Problem Sheet 5: 2D Elasticity - I

Important: For each of the problems, an Airy stress function is mentioned to help you attempt the problem "by hand". However, **this problem sheet is actually meant to give you practice in using SymPy within Jupyter Notebook**. When using SymPy, consider polynomials up to 4th, 5th or 6th degree (including lower degree terms) and use the framework demonstrated in [L43].

1. The Airy stress function required to find the stresses in the cantilever beam (of unit width) as shown in Figure 1 is $\varphi = C_1x^2 + C_2x^2y + C_3y^3 + C_4y^5 + C_5x^2y^3$. Find the values of the constants using the requirements that φ should satisfy the biharmonic equation and the boundary conditions (pointwise boundary conditions on the top and bottom of the beam and integrated boundary conditions on the ends $x = 0$ and $x = L$).

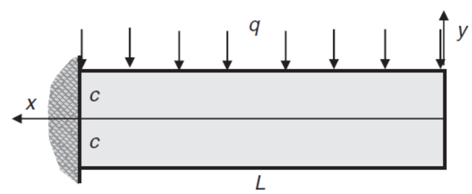

Figure 1

[Answer: $C_1 = -q/4$, $C_2 = -3q/8c$, $C_3 = q/20c$, $C_4 = -q/40c^3$, $C_5 = q/8c^3$]

2. Consider the cantilever beam (of unit width) loaded by uniform shear along its bottom edge as shown in Figure 2. Use appropriate conditions (pointwise boundary conditions on the horizontal surfaces and resultant conditions on the vertical surfaces) to find the stress field given that the required Airy stress function is of the form $\varphi = Axy + By^2 + Cy^3 + Dxy^2 + Exy^3$.

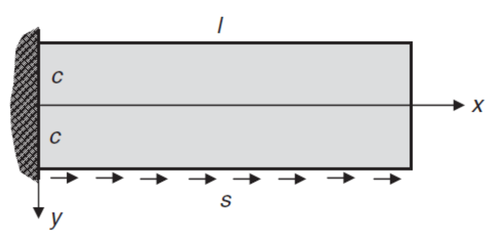

Figure 2

[Answer: $\sigma_{xx} = \frac{s}{2c}\left(1+\frac{3y}{c}\right)(l-x)$, $\sigma_{yy} = 0$, $\sigma_{xy} = -\frac{s}{4}\left(1+\frac{y}{c}\right)\left(1-\frac{3y}{c}\right)$]

3. To solve the problem of a cantilever beam (of unit width) carrying a uniformly varying loading as shown in Figure 3, the following Airy stress function form is proposed: $\varphi = C_1 xy + C_2 \frac{x^3}{6} + C_3 \frac{x^3 y}{6} + C_4 \frac{xy^3}{6} + C_5 \frac{x^3 y^3}{9} + C_6 \frac{xy^5}{20}$. Determine the values of the various constants such that all conditions on the problem are satisfied. Use resultant force boundary conditions at the beam-ends.

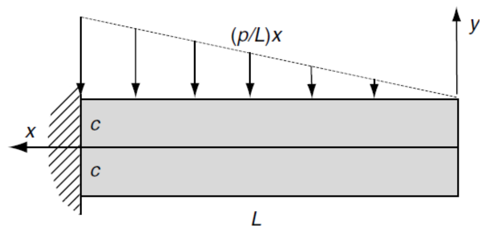

Figure 3

[Answer: $C_1 = -\frac{pc}{40L}$, $C_2 = -\frac{p}{2L}$, $C_3 = -\frac{3p}{4Lc}$, $C_4 = \frac{3p}{10Lc}$, $C_5 = \frac{3p}{8Lc^3}$, $C_6 = -\frac{p}{2Lc^3}$]

4. The cantilever beam (of unit width) shown in Figure 4 is subjected to a distributed shear stress $\tau_o x/l$ on the upper face. The following Airy stress function is given

$$\varphi = c_1 y^2 + c_2 y^3 + c_3 y^4 + c_4 y^5 + c_5 x^2 + c_6 x^2 y + c_7 x^2 y^2 + c_8 x^2 y^3.$$

Determine the constants and find the stress distribution in the beam. Use resultant force boundary conditions at the ends.

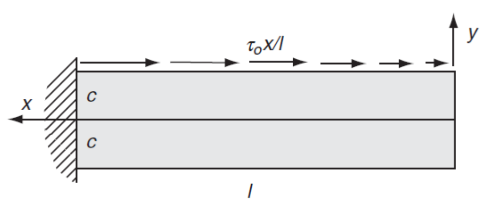

Figure 4

[Answer: $c_1 = \frac{\tau_o c}{12l}$, $c_2 = \frac{\tau_o}{20l}$, $c_3 = -\frac{\tau_o}{24cl}$, $c_4 = -\frac{\tau_o}{40c^2 l}$, $c_5 = -\frac{\tau_o c}{8l}$, $c_6 = -\frac{\tau_o}{8l}$, $c_7 = \frac{\tau_o}{8cl}$, $c_8 = \frac{\tau_o}{8c^2 l}$]