# Constitutive Relations

Balance of mass: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$ | $\rho \to 1$, $\vec{v}$ OR $\vec{u} \to 3$ | $\vec{v} := \frac{D\vec{u}}{Dt}$

1 eqn

Balance of lin. mom. $\nabla \cdot \underset{\approx}{\sigma} + \rho \vec{b} = 0$ | $\underset{\approx}{\sigma} \to 6$ | $\underset{\approx}{\sigma} = \underset{\approx}{\sigma}^T$

3 eqns.

4 eqns BUT 10 Unknowns

$\boxed{\text{Linear, Elastic, Isotropic Materials}}$

Constitutive Relations

$$\left.\begin{aligned} \sigma_{ij} &= \lambda \varepsilon_{kk} \delta_{ij} + 2G\, \varepsilon_{ij} \\ \underset{\sim}{\sigma} &= \lambda \left(\operatorname{tr} \underset{\sim}{\varepsilon}\right) \underset{\sim}{I} + 2G\, \underset{\sim}{\varepsilon} \end{aligned}\right\} \rightarrow 6$$

Material constants

$\delta_{ii} = 3$

$\hookrightarrow \delta_{11} + \delta_{22} + \delta_{33}$

$\delta_{11} = 1$

$\delta_{22} = 1$

$\delta_{33} = 1$

6 eqns $\longrightarrow$ 6 more unknowns in the form of $\underset{\sim}{\varepsilon}$ components

But not to worry! Because we have $\underline{\underline{6}}$ STRAIN-DISPLACEMENT RELNS.

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \longrightarrow 6 \text{ more eqns}$$

In 1st year Mechanics

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

$$\varepsilon_{xy} = \frac{1}{2G}\sigma_{xy}$$

$$\varepsilon_{yz} = \frac{1}{2G}\sigma_{yz}$$

$$\varepsilon_{zx} = \frac{1}{2G}\sigma_{zx}$$

$x \to 1$

$y \to 2$

$z \to 3$