Assignment Sheet in Fluid Mechanics
Dimensional Analysis and Similarity

1. The velocity v of a capillary wave on the surface of a liquid depends on the wave length λ, the density ρ, and the surface tension γ of the liquid. Determine the relation between v and these parameters.

   $[v\,(\rho\lambda/\gamma)^{1/2} = \text{constant}]$

2. The thickness δ of the boundary layer for a fluid passing over a flat plate depends on the distance x from the leading edge of the plate, the free-stream velocity U of the flow, the density ρ, and the dynamic viscosity of the fluid. Determine the relation between δ and these parameters. Think carefully about how this result compares with what you know from the chapter on laminar boundary layer?

   $$\left[\frac{\delta}{x} = f\left(\frac{\rho U x}{\mu}\right)\right]$$

3. Flow through an open-channel (not closed at the top) is determined primarily by inertia forces and gravity. The motion of water waves in such a channel is to be studied in a laboratory using a model one-twelfth the size of the channel. Determine the time for a wave in the channel to travel 10 m if it takes 15 seconds for the wave to travel this distance in the model. [4.33 s]

4. A model of a plane is built to a scale of 1/15 and is tested in a wind tunnel. If the plane is designed to travel at 800 km/h at an altitude of 5 km, determine the required density of the air in the wind tunnel so that the Reynolds and Mach numbers are the same. Assume the temperature is the same in both cases and the speed of sound in air at this temperature is 340 m/s. Take viscosity of air to be the same in both cases too (because temperature is assumed to be the same). [11.0 kg/m^3]

5. Imagine you are a young engineer assigned to work with the great scientific officer Mr Spock aboard the spacecraft *Starship Enterprise* for galactic exploration. This spacecraft is shaped like a disc (which is great for interstellar travel but leads to enormous drag in a planetary atmosphere). Mr Spock has asked you to prepare a simple way to convey the relationship between the drag force (F) on the spacecraft, the speed of the spacecraft (V), the total area (A) of the disc-like spacecraft, the density of the medium (ρ) through which it is flying, and the dynamic viscosity (μ) of the same medium . This is basically to help out Captain James Kirk who is a great commander but who can't be bothered with too many complicated equations.

   (a) Use your knowledge of the Buckingham pi theorem to obtain a functional relationship among the important non-dimensional groups involving the variables mentioned by Spock.

   (b) Following attack by the villain Krall, *Starship Enterprise* is swiftly and dangerously going to crash land into a nearby planet. The test data shows that as long as the speed of descent is below a critical value V_{crit} (for the test), all crew members are safe. Your job is to find out the corresponding speed in the actual condition in terms of V_{crit}. Density of the air outside is twice the density of the test medium.

   Before doing the calculations, you quickly go and consult Mr Spock who points out that at the high speed at which the Starship is descending, a dynamic similarity between the available test data and the real conditions outside the spacecraft can be maintained even without considering the non-dimensional group involving the dynamic viscosity (found in part (a)). He also points out that the ratio of the drag force to the area can be viewed as an effective shear stress which corresponding to the critical condition that you are interested in is the same for test data and actual condition. Take his advice and find the critical speed in the actual crash-landing condition to help Captain Kirk decide on whether to evacuate or not.

6. In one of his most popular songs, the Nobel laureate from 2016 in literature, Bob Dylan asks *"How many years must a mountain exist before it is washed to the sea?"* While such a question might seem to make only poetic sense, it is deeply connected to physical reality and can be studied through systematic methods under the purview of *non-Newtonian* fluid mechanics. In such a flow, the stress depends non-linearly on the velocity gradient. Using the Buckingham Pi theorem and the methods of dimensional analysis express the characteristic stress (σ_0) of such a flow as a function of the velocity scale (V), the length scale (D), and a special non-Newtonian fluid parameter called the flow consistency index (K). Note that K has the units of $\text{Pa} \cdot \text{s}^n$, where n is the flow behaviour index (n is just a non-negative number and is itself dimensionless). In order to present this result to a practising engineer, it is better to express in terms of the familiar looking expression (from Newtonian fluid mechanics) involving a product of an effective dynamic viscosity and V/D (which has the same dimension as that of velocity gradient). For this special case, extract a form of the effective dynamic viscosity in terms of K, V, D, and n. What is the expression for σ_0 when $n = 1$?

7. A constant-property Newtonian fluid is confined between two long concentric circular cylinders. The radii of the inner and the outer cylinders are a and b respectively. The outer cylinder is at rest. The torque per unit length in the axial direction, T', required to turn the inner cylinder at a constant angular speed, is a function of the angular velocity Ω, cylinder radii a, b, and the dynamic viscosity μ of the fluid.

   (a) Determine the number of independent non-dimensional parameters characterizing the problem.

   (b) Use the method of repeating variables to determine the relevant non-dimensional parameters.

   (c) The torque per unit length T', was found to be 1 N in an experiment when Ω was 10 rad/s, $a = 0.5$ m and $b = 0.8$ m. Is it possible to predict the torque per unit length T', using this test data and dimensional analysis, if the fluid between the two coaxial cylinders is the same and

   (i) $\Omega = 10$ rad/s, $a = 1$ m, and $b = 2$ m,

   (ii) $\Omega = 5$ rad/s, $a = 1$ m, and $b = 1.6$ m ?

   If your answer to part (i) or part (ii) is yes, determine the value of T', for each of the cases (i) and (ii). If your answer to part (i) or part (ii) is no, explain why T' cannot be determined using the given test data and dimensional analysis, for each of the cases (i) and (ii).

8. In the 2016 movie *Passengers* a futuristic spacecraft is shown carrying thousands of passengers for migration to another planet. As part of extensive amenities provided for the long travel, there is a swimming pool for the passengers. Due to some malfunction, the artificial gravity aboard the spacecraft gets switched off. The water in the swimming pool is then lifted off into a huge blob of water. This blob is, however, unstable and gets broken up into smaller blobs.

   (a) Starting with dimensional analysis, prescribe a method for estimating the number of the smaller blobs if the stable blob diameter (D) is determined by the density (ρ), the dynamic viscosity (μ), and the surface tension (γ). Note that the length, breadth, and depth of the pool are L, B, and H, respectively. Assume that initially the pool was completely filled.

   (b) Following the mishap which almost resulted in the casualty of a lady who was taking a swim at that time, imagine that engineers have started to investigate the complicated fluid mechanical behaviour that follows the gravity switch-off by first looking into cases with very low gravity.
   As part of this engineering team, you have figured out that the dynamics is governed by ρ, γ, μ, the acceleration due to gravity (g), the characteristic velocity of the water (V), and the diameter (D)

of the water blob. From experience, you already know that one of the non-dimensional numbers that will govern the dynamics is the Reynolds number ($Re = \rho VD/\mu$).

(i) Confirm that besides the Reynolds number, *two other* non-dimensional numbers are required to capture the dynamics, and figure out the non-dimensional number involving the acceleration due to gravity taking ρ, γ, D as repeating variables. This number is referred to as the Bond number (Bo).

(ii) In order to carry out tests on a model set-up, it is required to establish dynamical similarity. Instead of using dimensional analysis, *use a proper combination* of the Reynolds number (Re), the Bond number (Bo), and the non-dimensional number associated with the stable blob diameter found in part (a) to show that the third non-dimensional number is $\pi_3 = \dfrac{\rho V^4}{\gamma g}$ so that the functional relation required for dynamical similarity is $\dfrac{\rho V^4}{\gamma g} = f(Re, Bo)$.

(iii) Explain clearly *why* the above relation helps establish dynamical similarity. *Hint:* Interpret the left-hand-side as the ratio of forces.