Problem Sheet 4: 2D Elasticity

1. Consider the ramps present in the Nalanda Classroom Complex on the side of Shiru Cafe. Suppose a long queue of students is standing on one such stretch of ramp. Can the ramp be considered to be under plane strain conditions? Justify why or why not.

2. The thin-walled pressure vessel walls discussed in First-Year Mechanics are considered to be in plane stress conditions. Explain why this is an *assumption*, and justify its validity.

3. In the Avenger movies, Captain America's vibranium shield can be thought of to be cut out of a spherical thin-walled pressure vessel. Now, look at this clip from one of the early movies *"Captain America: The First Avenger"*. Referring to the bullet firing scene starting at the 40 second mark, do you think the shield is under plane stress at the times of impact?

4. Consider the following polynomial form for the stress function:

$$\varphi = A_{40}x^4 + A_{31}x^3y + A_{22}x^2y^2 + A_{13}xy^3 + A_{04}y^4$$

   Determine the condition(s) required to satisfy the biharmoic equation.

   [$3A_{40} + A_{22} + 3A_{04} = 0$]

5. Consider the following polynomial form for the stress function:

$$\varphi = A_{50}x^5 + A_{41}x^4y + A_{32}x^3y^2 + A_{23}x^2y^3 + A_{14}xy^4 + A_{05}y^5$$

   Determine the condition(s) required to satisfy the biharmoic equation.

   [$5A_{50} + A_{32} + A_{14} = 0$; $A_{41} + A_{23} + 5A_{05} = 0$]

6. An annular region lies between $r = R_1$ and $r = R_2$. The inner and the outer periphery are loaded by uniform pressure of magnitude p_1 and p_2, respectively. Considering either plane strain or plane stress condition together with the assumption of axisymmetry, the relevant stress equilibrium equation is

$$\frac{\mathrm{d}\sigma_{rr}}{\mathrm{d}r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0.$$

   Again due to axisymmetry, we have $\varepsilon_{rr} = \dfrac{\mathrm{d}u}{\mathrm{d}r}$ and $\varepsilon_{\theta\theta} = \dfrac{u}{r}$, where $u \equiv u(r)$ is the diplacement field in the radial direction.

   (a) Considering plane strain conditions what is the change in the radial thickness? And, what is σ_{zz}?

(b) Considering plane stress conditions what is the change in the radial thickness? And, what is the change in the axial thickness?

7. Consider a cantilever beam of rectangular cross-section (width: b and height: $2h$) is loaded by a vertically downward tip loading of magnitude P.

(a) A first year Mechanics student decides that the shear stress value over the face of any cut-section (perpendicular to the longitudinal axis of the beam) can be simply found by dividing the shear force (at that cut-section) by the area of the cross-section. Explain to him/her why this would be wrong.

(b) Using the stress equilibrium equations, and the first year formulae $\sigma_{xx} = \dfrac{My}{I}$ and $V = \dfrac{\mathrm{d}M}{\mathrm{d}x}$, determine the distribution of the shear stress on the cross-section? Here, σ_{xx} is the bending stress, M is the bending moment, I is the second moment of area (or, moment of inertia), and V is the shear force.

(c) Verify that the following Airy stress function generates the same shear stress as found in part (b): $\varphi = \dfrac{3P}{4h}\left(xy - \dfrac{xy^3}{3h^2}\right)$.

(d) Explain the significance of the non-zero σ_{xx} that the Airy stress function given in part (c) generates.

(e) Suppose there is an axial load T applied in addition to the load P. Then what change should be made to the Airy stress function so as to generate a proper stress field.