## MECHANICS (ME10001)

Tutorial 9: Torsion

1. An aluminum tube of 40 mm inner diameter and 50 mm outer diameter is rigidly fixed to an aluminum circular flange. The assembly is fixed to a wall by two steel bolts of 10 mm diameter. The allowable shear stress and bearing stress of steel are 45 MPa and 90 MPa respectively. For aluminum, the allowable shear stress and bearing stress are 30 MPa and 60 MPa , respectively. (a) Determine the maximum torsion $\mathbf{T}$ the system can withstand and
 (b) the angle of twist of the tube for the $\mathbf{T}$ computed in part (a). $G_{\text {Aluminum }}=28 \mathrm{GPa}$. [(a) $282.74 \mathrm{~N}-\mathrm{m}$ (b) $0.32^{\circ}$ ]
2. What must be the length of a 5 mm diameter aluminum wire so that it could be twisted through one complete revolution without exceeding a shear stress of 42 MPa . Given $G_{\text {Aluminum }}=27 \mathrm{GPa}$. [10.1 m]
3. Knowing that each portion of the shaft $A D$ consists of a solid circular rod, determine (a) the portion of the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress. [(a) Shaft $B C$ (b) 57.3 MPa$]$

4. A solid aluminum shaft of 50 mm diameter is to be replaced by a hollow steel shaft of 50 mm outside diameter. Both the shafts are 1000 mm in length and should undergo same angle of twist by the application of $200 \mathrm{~N}-\mathrm{m}$ torque at the ends. Shear modulus of elasticity for steel is three times that of the aluminum. Find the dimensions of the steel shaft. [ 45.18 mm ]
5. Two gears are attached to two 50 mm diameter steel shafts. The gear at $B$ has a 200 mm pitch diameter; the gear at $C$, a 400 mm diameter pitch diameter. Through what angle will end $A$ turn if a torque of $560 \mathrm{~N}-\mathrm{m}$ is applied at $A$ and end $D$ of the second shaft is prevented from rotating? Given $\mathrm{G}=$ 84 MPa [0.195 rad]
6. The preliminary design of a large shaft connecting a motor to a generator calls for the use of hollow shaft with inner and outer diameter of 100 mm and 150 mm , respectively. Knowing that the allowable shearing stress is 85 MPa , determine the maximum torque which may be transmitted (a) by the shaft as designed, (b) by a solid shaft of the same weight, (c) by a hollow shaft of same weight and of 200 mm outside diameter. [(a) $45.2 \mathrm{kN}-\mathrm{m}$ (b) $23.3 \mathrm{kN}-\mathrm{m}$ (c) $70.4 \mathrm{kN}-\mathrm{m}]$

7. The torque shown are exerted on pulleys $B, C$ and $D$. Knowing that the entire shaft is made of steel ( $G$ $=27 \mathrm{GPa}$ ), determine the angle of twist between (a) $C$ and $B$, (b) $D$ and $B$. [(a) $8.54^{\circ}$ (b) $2.11^{\circ}$ ]

8. Two solid steel shafts, each of 30 mm diameter, are connected by gear shown. Knowing that $G=77 \mathrm{GPa}$, determine the angle through which end $A$ rotates when a $200 \mathrm{~N}-\mathrm{m}$ torque $\mathbf{T}$ is applied at $A$. [ $\phi_{A}=3.79^{\circ}$ ]

9. The design specifications of 1.2 m long solid circular transmission shaft require that the angle of twist of the shaft not exceed $4^{\circ}$ when a torque of $680 \mathrm{~N}-\mathrm{m}$ is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 83 MPa and a modulus of rigidity of 77 GPa . [d $=35.2 \mathrm{~mm}$ ]
10. Two 5 mm thick cylinders AC and BD are glued together at the overlapping portion BC. (a) Determine the maximum torque the system can withstand, without exceeding the shear stress limit of 5 MPa at the glued joint. (b) Assuming no slippage at the glued joint, at this torque, what would be the angle of twist of end A with respect to $\mathrm{B} ? \mathrm{G}=80 \mathrm{GPa}$ for the cylinder materials. Dimensions are in mm .
 [(a) $\mathbf{T}=785.4 \mathrm{~N}-\mathrm{m}$ (b) $\phi_{A B}=0.256^{\circ}$ ]
11. The composite shaft shown is to be twisted by applying the torques shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 39 GPa for the brass, determine the largest angle of twist of end $B$ relative to end $A$ if the following allowable stresses are not to be exceeded $\tau_{\text {steel }}=100 \mathrm{MPa}$ and $\tau_{\text {brass }}=55 \mathrm{MPa} .\left[\phi_{\text {all }}=7.27^{\circ}\right]$

12. A 120 mm diameter solid steel shaft transmits 400 kW at 2 Hz . (a) Determine the maximum shear stress, (b) what would be the required shaft diameter to operate at 4 Hz . At the same maximum stress? [(a) 93.72 MPa (b) 95.24 mm ]
