Boundary Conditions

At the end of the previous chapter on Material Behaviour, we had arrived at a closed system of equations i.e.the number of unknowns was equal to the number of equations.

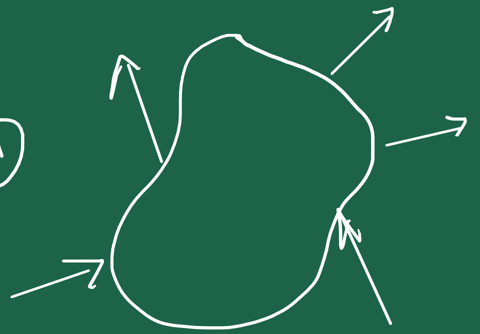
However,these equations can be utilized to solve various specific physical situations only when proper boundary conditions that are specific to those physical situations are mentioned.

For various physical situations, our governing equations remain the same. But it is depending on what boundary conditions for that physical situation that we are going to end up with different solutions.

Boundary conditions can be specified over the surface of the body in different ways:

1. Over the entire surface of the body tractions are specified.
2. Over the entire surface of the body displacements are specified.
3. Over a part of the surface tractions are specified and over the rest of the surface displacements are specified.

1



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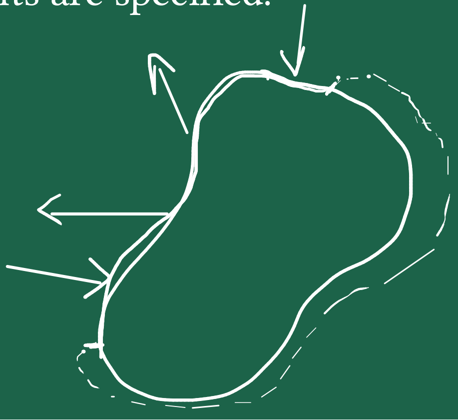


Example :

At $x=a$: $\vec{T} = \underset{\approx}{\sigma}^{T} \cdot \hat{n}$ $(n_x = 1,\ n_y = 0)$

$$\underset{F}{T_x} = \underset{1}{\sigma_{xx}}\, \underset{1}{\cancel{n_x}} + \sigma_{xy}\, \underset{0}{\cancel{n_y}} \Rightarrow F = \sigma_{xx}$$

$$\underset{0}{T_y} = \sigma_{xy}\, \underset{1}{\cancel{n_x}} + \sigma_{yy}\, \underset{0}{\cancel{n_y}} \Rightarrow 0 = \sigma_{xy}$$

At $y=b$: $n_x = 0$, $n_y = 1$

$0 = \sigma_{xy}$

$0 = \sigma_{yy}$

At $y=0$: $n_x = 0$, $n_y = -1$

$$\underset{0}{T_x} = \sigma_{xx}\, \underset{0}{\cancel{n_x}} + \sigma_{xy}\, \underset{-1}{\cancel{n_y}}$$

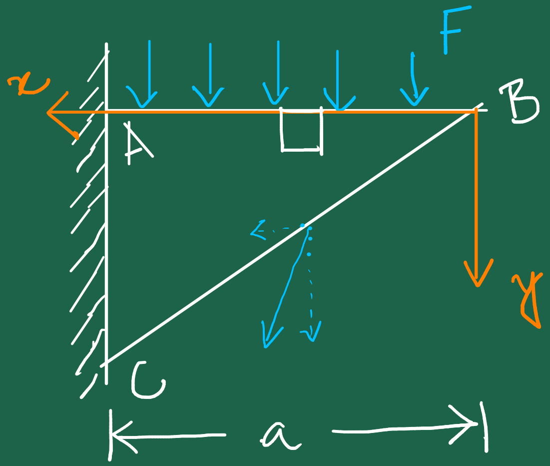
$\Rightarrow \sigma_{xy} = 0$

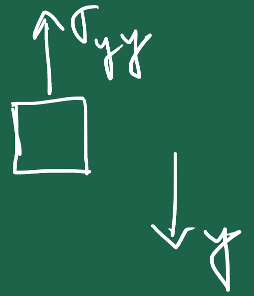
$$\underset{0}{T_y} = \sigma_{xy}\, \underset{0}{\cancel{n_x}} + \sigma_{yy}\, \underset{-1}{\cancel{n_y}}$$

$\Rightarrow \sigma_{yy} = 0$

At $x = 0$:

$u = 0$ and $v = 0$





At $y=0$: $n_x = 0$, $n_y = -1$

$$\underset{0}{\cancel{T_x}} = \sigma_{xx}\,\underset{0}{\cancel{n_x}} + \sigma_{xy}\,\underset{-1}{\cancel{n_y}}$$

$$\Rightarrow \quad \sigma_{xy} = 0$$

$$\underset{F}{\cancel{T_y}} = \sigma_{xy}\,\underset{0}{\cancel{n_x}} + \sigma_{yy}\,\underset{-1}{\cancel{n_y}}$$

$$\Rightarrow \quad \sigma_{yy} = -F$$

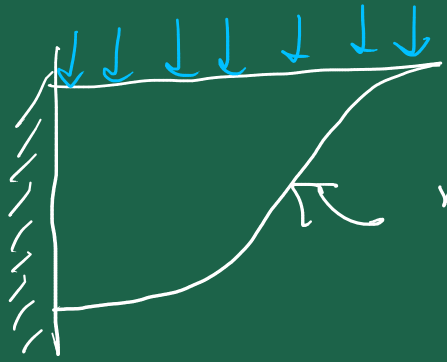
Along BC : $n_x \neq 0$, $n_y \neq 0$ (n_x & n_y values are const.)

$$\underset{0}{\cancel{T_x}} = \sigma_{xx}\,\underset{\checkmark}{n_x} + \sigma_{xy}\,\underset{\checkmark}{n_y}$$

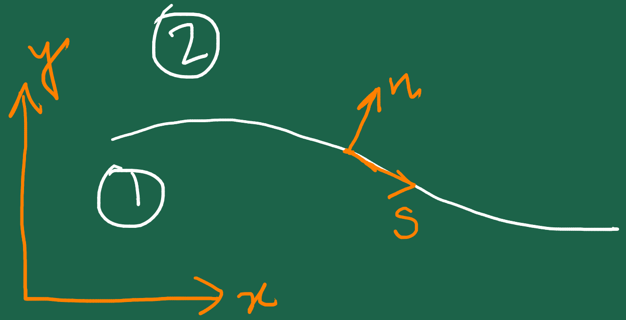
$$\underset{0}{\cancel{T_y}} = \sigma_{xy}\,\underset{\checkmark}{n_x} + \sigma_{yy}\,\underset{\checkmark}{n_y}$$

Along AC or $x = a$

$u = 0$ and $v = 0$

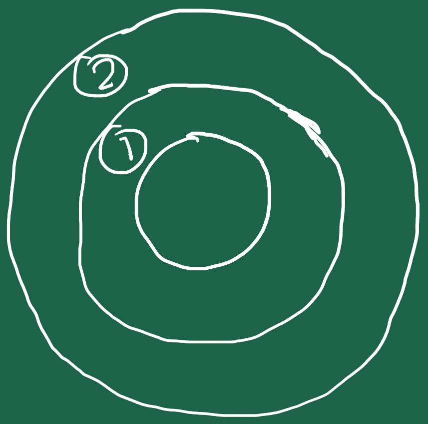


n_x and n_y values are <u>not</u> const.



Perfectly bonded interface

Both the displacements and the tractions are considered to be continuous.



$$u_r^{(1)}\Big|_{in} = u_r^{(2)}\Big|_{in}$$

$$\sigma_{rr}^{(1)}\Big|_{in} = \sigma_{rr}^{(2)}\Big|_{in}$$

Think about $\sigma_{r\theta}$ and σ_{rz}!