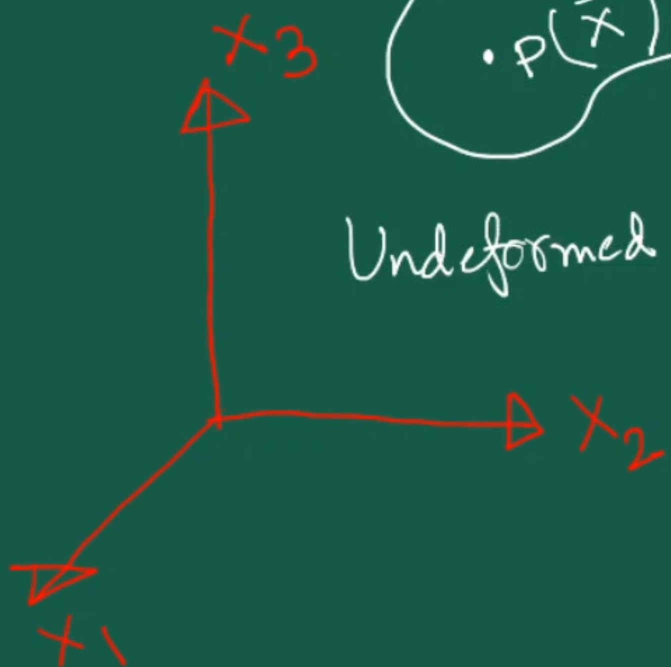
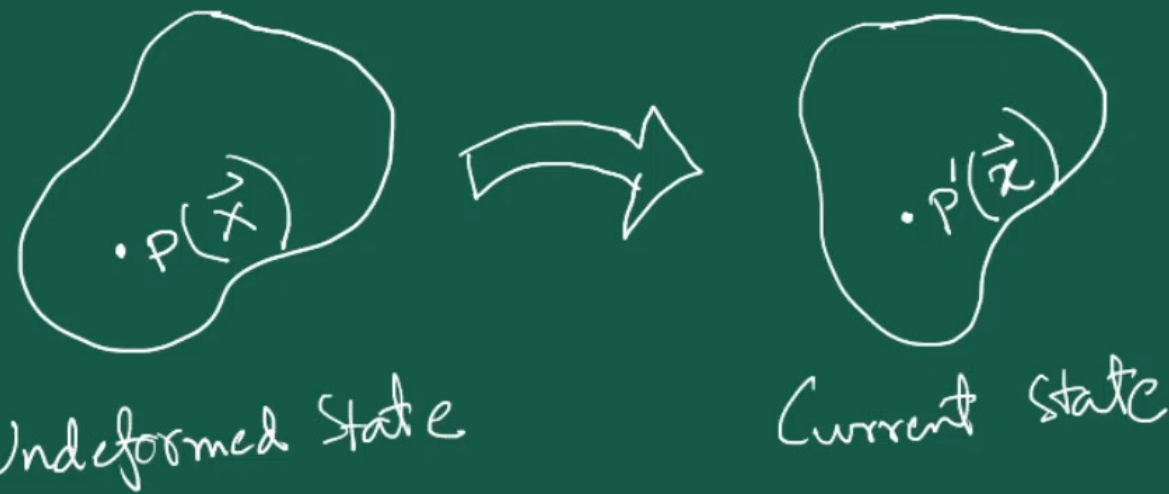
KINEMATICS

Deformation and Displacement





$$\left.\begin{aligned} x_1 &\equiv x_1(X_1, X_2, X_3, t) \\ x_2 &\equiv x_2(X_1, X_2, X_3, t) \\ x_3 &\equiv x_3(X_1, X_2, X_3, t) \end{aligned}\right\} \rightarrow \begin{aligned} \vec{x} &\equiv \vec{x}(\vec{X}, t) \\ x_i &\equiv x_i(X_j, t) \end{aligned}$$

Deformation and Displacement . . . contd.

Definition of Displacement

Displacement vector: $\vec{u}$

$$\vec{u} := \vec{x}(\vec{X}, t) - \vec{X}$$

The dependence on t will be implicitly understood from now on

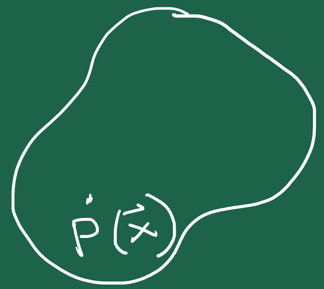
$$\vec{u} = \vec{x}(\vec{X}) - \vec{X}$$

$$u_1 = x_1(X_1, X_2, X_3) - X_1$$

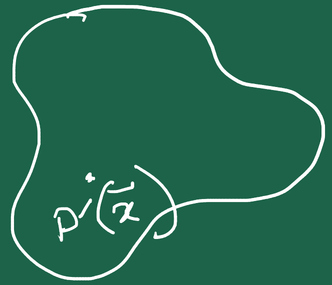
$$u_2 = x_2(X_1, X_2, X_3) - X_2$$

$$u_3 = x_3(X_1, X_2, X_3) - X_3$$

$$\vec{u}(\vec{X}) = \vec{x}(\vec{X}) - \vec{X}$$

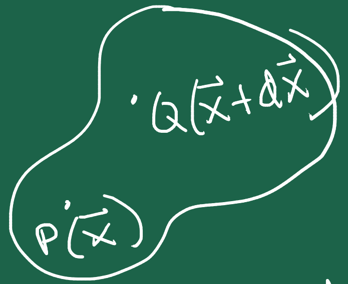


Undeformed State

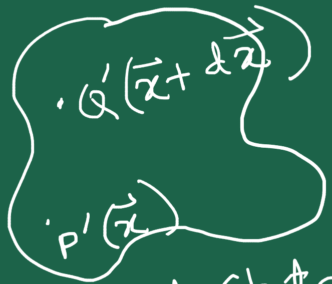


Current State

Quantification of Deformation



Undeformed State



Current State

$$\vec{u}(\vec{X}+d\vec{X}) = (\vec{x}+d\vec{x}) - (\vec{X}+d\vec{X})$$

But, $\vec{u}(\vec{X}) = \vec{x} - \vec{X}$

$$\vec{u}(\vec{X}+d\vec{X}) = \vec{u}(\vec{X}) + d\vec{x} - d\vec{X}$$

$$\Rightarrow d\vec{x} = d\vec{X} + \vec{u}(\vec{X}+d\vec{X}) - \vec{u}(\vec{X})$$

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Quantification of Deformation ... contd.

$$d\vec{x} = d\vec{X} + \vec{u}(\vec{X} + d\vec{X}) - \vec{u}(\vec{X})$$

$$dx_1 = dX_1 + u_1(X_1 + dX_1, X_2 + dX_2, X_3 + dX_3) - u_1(X_1, X_2, X_3)$$

$$dx_2 = dX_2 + u_2(X_1 + dX_1, X_2 + dX_2, X_3 + dX_3) - u_2(X_1, X_2, X_3)$$

$$dx_3 = dX_3 + u_3(X_1 + dX_1, X_2 + dX_2, X_3 + dX_3) - u_3(X_1, X_2, X_3)$$

Brief refresher on Taylor series expansion

$$f(x + dx) = f(x) + \frac{dx}{\lfloor 1} f'(x) + \frac{(dx)^2}{\lfloor 2} f''(x) + \cdots$$

$$f(x + dx, y + dy) = f(x, y) + \frac{dx}{\lfloor 1} \frac{\partial f}{\partial x} + \frac{dy}{\lfloor 1} \frac{\partial f}{\partial y} + \frac{(dx)^2}{\lfloor 2} \frac{\partial^2 f}{\partial x^2} + \frac{(dy)^2}{\lfloor 2} \frac{\partial^2 f}{\partial y^2} + 2 \frac{dx\, dy}{\lfloor 2} \frac{\partial^2 f}{\partial x \partial y} + \cdots$$

$$f(x+dx,\, y+dy,\, z+dz) = f(x,y,z) + \frac{dx}{1!}\frac{\partial f}{\partial x} + \frac{dy}{1!}\frac{\partial f}{\partial y} + \frac{dz}{1!}\frac{\partial f}{\partial z} + \dots$$

$$dx_1 = dX_1 + \cancel{u_1(X_1,X_2,X_3)} + dX_1\frac{\partial u_1}{\partial X_1} + dX_2\frac{\partial u_1}{\partial X_2} + dX_3\frac{\partial u_1}{\partial X_3} + \dots - \cancel{u_1(X_1,X_2,X_3)}$$

$$dx_1 \approx dX_1 + \left(dX_1\frac{\partial}{\partial X_1} + dX_2\frac{\partial}{\partial X_2} + dX_3\frac{\partial}{\partial X_3}\right)u_1$$

$$dx_2 \approx dX_2 + \left(dX_1\frac{\partial}{\partial X_1} + dX_2\frac{\partial}{\partial X_2} + dX_3\frac{\partial}{\partial X_3}\right)u_2$$

$$dx_3 \approx dX_3 + \left(dX_1\frac{\partial}{\partial X_1} + dX_2\frac{\partial}{\partial X_2} + dX_3\frac{\partial}{\partial X_3}\right)u_3$$

$$dX_1\frac{\partial}{\partial X_1} + dX_2\frac{\partial}{\partial X_2} + dX_3\frac{\partial}{\partial X_3} \equiv d\vec{X}\cdot\nabla$$

$$d\vec{X} \equiv (dX_1,\, dX_2,\, dX_3)$$

$$\nabla \equiv \left(\frac{\partial}{\partial X_1},\, \frac{\partial}{\partial X_2},\, \frac{\partial}{\partial X_3}\right)$$

$$\left.\begin{aligned} dx_1 &= dX_1 + d\vec{X}\cdot\nabla\, u_1 \\ dx_2 &= dX_2 + d\vec{X}\cdot\nabla\, u_2 \\ dx_3 &= dX_3 + d\vec{X}\cdot\nabla\, u_3 \end{aligned}\right\} \rightarrow \boxed{d\vec{x} = d\vec{X} + (d\vec{X}\cdot\nabla)\,\vec{u}}$$

KINEMATICS ... CONTD.

STRAIN

Definition of Engineering Strain (ε_E)

$$\varepsilon_E := \frac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|}$$

Because it is difficult to deal with sq. roots, consider the squares of the magnitudes

$$|d\vec{x}|^2 = d\vec{x} \cdot d\vec{x}$$

$$|d\vec{X}|^2 = d\vec{X} \cdot d\vec{X}$$

$$|d\vec{x}|^2 = d\vec{x} \cdot d\vec{x} = (dx_1)^2 + (dx_2)^2 + (dx_3)^2$$

$$= \left(dX_1 + dX_1\frac{\partial u_1}{\partial X_1} + dX_2\frac{\partial u_1}{\partial X_2} + dX_3\frac{\partial u_1}{\partial X_3}\right)^2$$

$$+ \left(dX_2 + dX_1\frac{\partial u_2}{\partial X_1} + dX_2\frac{\partial u_2}{\partial X_2} + dX_3\frac{\partial u_2}{\partial X_3}\right)^2$$

$$+ \left(dX_3 + dX_1\frac{\partial u_3}{\partial X_1} + dX_2\frac{\partial u_3}{\partial X_2} + dX_3\frac{\partial u_3}{\partial X_3}\right)^2$$

$$|d\vec{x}|^2 = \underbrace{(dx_1)^2 + (dx_2)^2 + (dx_3)^2}_{|d\vec{x}|^2} + 2\Big[(dx_1)^2 E_{11} + (dx_2)^2 E_{22} + (dx_3)^2 E_{33} + 2dx_1 dx_2 E_{12} + 2dx_2 dx_3 E_{23} + 2dx_3 dx_1 E_{31}\Big]$$

where

$$E_{11} = \frac{\partial u_1}{\partial x_1} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_1}\right)^2 + \left(\frac{\partial u_3}{\partial x_1}\right)^2\right]$$

$$E_{22} = \frac{\partial u_2}{\partial x_2} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial x_2}\right)^2 + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_3}{\partial x_2}\right)^2\right]$$

$$E_{33} = \frac{\partial u_3}{\partial x_3} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2 + \left(\frac{\partial u_3}{\partial x_3}\right)^2\right]$$

$$E_{12} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_1}\frac{\partial u_3}{\partial x_2}\right]$$

$$E_{23} = \frac{1}{2}\left[\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} + \frac{\partial u_1}{\partial x_2}\frac{\partial u_1}{\partial x_3} + \frac{\partial u_2}{\partial x_2}\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\frac{\partial u_3}{\partial x_3}\right]$$

$$E_{31} = \frac{1}{2}\left[\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} + \frac{\partial u_1}{\partial x_3}\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_3}\frac{\partial u_2}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\frac{\partial u_3}{\partial x_1}\right]$$

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STRAIN . . . CONTD.

$$|d\vec{x}|^2 = |d\vec{X}|^2 + 2\, dX_i\, dX_j\, E_{ij}$$

where $$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right)$$

[Check for yourself by putting diff. values of i & j]

$E_{ij} \equiv \underset{\approx}{E}$: Finite Strain Tensor

$E_{ij} \equiv \underset{\approx}{E}$: 2nd order tensor

$E_{ij} = E_{ji}$: $\underset{\approx}{E}$ is symmetric

$E_{ij} \equiv \underset{\approx}{E}$ can be represented as a 3×3 matrix

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12} & E_{22} & E_{23} \\ E_{13} & E_{23} & E_{33} \end{bmatrix}$$

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Very Important: If $\left|\frac{\partial u_i}{\partial x_j}\right| \ll 1$, $E_{ij} \approx \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) =: \varepsilon_{ij}$

$\varepsilon_{ij} \equiv \underset{\approx}{\varepsilon}$: Infinitesimal Strain Tensor

$\varepsilon_{ij} = \varepsilon_{ji}$: Symmetric

$x_1 \to x$, $x_2 \to y$, $x_3 \to z$

$u_1 \to u$, $u_2 \to v$, $u_3 \to w$

$$\left.\begin{aligned} &\varepsilon_{11} \to \varepsilon_{xx} = \frac{\partial u}{\partial x}; \quad \varepsilon_{22} \to \varepsilon_{yy} = \frac{\partial v}{\partial y}; \quad \varepsilon_{33} \to \varepsilon_{zz} = \frac{\partial w}{\partial z} \end{aligned}\right\} \text{Normal strain}$$

$$\varepsilon_{12} \to \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \varepsilon_{yx} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\gamma_{yx}$$

$$\varepsilon_{23} \to \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \varepsilon_{zy} = \frac{1}{2}\gamma_{zy} = \frac{1}{2}\gamma_{yz}$$

$$\left.\varepsilon_{31} \to \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \varepsilon_{xz}\right\} \text{Shear Strain}$$

$$= \frac{1}{2}\gamma_{xz} = \frac{1}{2}\gamma_{zx}$$

The expressions of $\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{zx}$ in terms of the derivatives of the displacement vector components are referred to as <u>STRAIN-DISPLACEMENT RELATIONS</u>.

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) & \dfrac{\partial w}{\partial z} \end{bmatrix}$$

$$|d\vec{x}|^2 = |d\vec{X}|^2 + 2\,dX_i\,dX_j\,E_{ij} \xrightarrow[\left|\frac{\partial u_i}{\partial x_j}\right| << 1]{\text{If}} |d\vec{x}|^2 = |d\vec{X}|^2 + 2\,dX_i\,dX_j\,\varepsilon_{ij}$$

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STRAIN ... CONTD.

$$|d\vec{x}|^2 = |d\vec{X}|^2 + 2dX_i\,dX_j\,\varepsilon_{ij}$$

$$\left|\vec{a} = |\vec{a}|\,\hat{e}_a\right.$$

$dX_i = |d\vec{X}|\,N_i$, where N_i is the unit vector along dX_i

$dX_j = |d\vec{X}|\,N_j$, " N_j " " " " " dX_j

$$|d\vec{x}|^2 = |d\vec{X}|^2 + 2|d\vec{X}|^2 N_i N_j \varepsilon_{ij}$$

Note : $N_i N_j \varepsilon_{ij} \equiv \hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N} \equiv \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$

$$|d\vec{x}|^2 = |d\vec{X}|^2 \left(1 + 2\hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}\right)$$

$$\frac{|d\vec{x}|^2}{|d\vec{X}|^2} = 1 + 2\hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}$$

But, $\varepsilon_E = \dfrac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|} \Rightarrow \dfrac{|d\vec{x}|}{|d\vec{X}|} = 1 + \varepsilon_E$

$$\therefore \quad (1+\varepsilon_E)^2 = 1 + 2\hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}$$

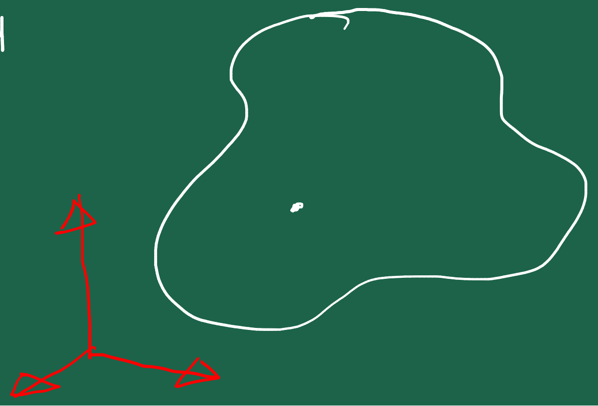
$$\Rightarrow 1 + 2\varepsilon_E + \varepsilon_E^2 = 1 + 2\hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}$$

small deformations $\rightarrow \varepsilon_E \ll 1 \Rightarrow \varepsilon_E^2$ can be neglected

But, overall $\left|\dfrac{\partial u_i}{\partial x_j}\right| \ll 1$

$$\therefore \quad 1 + 2\varepsilon_E \approx 1 + 2\hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}$$

$$\Rightarrow \boxed{\varepsilon_E = \hat{N}^T \cdot \underset{\approx}{\varepsilon} \cdot \hat{N}}$$



KINEMATICS ... CONTD.

Rigid body rotation

$$d\vec{x} = d\vec{X} + d\vec{X}\cdot\nabla\vec{u}$$

$$\updownarrow$$

$$dx_i = dX_i + dX_j \frac{\partial u_i}{\partial X_j}$$

$$= dX_i + dX_j \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)$$

$$+ dX_j \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} - \frac{\partial u_j}{\partial X_i}\right)$$

$$dX_j \frac{\partial}{\partial X_j} \equiv dX_1 \frac{\partial}{\partial X_1} + dX_2 \frac{\partial}{\partial X_2} + dX_3 \frac{\partial}{\partial X_3}$$

$$\equiv d\vec{X}\cdot\nabla$$

$\frac{\partial u_i}{\partial X_j} \leftrightarrow$ 3x3 matrix

$\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) \rightarrow$ Strain tensor ; $\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} - \frac{\partial u_j}{\partial X_i}\right) \rightarrow$ Rotation tensor

Symmetric

Anti-Symmetric

Rotation tensor $\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) =: \Omega_{ij}$

$$dx_i = dX_i + dX_j \varepsilon_{ij} + dX_j \Omega_{ij}$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \equiv \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ -\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) & -\frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$dx_i = dX_i + dX_j\,\varepsilon_{ij} + dX_j\,\Omega_{ij}$$

Consider the special case when $\varepsilon_{ij} = 0$

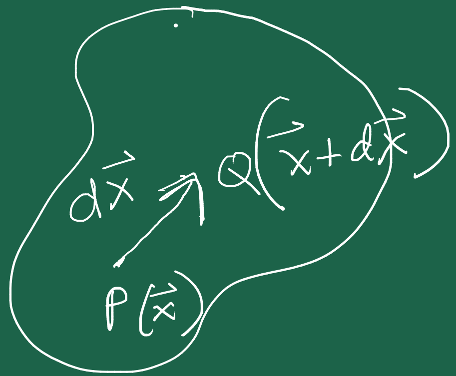
$$dx_i = dX_i + dX_j\,\Omega_{ij}$$

$$d\vec{x} = d\vec{X} + \vec{\omega} \times d\vec{X}$$

$$\vec{\omega} \equiv \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} ; \quad d\vec{X} \equiv \begin{bmatrix} dX_1 \\ dX_2 \\ dX_3 \end{bmatrix}$$

The vector $\vec{\omega} \times d\vec{X}$ must be perpendicular to $d\vec{X}$

If $|\vec{\omega}|$ and $|d\vec{X}|$ are small, then $|\vec{\omega} \times d\vec{X}|$ must also be very small



Then, we can conclude $d\vec{x} \approx d\vec{X}$

KINEMATICS ... CONTD

Principal Strains

$$\varepsilon_E = \hat{N}^T \cdot \underset{\sim}{\varepsilon} \cdot \hat{N} \equiv N_i N_j \varepsilon_{ij}$$

Constrained extremization problem; constraint being $\hat{N}^T \cdot \hat{N} = 1$ or $N_k N_k = 0$

$$\frac{\partial \varepsilon_E}{\partial N_K} = 0 \quad \times$$ ← Constraint is not imposed

$$L = \varepsilon_E + \lambda(1 - N_K N_K)$$ ← Constraint included

$$\frac{\partial L}{\partial N_K} = 0$$

$$\frac{\partial}{\partial N_K}\left(\varepsilon_E + \lambda(1 - N_K N_K)\right) = 0$$

$$\Rightarrow \frac{\partial}{\partial N_K}\left(N_i N_j \varepsilon_{ij}\right) - \lambda \frac{\partial}{\partial N_K}\left(N_K N_K\right) = 0$$

$$\Rightarrow \varepsilon_{ij}\left(\frac{\partial N_i}{\partial N_K} N_j + N_i \frac{\partial N_j}{\partial N_K}\right) - 2\lambda N_K = 0$$

$$\Rightarrow \varepsilon_{ij}\left(N_j \frac{\partial}{\partial N_K}\left(N_K \delta_{iK}\right) + N_i \frac{\partial}{\partial N_K}\left(N_K \delta_{jK}\right)\right) - 2\lambda N_K = 0$$

$$\Rightarrow \varepsilon_{ij}\left(N_j \delta_{iK} + N_i \delta_{jK}\right) - 2\lambda N_K = 0$$

$$\Rightarrow \varepsilon_{Kj} N_j + \varepsilon_{iK} N_i - 2\lambda N_K = 0$$

$$\Rightarrow \varepsilon_{jK} N_j + \varepsilon_{iK} N_i - 2\lambda N_K = 0 \qquad \left[\text{Since } \varepsilon_{Kj} = \varepsilon_{jK}\right]$$

$$\Rightarrow \varepsilon_{pk} N_p + \varepsilon_{pk} N_p - 2\lambda N_k = 0$$

[Replacing the dummy indices j & i by p]

$$\Rightarrow 2\varepsilon_{pk} N_p - 2\lambda N_k = 0$$

$$\Rightarrow \varepsilon_{pk} N_p - \lambda N_k = 0$$

$$\Rightarrow \varepsilon_{pk} N_p - \lambda \delta_{pk} N_p = 0$$

$$\Rightarrow \left(\varepsilon_{pk} - \lambda \delta_{pk}\right) N_p = 0$$

$$\Updownarrow$$

$$\left(\left[\underset{\approx}{\varepsilon}\right] - \lambda\left[\underset{\approx}{I}\right]\right)\left[\hat{N}\right] = 0 \Leftrightarrow \begin{bmatrix} \varepsilon_{11} - \lambda & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} - \lambda & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} - \lambda \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = 0$$

For non-trivial solutions, we must have

$$\begin{vmatrix} \varepsilon_{11}-\lambda & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22}-\lambda & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33}-\lambda \end{vmatrix} = 0$$

eigenvalues

The roots of the above eqn, i.e. 3 values of λ will be the extreme values of the engineering strain, and corresponding to each such extreme value we can find a set of $\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$ ← eigenvectors (3 such sets corresponding to each λ)

KINEMATICS ... CONTD.

STRAIN COMPATIBILITY

What if we are given the strain field (ε_{ij}) and we are asked to find the displacement components?

$$\frac{\partial u}{\partial x} = \varepsilon_{xx} \quad , \quad \frac{\partial v}{\partial y} = \varepsilon_{yy} \quad , \quad \frac{\partial w}{\partial z} = \varepsilon_{zz} \quad ,$$

$$\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \varepsilon_{xy} \, , \quad \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \varepsilon_{yz} \, , \quad \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \varepsilon_{zx}$$

Illustration of an incompatible strain field

Given: $\varepsilon_{xx} = y^2 \quad , \quad \varepsilon_{xy} = 0 \quad , \quad \varepsilon_{yy} = 0$

Find: u, v, w

$$\frac{\partial u}{\partial x} = \varepsilon_{xx} = y^2$$

$$\Rightarrow \quad u = xy^2 + f(y)$$

$$\frac{\partial v}{\partial y} = \varepsilon_{yy} = 0$$

$$\Rightarrow \quad v = g(x)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(2xy + \frac{df}{dy} + \frac{dg}{dx}\right)$$

But ε_{xy} is given to be 0.

$$\therefore \frac{1}{2}\left(2xy + \frac{df}{dy} + \frac{dg}{dx}\right) = 0 \Rightarrow \underbrace{2xy + \frac{df}{dy} + \frac{dg}{dx} = 0}$$

Can <u>never</u> be satisfied → Problem is with the $2xy$ part

If $2xy$ were absent

$$\frac{df}{dy} + \frac{dg}{dx} = 0$$

$$\Rightarrow \frac{df}{dy} = -\frac{dg}{dx} = K$$