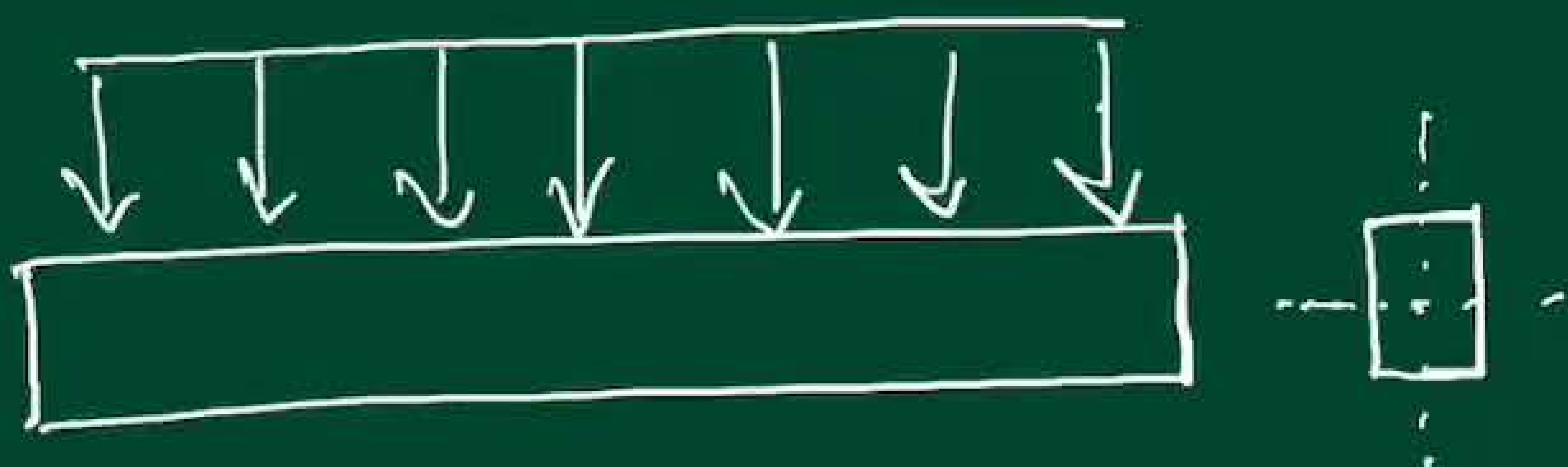
SPECIAL SCREEN-RECORDED LECTURES DURING COVID-19 SITUATION

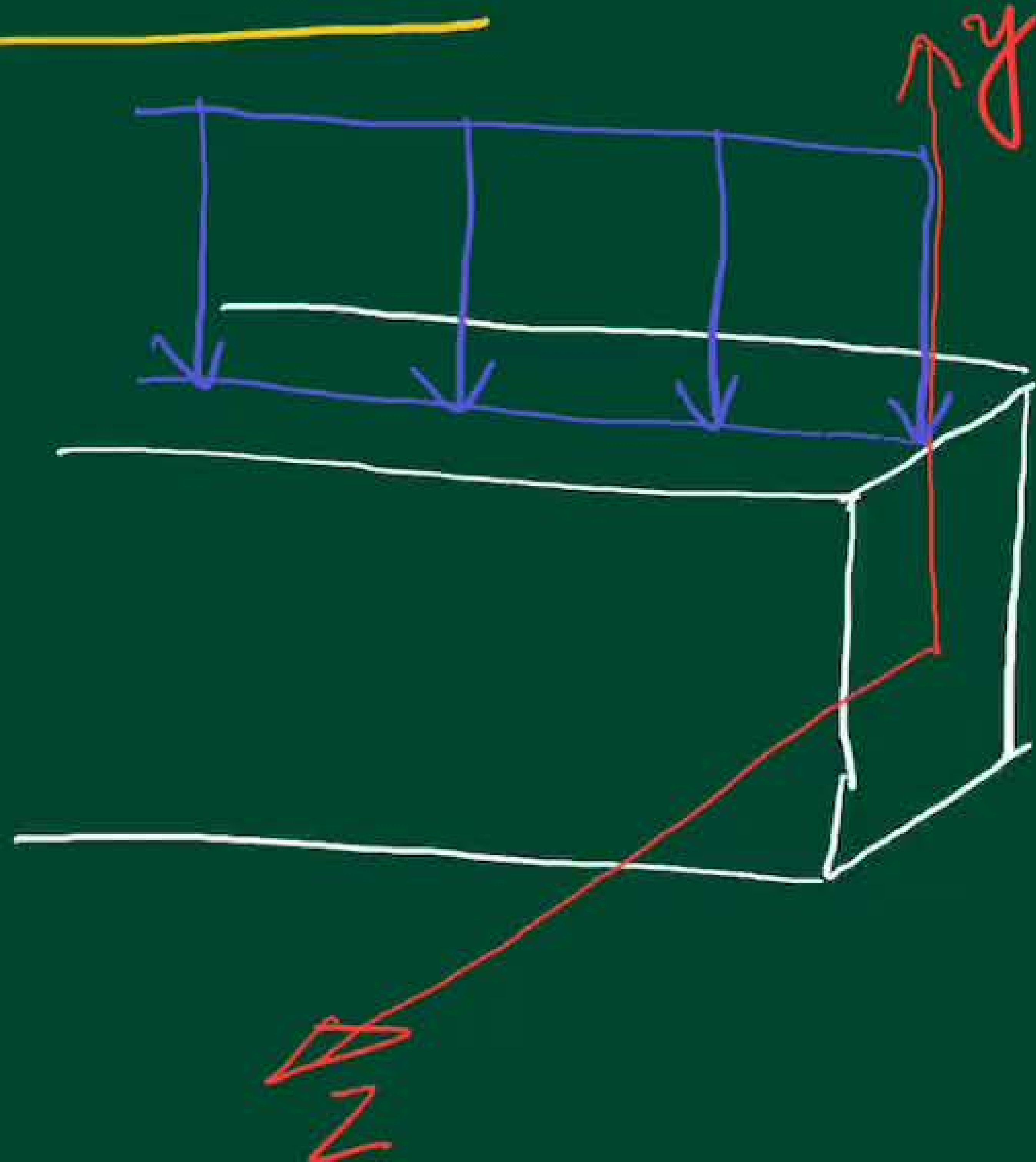
ADVANCED MECHANICS OF SOLIDS (ME60402)

MECHANICAL ENGINEERING DEPT., IIT KHARAGPUR

UNSYMMETRIC BENDING OF BEAMS

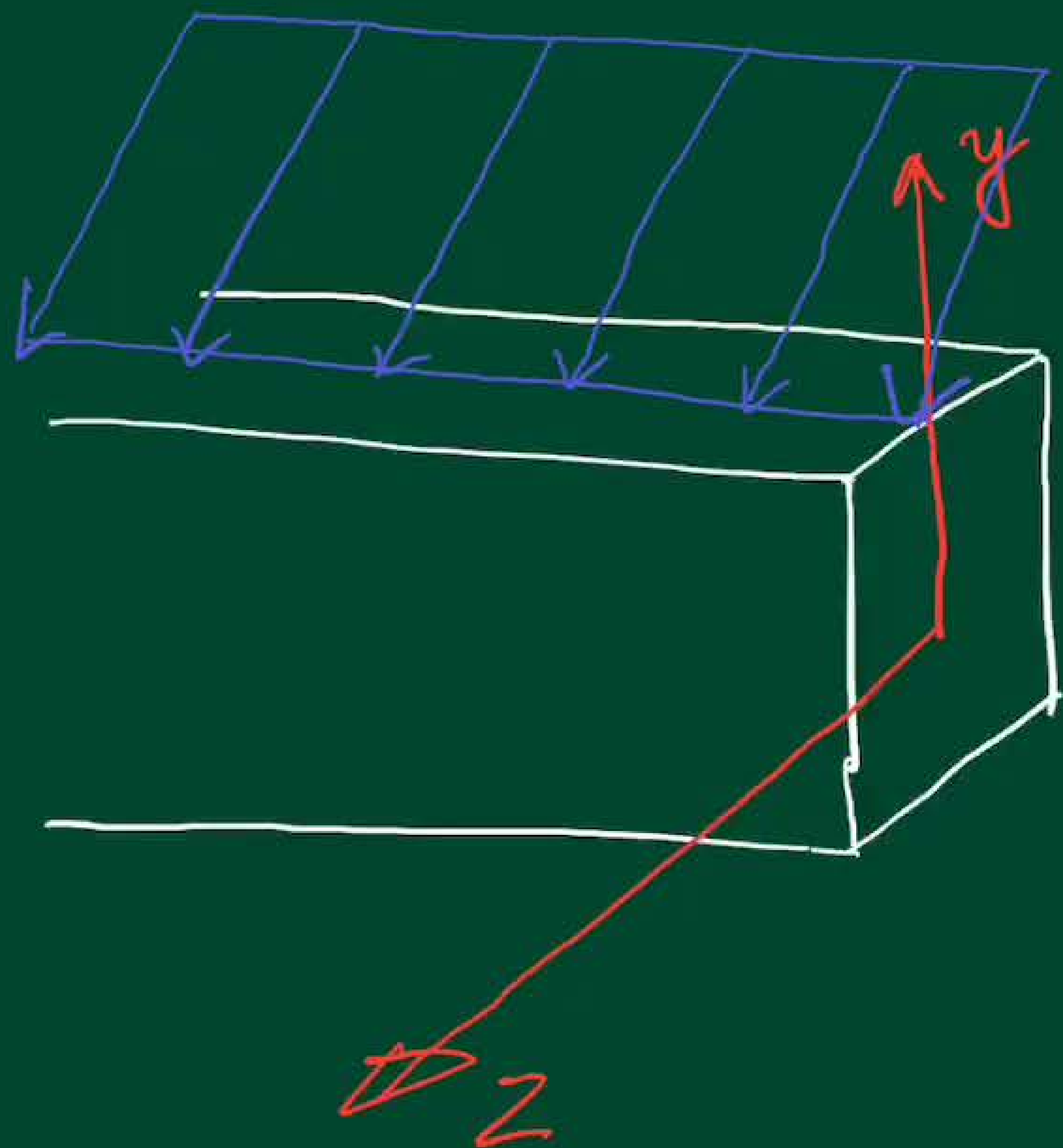
(OR SKEW)



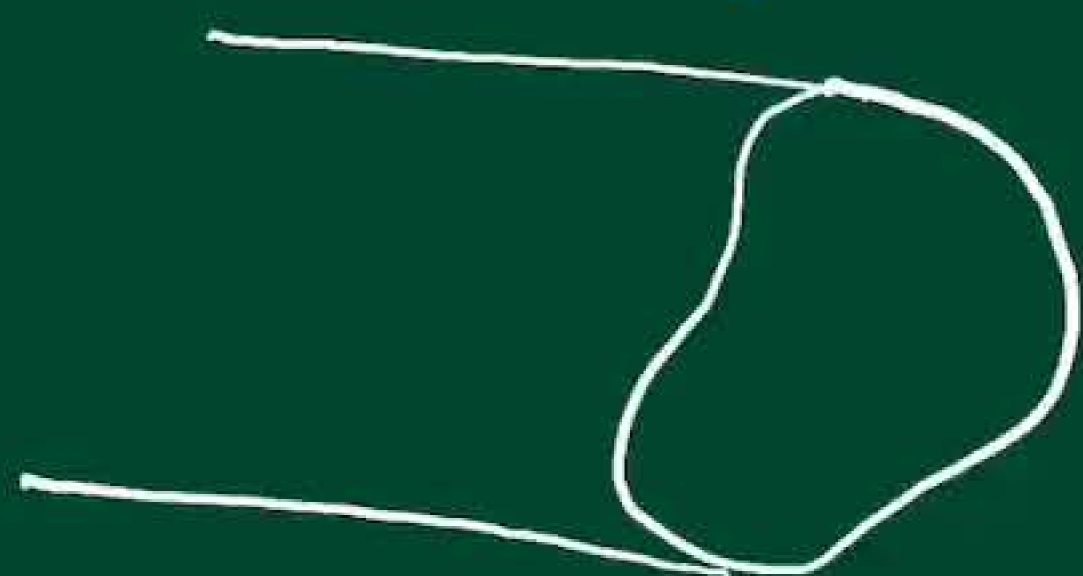


SYMMETRIC BENDING OF BEAMS

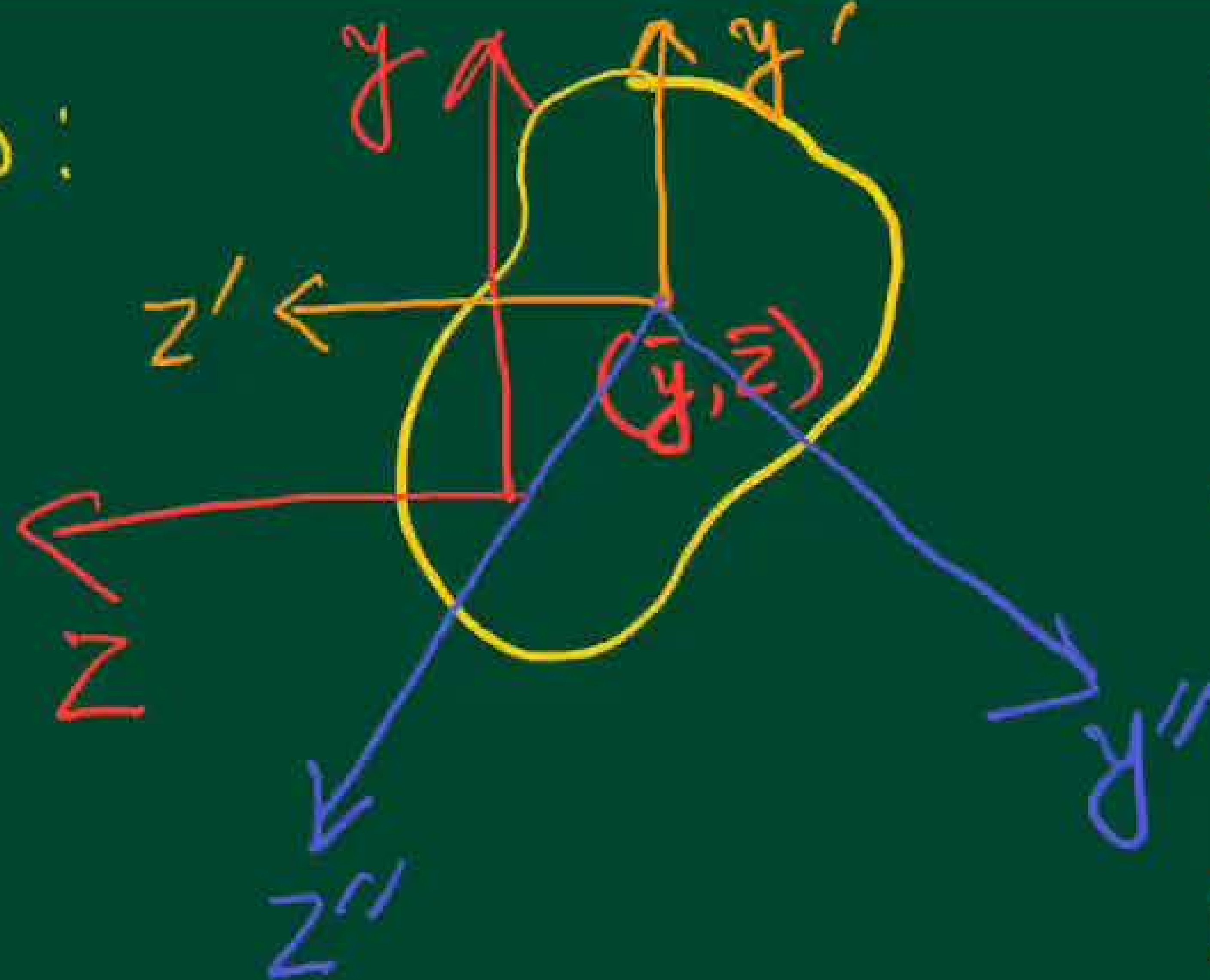
1st CASE OF UNSYMMETRIC BENDING : C/S IS SYMMETRIC BUT LOADING IS UNSYMMETRIC



2nd CASE OF UNSYMMETRIC BENDING : C/S IS UNSYMMETRIC



CENTROID:



$$\bar{y} = \frac{\int y \, dA}{A}$$

$$\bar{z} = \frac{\int z \, dA}{A}$$

$$0 = \int y' \, dA$$

$$0 = \int z' \, dA$$

y', z' are referred to as <u>centroidal axes</u>

y'', z'' " " " " " "

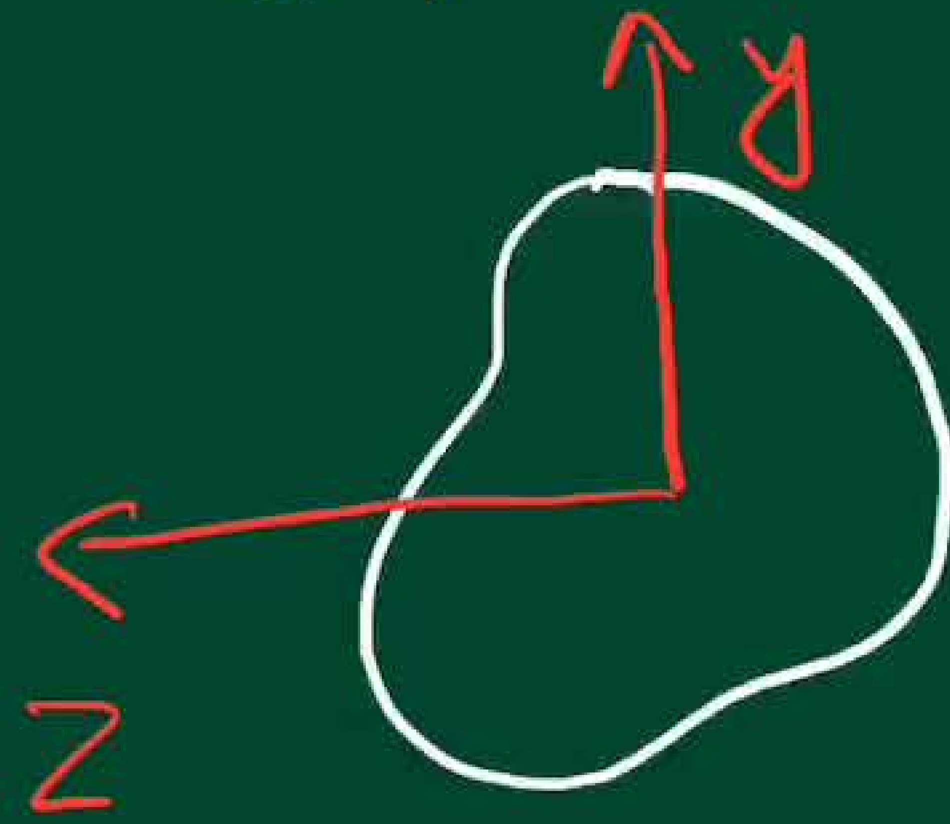
Principal Axes

2nd moment of area (or, Moment of inertia)

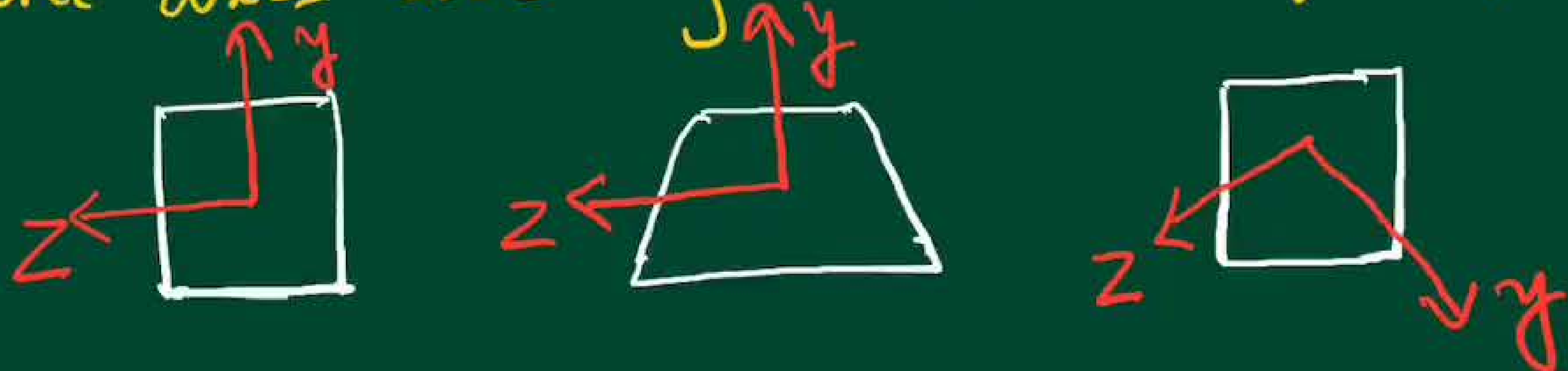
$$I_y := \int z^2 \, dA$$

$$I_z := \int y^2 \, dA$$

$$I_{yz} := \int yz \, dA$$

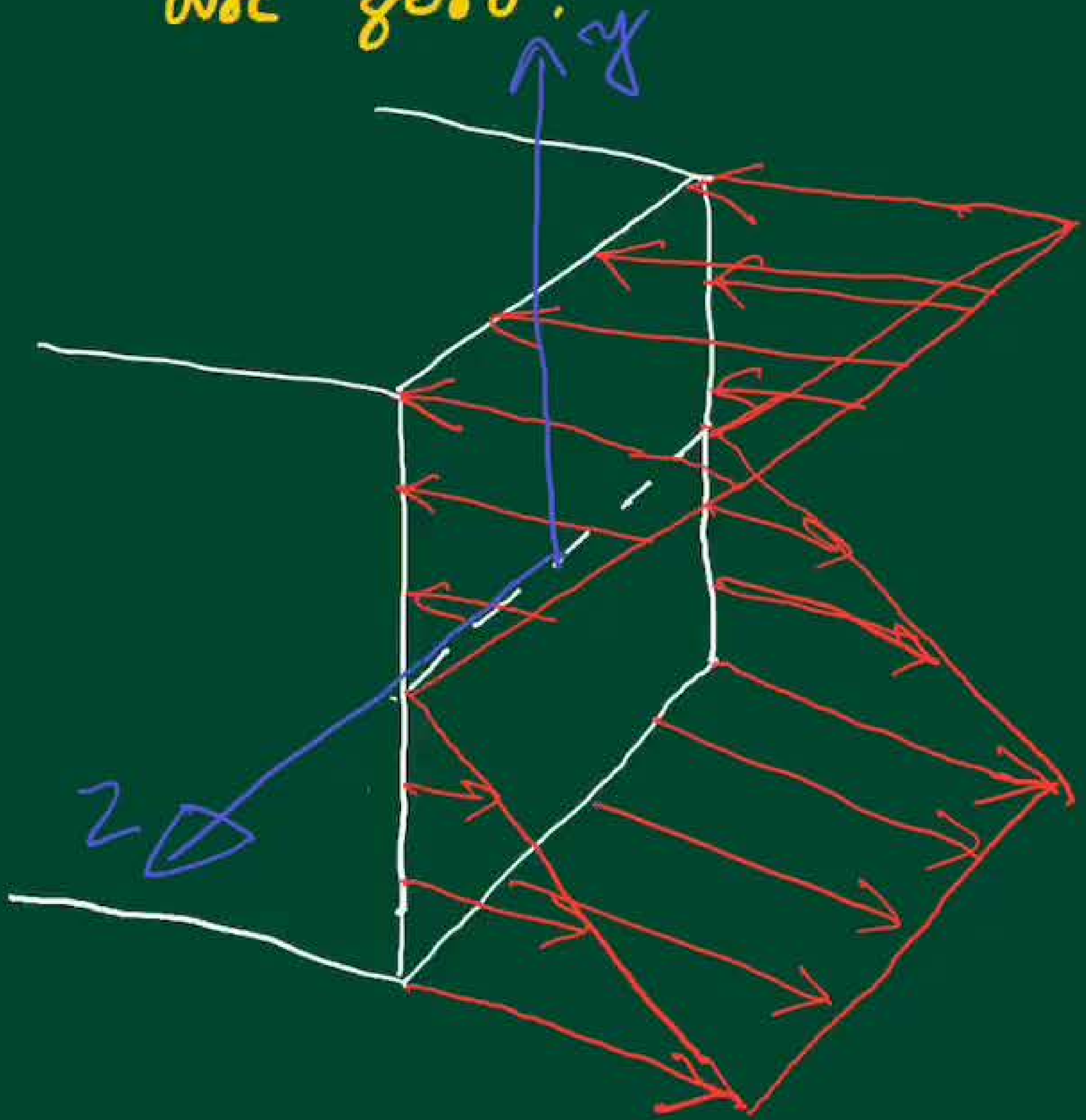


If the axes are so chosen that $I_{yz} = 0$ then the axes are referred to as principal axes.



Neutral Axis (N/A)

The axis in a c/s along which the bending stresses are zero.



Symmetric Bending

The N/A of a c/s under pure bending always passes through the centroid.

Zero axial forces :

$$\int \sigma_{xx}\, dA = 0$$

$$\Rightarrow \int E\, \varepsilon_{xx}\, dA = 0$$

$$\Rightarrow \int E\left(-\frac{y}{R}\right) dA = 0$$

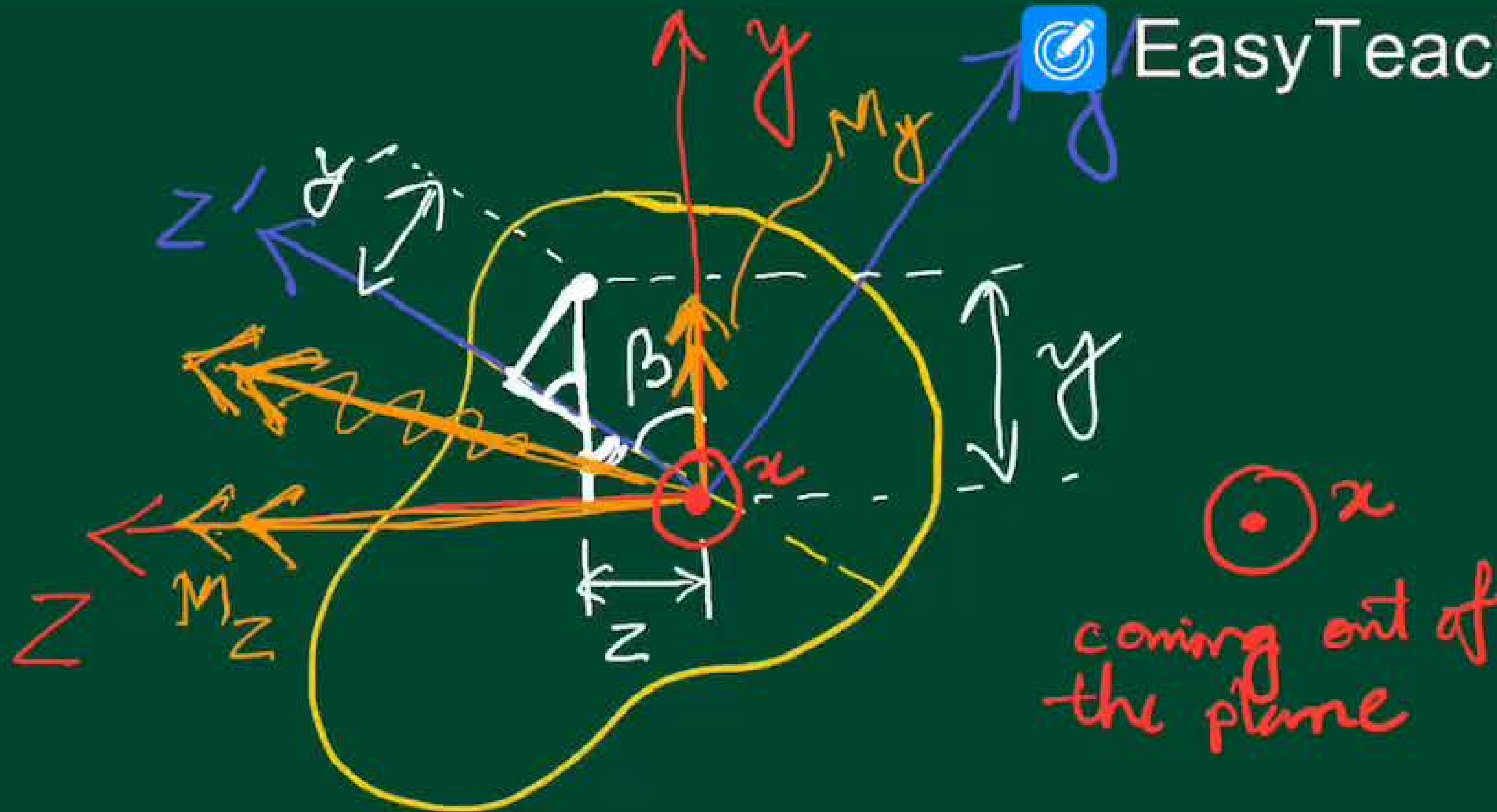
$$\Rightarrow \int y\, dA = 0$$

$$\bar{y} = \frac{\int y\, dA}{A}$$

Unsymmetric bending

$$y = \frac{y'}{\sin\beta} + \frac{z}{\tan\beta}$$

$$\Rightarrow y' = y\sin\beta - z\cos\beta$$

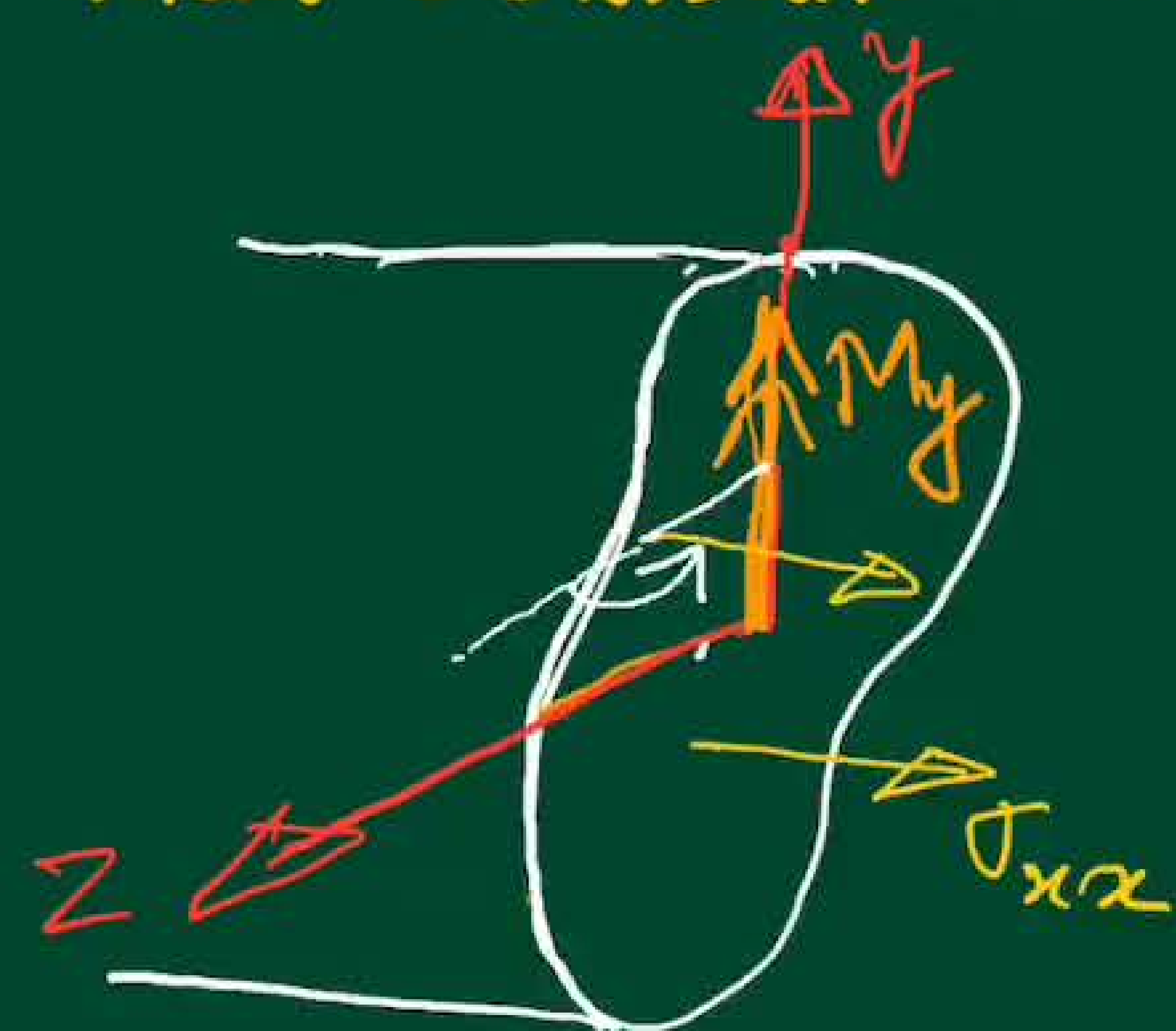


We understand that under pure bending the bending stresses denoted by σ_{xx} will have a linear distribution about the z' axis (i.e. about the N/A)

$$\sigma_{xx} = K y'$$

$$M_z = -\int \sigma_{xx}\, y\, dA$$

$$M_y = +\int \sigma_{xx}\, z\, dA$$



EasyTeach

$$M_z = -\int \sigma_{xx}\, y\, dA$$

$$\Rightarrow -M_z = \int K y' y\, dA$$

$$\Rightarrow -M_z = \int K(y\sin\beta - z\cos\beta)\, y\, dA$$

$$\Rightarrow -M_z = K\sin\beta \int y^2\, dA - K\cos\beta \int yz\, dA$$

$$\Rightarrow -M_z = K\sin\beta\, I_z - K\cos\beta\, I_{yz} \quad \text{--- (1)}$$

$$M_y = \int \sigma_{xx}\, z\, dA$$

$$\Rightarrow M_y = \int K(y\sin\beta - z\cos\beta)\, z\, dA$$

$$\Rightarrow M_y = K\sin\beta\, I_{yz} - K\cos\beta\, I_y \quad \text{--- (2)}$$

EasyTeach

$$① \div ②$$

$$\Rightarrow -\frac{M_z}{M_y} = \frac{\sin\beta\, I_z - \cos\beta\, I_{yz}}{\sin\beta\, I_{yz} - \cos\beta\, I_y}$$

$$= \frac{\tan\beta\, I_z - I_{yz}}{\tan\beta\, I_{yz} - I_y}$$

$$\therefore -M_z \tan\beta\, I_{yz} + M_z I_y = M_y \tan\beta\, I_z - M_y I_{yz}$$

$$\Rightarrow \tan\beta = \frac{M_z I_y + M_y I_{yz}}{M_z I_{yz} + M_y I_z}$$

After we know about β, we can go to Eq. ① or Eq. ② to find K.

And then we can find σ_{xx}.

$$\sigma_{xx} = \frac{M_y\left(y I_{yz} - z I_z\right) - M_z\left(z I_{yz} - y I_y\right)}{I_{yz}^2 - I_y I_z}$$

→ Generalized flexure formula

Find out σ_{xx} when y and z are principal axes
i.e. $I_{yz} = 0$

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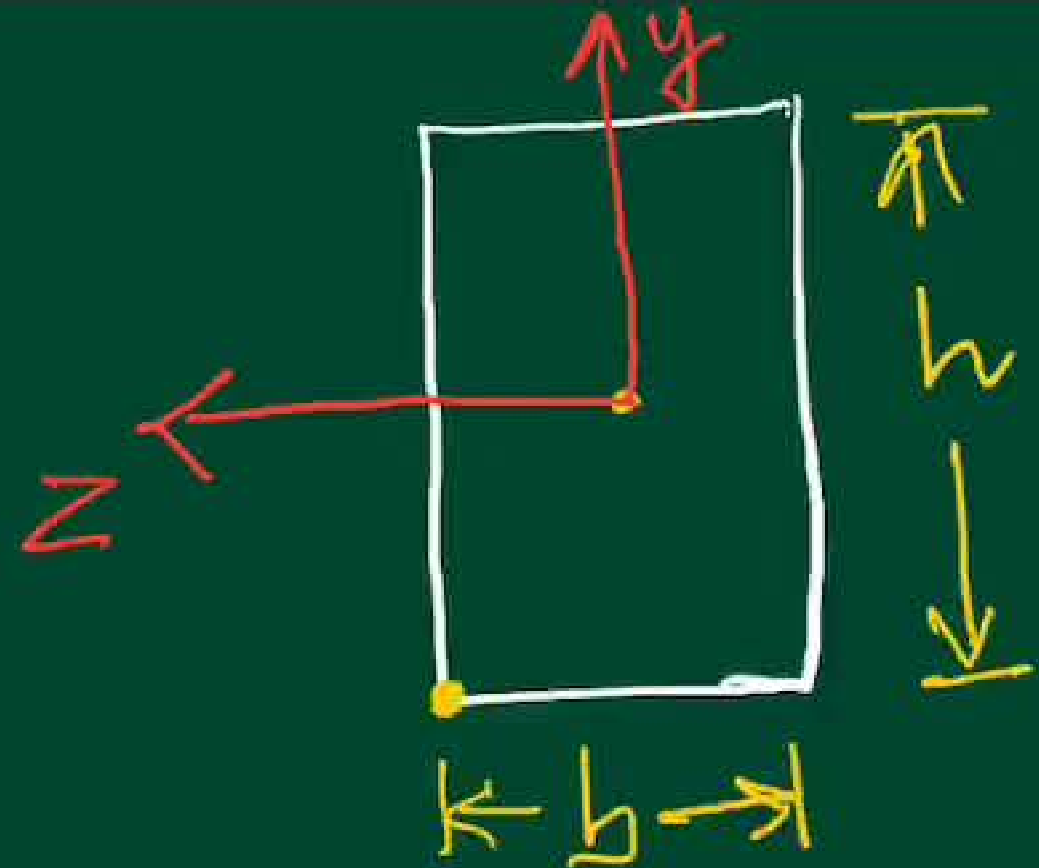
UNSYMMETRIC BENDING OF BEAMS ... CONTD.

Steps to follow while solving problems:

#1 Find the centroid of the c/s

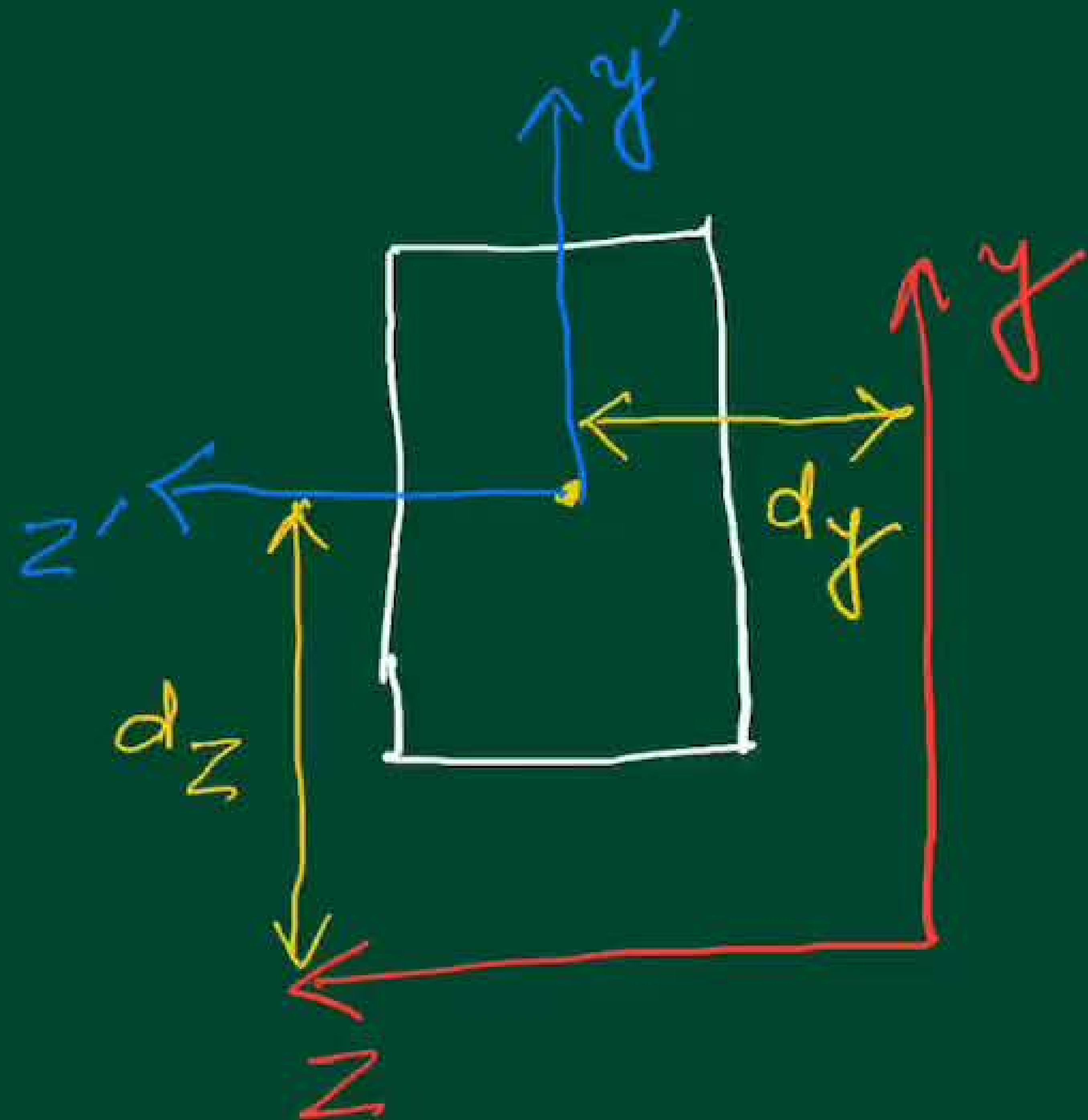
#2 Find the second moments of area



$$I_z = \frac{1}{12} bh^3$$

$$I_y = \frac{1}{12} b^3 h$$

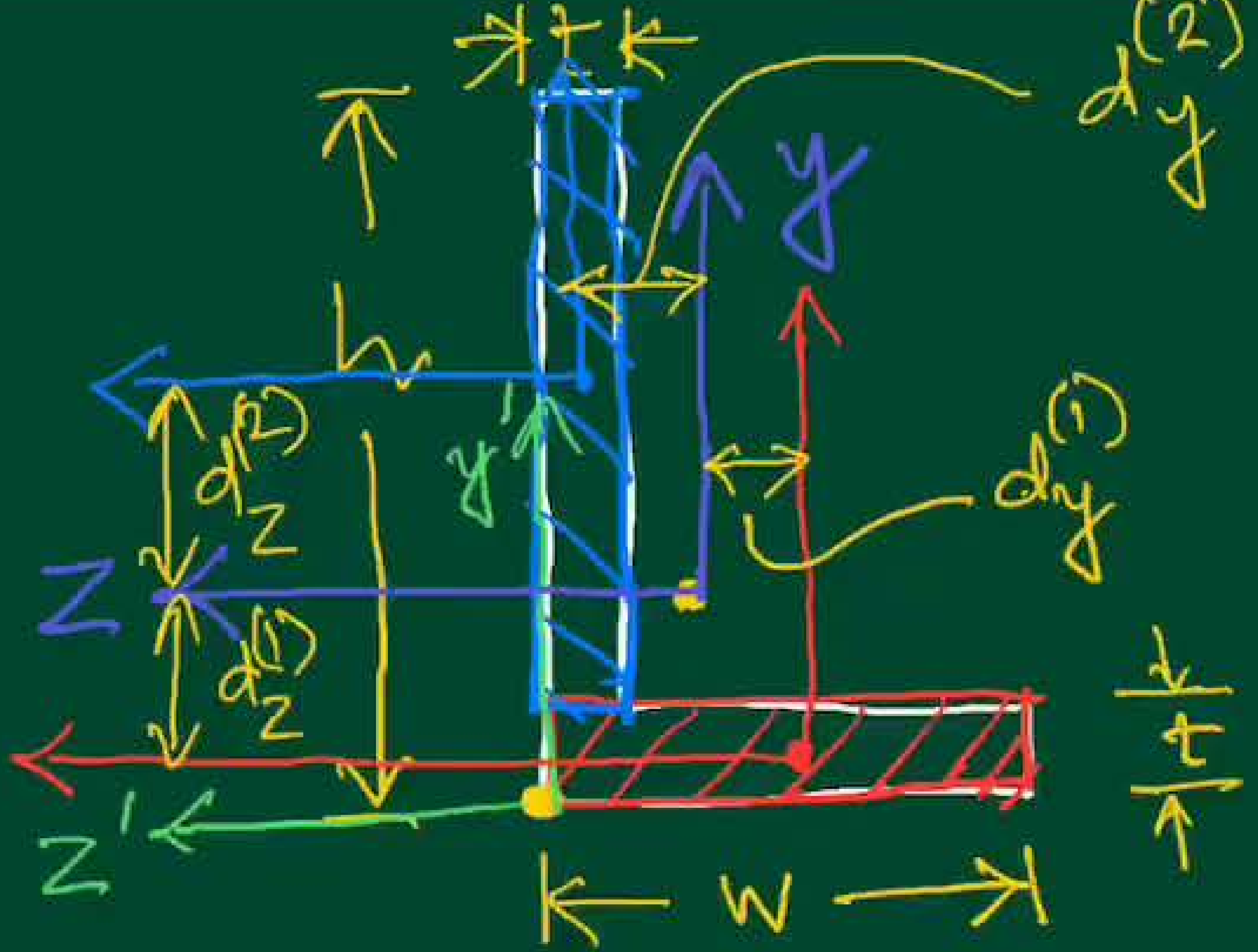
$$I_{yz} = 0$$



$$I_z = \frac{1}{12} bh^3 + bh\, d_z^2$$

$$I_y = \frac{1}{12} b^3 h + bh\, d_y^2$$

$$I_{yz} = 0 + bh\, d_y\, d_z$$



$$z_c = \frac{A_1 z_1' + A_2 z_2'}{A_1 + A_2}$$

$$y_c = \frac{A_1 y_1' + A_2 y_2'}{A_1 + A_2}$$

$$I_z = I_z^{(1)} + A_1 \left(d_z^{(1)}\right)^2 + I_z^{(2)} + A_2 \left(d_z^{(2)}\right)^2$$

$$I_y = I_y^{(1)} + A_1 \left(d_y^{(1)}\right)^2 + I_y^{(2)} + A_2 \left(d_y^{(2)}\right)^2$$

$$I_{yz} = \cancelto{0}{I_{yz}^{(1)}} + A_1 d_y^{(1)} d_z^{(1)} + \cancelto{0}{I_{yz}^{(2)}} + A_2 d_y^{(2)} d_z^{(2)}$$