6V}{bh^3}\,\frac{h^2}{4} = \frac{-3V}{2bh} = -\frac{3}{2}\left(\frac{V}{bh}\right)$

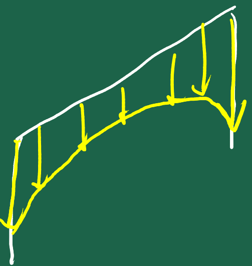
$\frac{V}{bh} : \tau_{avg}$

$\tau_{max} = \frac{3}{2}\,\tau_{avg}$

min → $\tau_{xy}\Big|_{y=\pm\frac{h}{2}} = 0$

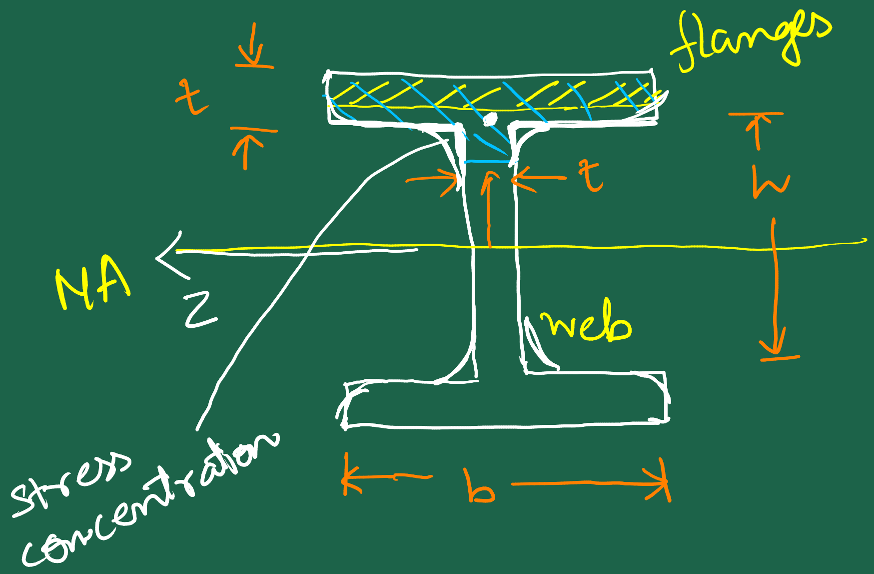


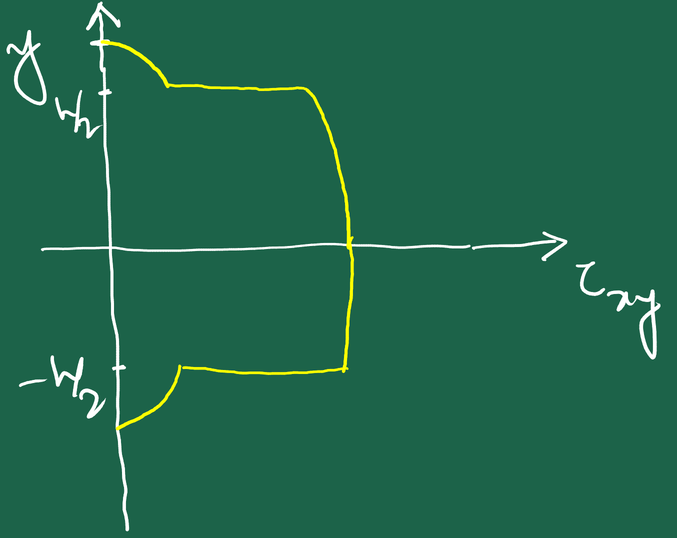


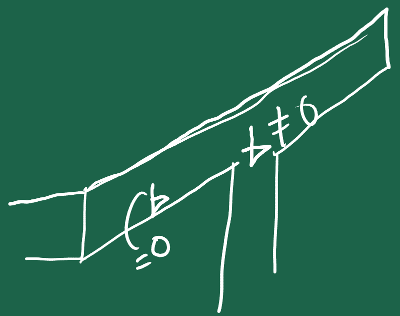


$|\tau_{xy}| = \frac{6|V|}{bh^3}\left(\frac{h^2}{4} - y^2\right)$

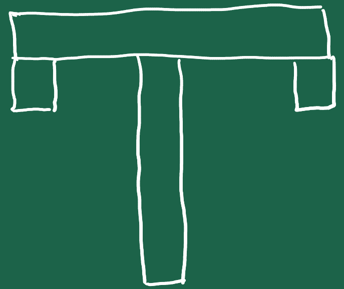
↳ valid along z direction if $\left(\frac{b}{h} < \frac{1}{4}\right)$


flanges
t
t
w
NA
z
web
stress concentration
b


y
w/2
-w/2
τxy


τ=0
τ≠0

Shear flow (along length)

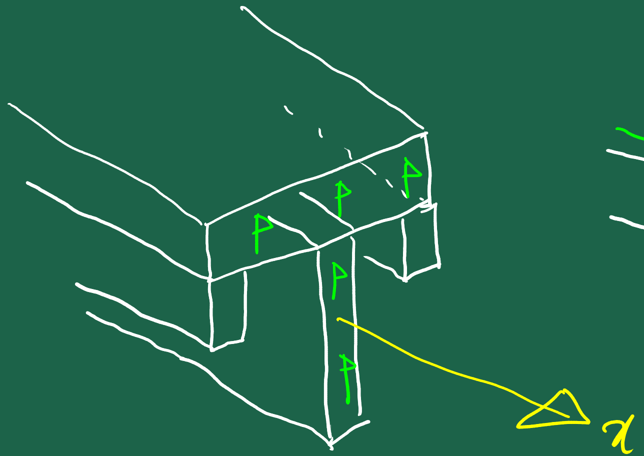


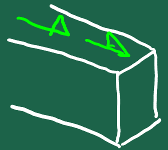
$$\tau_{yx} = -\frac{VQ}{It}$$

$$\Rightarrow \underbrace{(\tau_{yx}\, t)}_{\text{Force per unit length}} = -\frac{VQ}{I}$$

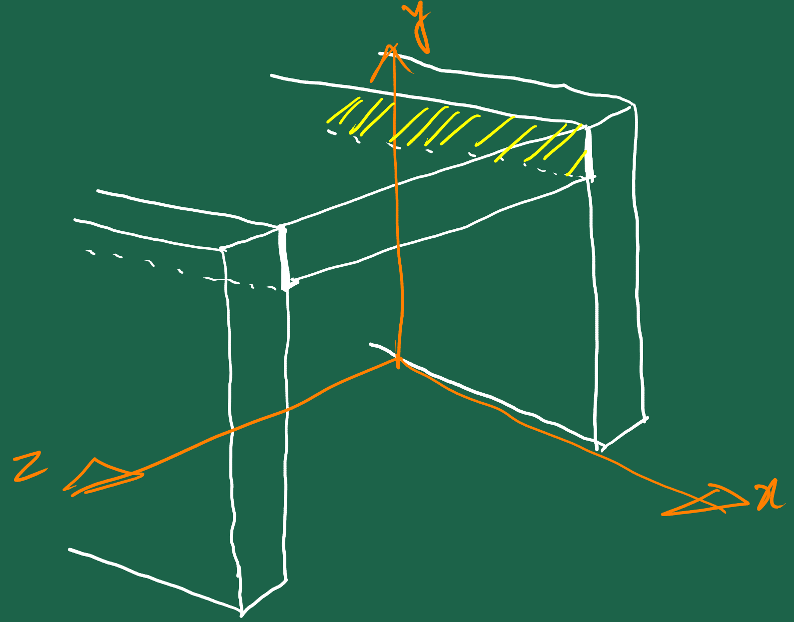
$$\boxed{\text{Force per unit length}} \downarrow$$

shear flow



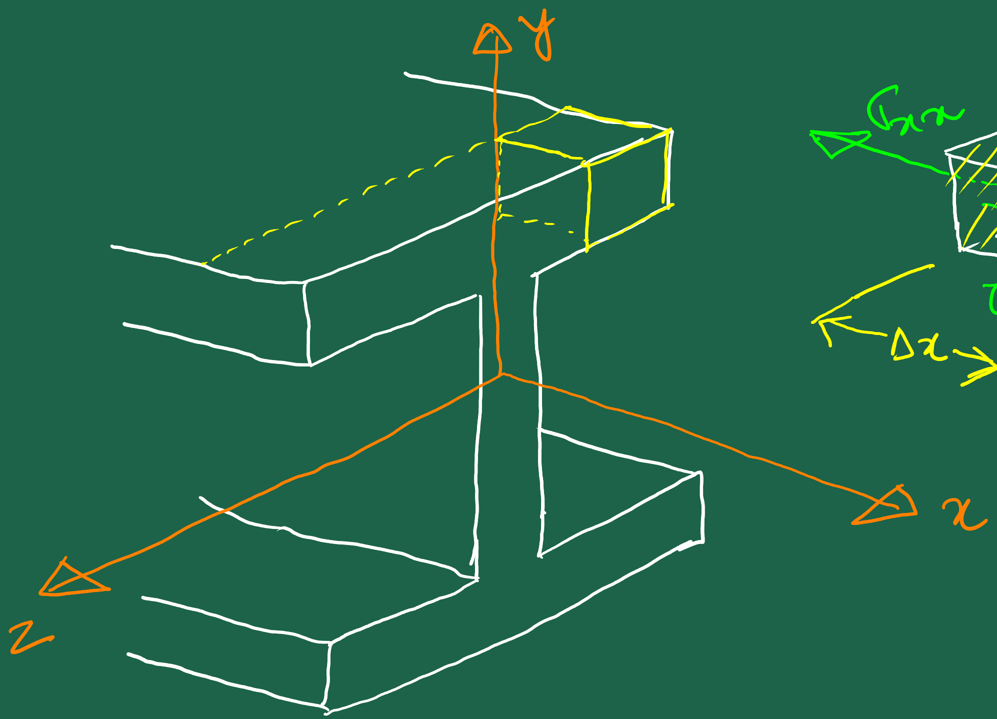


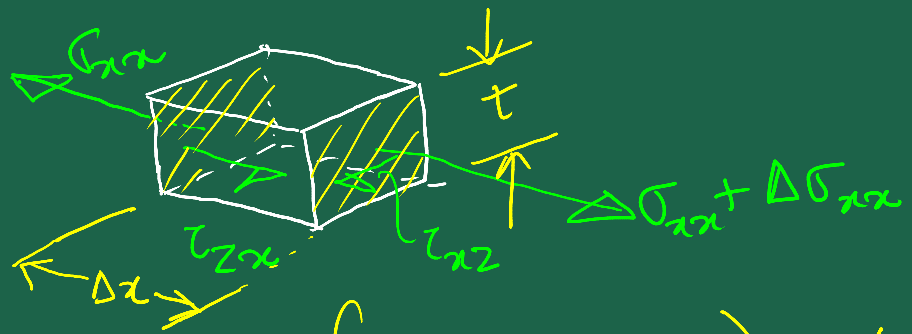
built-up beam



$$\underbrace{(\tau_{zx}\, t)}_{\text{shear flow}} = \frac{VQ}{I}$$

Shear flow distribution (within a c/s)





$$\int_{A'}\left(\sigma_{xx}+\Delta\sigma_{xx}\right)dA' + \tau_{zx}t\,\Delta x = \int_{A'}\sigma_{xx}\,dA'$$

$$\Rightarrow \quad \tau_{zx}t\,\Delta x = -\int_{A'}\Delta\sigma_{xx}\,dA'$$

$$\Rightarrow \quad \tau_{zx}t = -\int_{A'}\frac{\Delta\sigma_{xx}}{\Delta x}dA' \xrightarrow[\Delta x\to 0]{} -\int\frac{d\sigma_{xx}}{dx}dA'$$

EXTREMELY IMPORTANT !

Walls must be thin

$$\Rightarrow \underbrace{\tau_{zx} t}_{\text{Shear flow } q} = -\int_{A'} \frac{d}{dx}\left(-\frac{My}{I}\right) dA'$$

$$= \int_{A'} \frac{dM}{dx} \frac{y}{I} \, dA'$$

$$= \frac{V}{I} \int_{A'} y \, dA'$$

$$= \frac{VQ}{I}$$

Shear flow associated with τ_{xz} is going to be equal to that associated with τ_{zx}, i.e. $\frac{VQ}{I}$