PROBLEM SHEET 1: ENERGY METHODS

1. During lecture, it was shown that if a body occupying a volume V and subjected to body forces (per unit volume) ρb_i (where ρ is the mass density) and tractions $t_i = \sigma_{ji} n_j$ specified on a part of the surface S_1 together with displacements u_i specified on the remaining part of the surface S_2, is in static equilibrium, then the virtual work associated with the external forces is given by $\delta W_e = \int_V \sigma_{ij} \delta \varepsilon_{ij} \, \mathrm{d}V$. This statement is the necessity condition for equilibrium. Show that the converse also holds; thus: If we have $\delta W_e = \int_V \sigma_{ij} \delta \varepsilon_{ij} \, \mathrm{d}V$, then the body will be in equilibrium, i.e. $\rho b_i + \dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$ *and* $t_i = \sigma_{ji} n_j$.

   Hint: See §3.2 in Dym and Shames.

2. A cantilever beam is loaded as shown in Figure 1 by a point load P. Determine the vertical deflection at the point of application of the load. Also, determine the vertical deflection at the cantilever tip.

   $$\left[d_P = \frac{P(L-b)^3}{3EI}, \; d_{\text{tip}} = \frac{P(L^3 - b^3)}{3EI} - \frac{Pb(L^2 - b^2)}{2EI} \right]$$

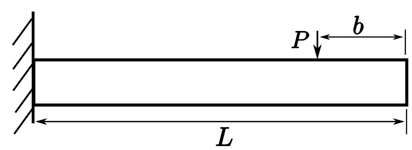


Figure 1

3. A uniformly loaded beam is fixed at both ends as shown in Figure 2. Without taking advantage of symmetry and using appropriate energy methods, determine the reaction components on the left side of the beam due to the uniformly distributed load q_0 [N/m].

   $$\left[R = \frac{q_0 L}{2}, \; M = \frac{q_0 L^2}{12} \right]$$

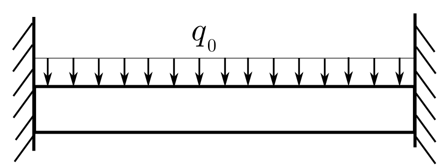


Figure 2

4. The uniform semi-circular beam made of Hookean material is loaded by a vertical force P as shown in Figure 3. Using Castigliano's theorem, determine the horizontal displacement at the free end.
$$\left[\frac{Pa^3(\pi-1)}{2EI}\right]$$

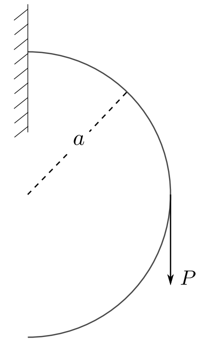


Figure 3

5. Consider the beam shown in Figure 4 where q_0 [N/m] represents a uniformly distributed load. The beam material behaves according to the stress-strain law $\sigma_{xx} = k\varepsilon_{xx}^{1/3}$. The beam has a rectangular cross-section of width b and height h. Find the algebraic equation that needs to be solved in order to find the reaction force at the left end. How does this equation change with the values of k, b, and h?
[The algebraic equation resulting from $\displaystyle\int_0^L \left(R - \frac{q_0 x}{2}\right)^3 x^4 \; \mathrm{d}x = 0$. Equation does not change.]

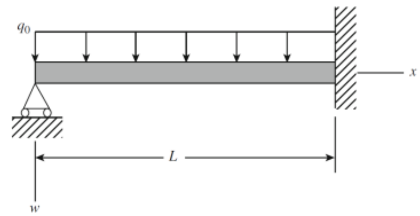
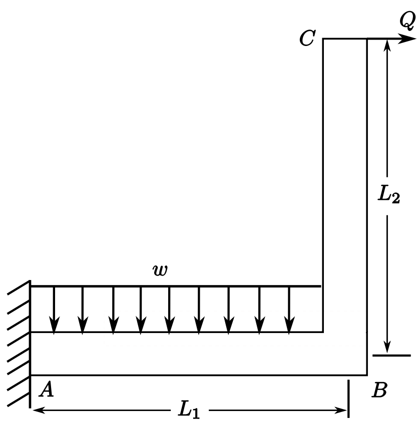


Figure 4

6. For the structure shown in Figure 5, the material has Young's modulus E. The second moment of area I is the same throughout. The structure is loaded by a horizontal force Q at C and by a uniform load w [N/m] distributed over AB. Determine the horizontal and vertical displacement components of point C in terms of Q, w, L_1, L_2, E, and I. Note that the cross-sectional dimensionals are much smaller than L_1 and L_2.



$$\left[d_{\text{hor}} = \frac{QL_2^3}{3EI} + \frac{QL_1L_2^2}{EI} + \frac{wL_1^3L_2}{6EI},\; d_{\text{vert}} = \frac{QL_1^2L_2}{2EI} + \frac{wL_1^4}{8EI}\text{ (down)}\right]$$

Figure 5