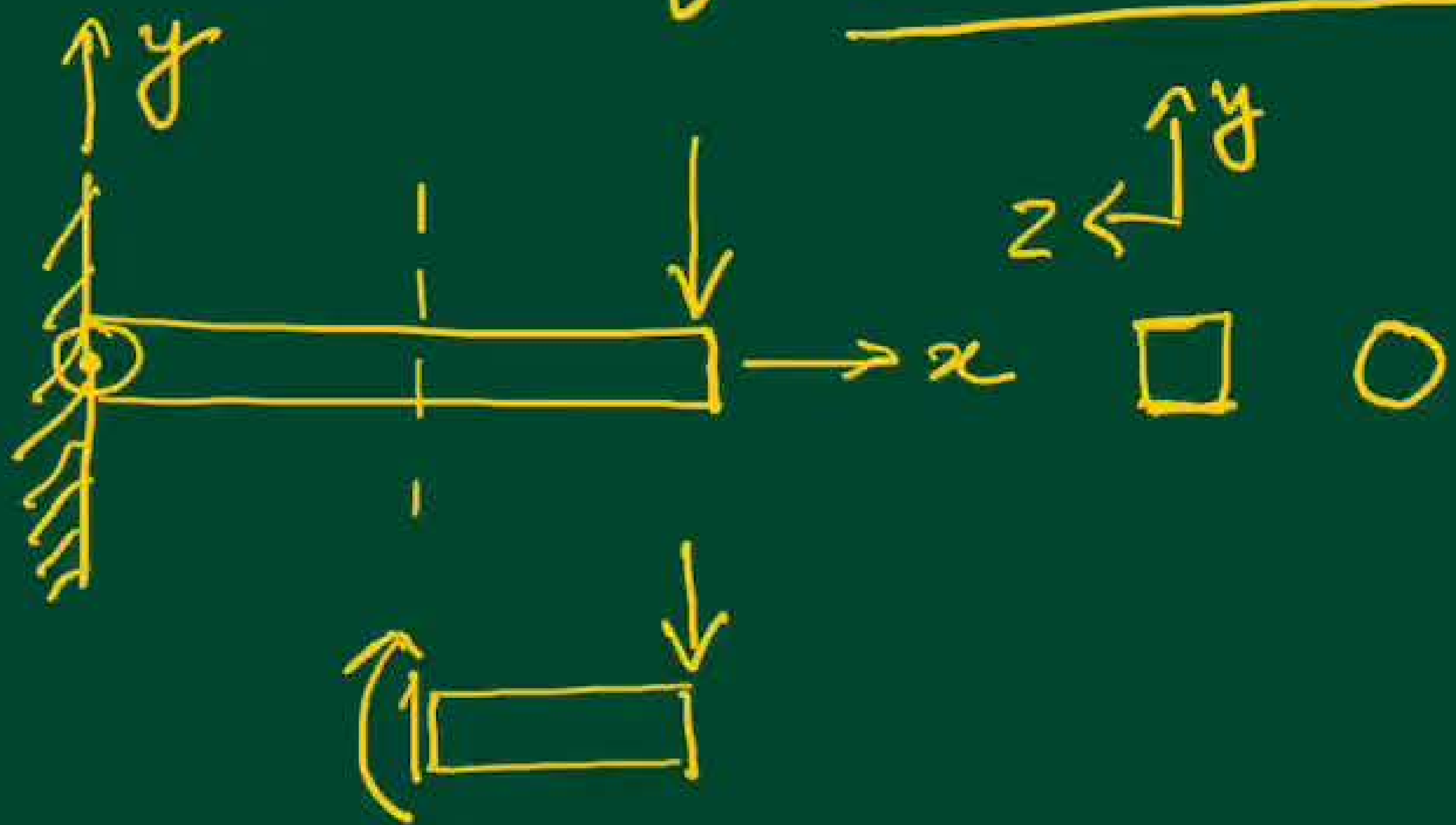
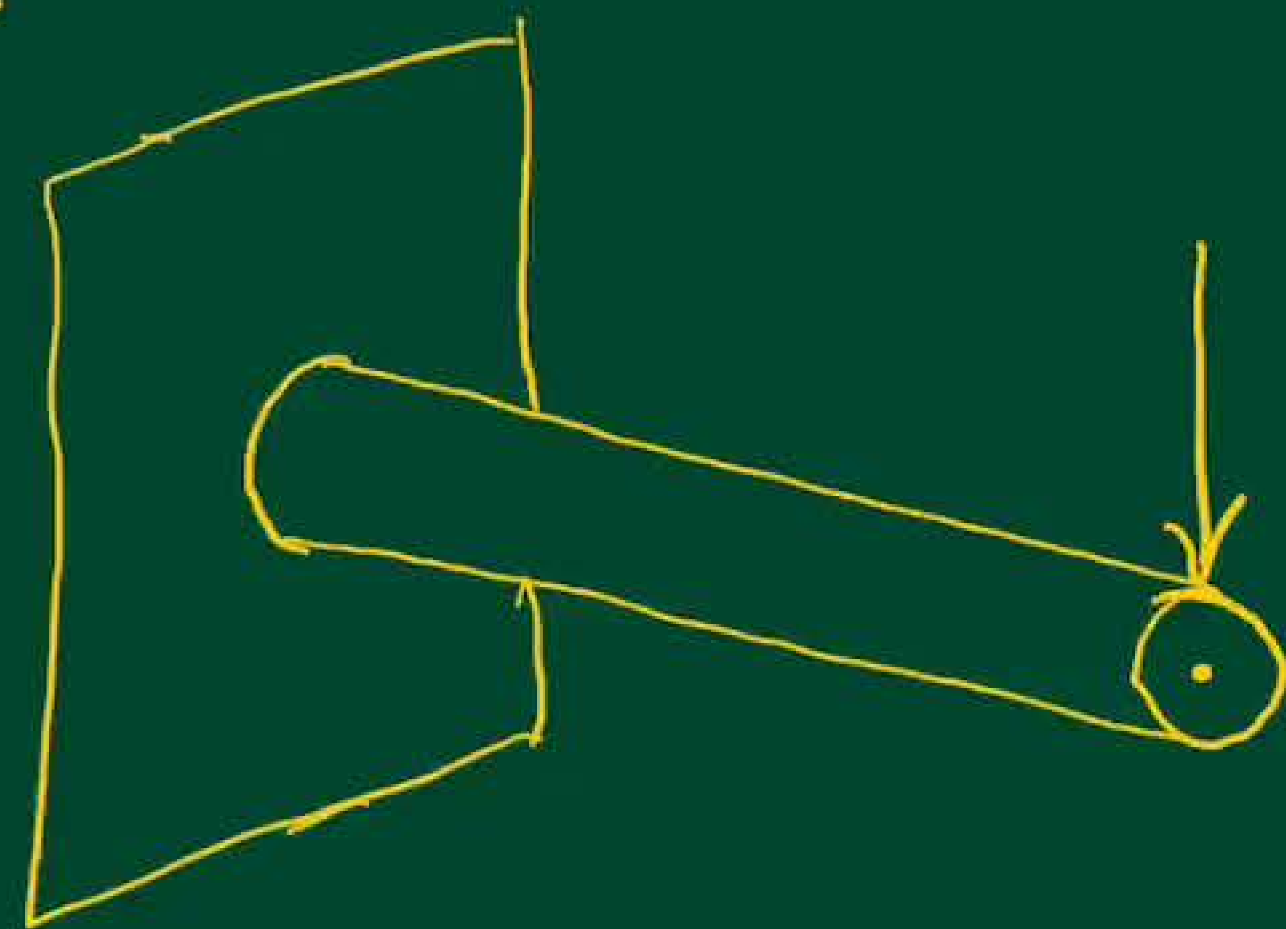
ADVANCED MECHANICS OF SOLIDS (ME60402)

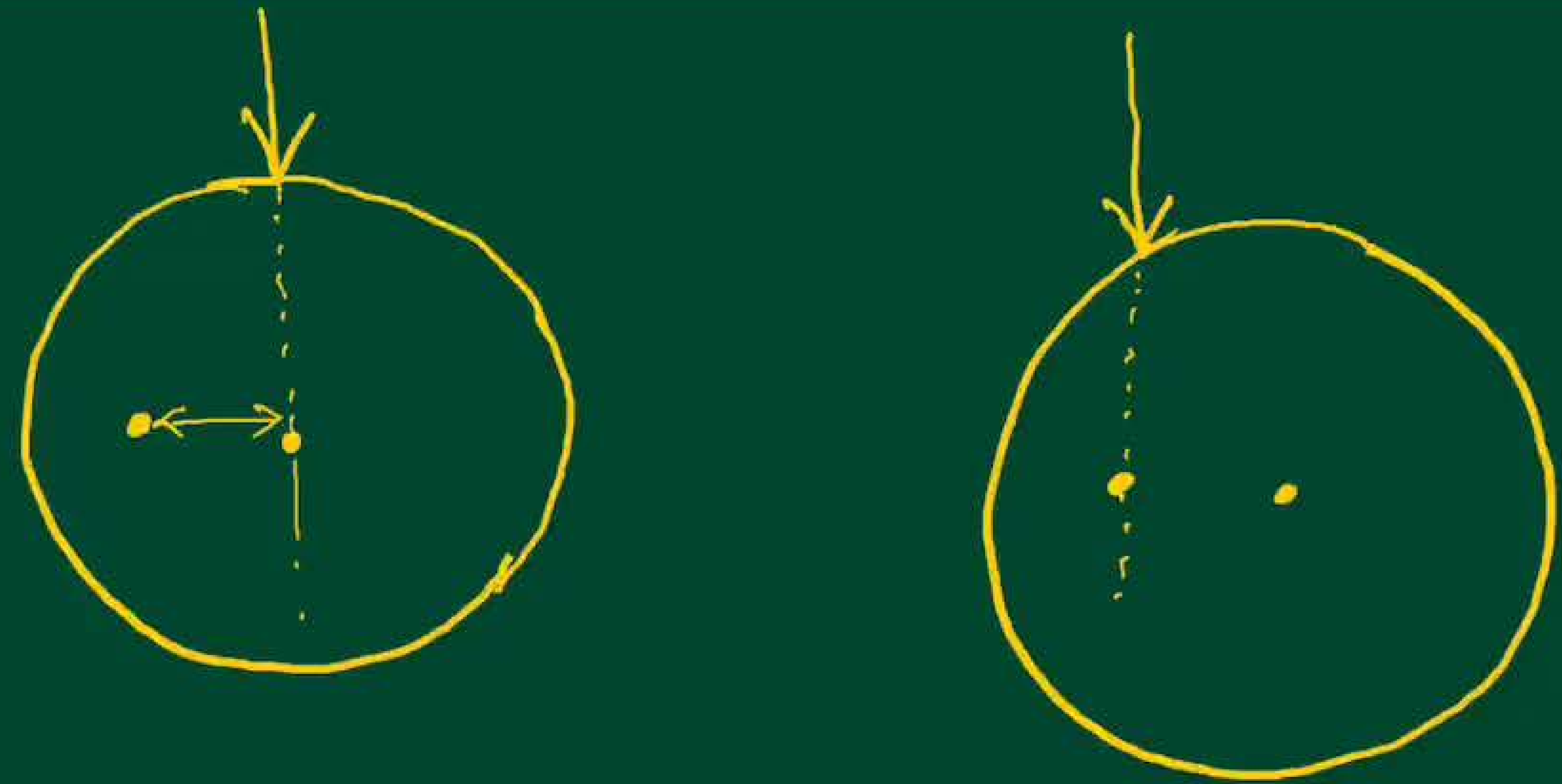
MECHANICAL ENGINEERING DEPARTMENT, IIT KHARAGPUR

SHEAR CENTRE

Previous lectures on Unsymmetric Bending pertained to the case of pure bending.







In general, transverse forces on a beam will induce bending <u>and</u> twisting.

Imp. question: Is it possible to have a case when the transverse forces induce <u>only</u> bending?

Ans: Yes, when the transverse force passes through a special point called the SHEAR CENTRE.

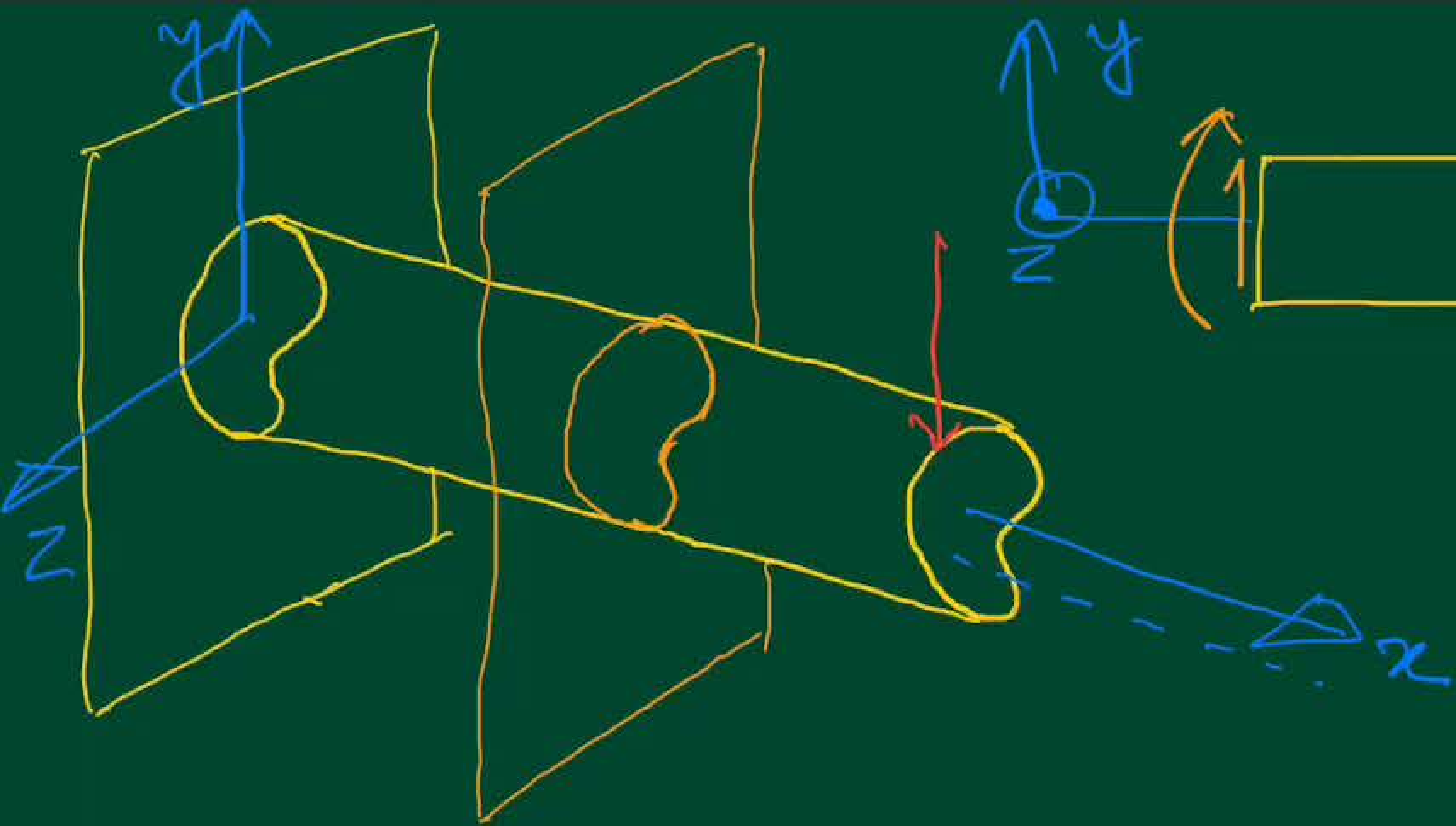
EasyTeach

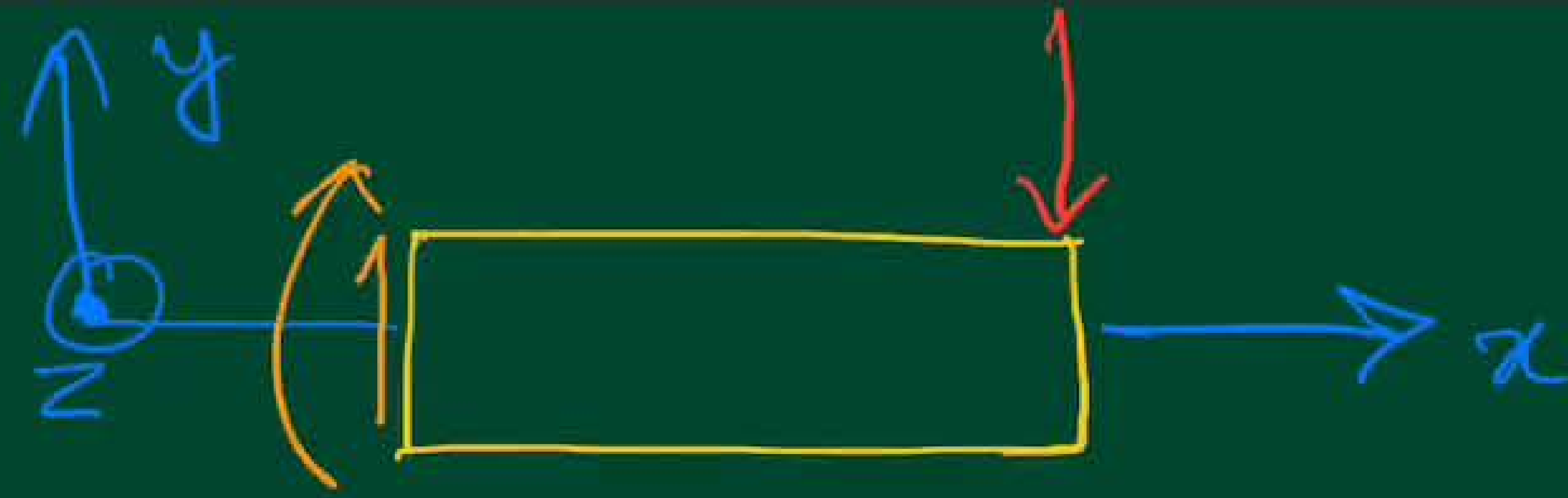
Extremely important points to note

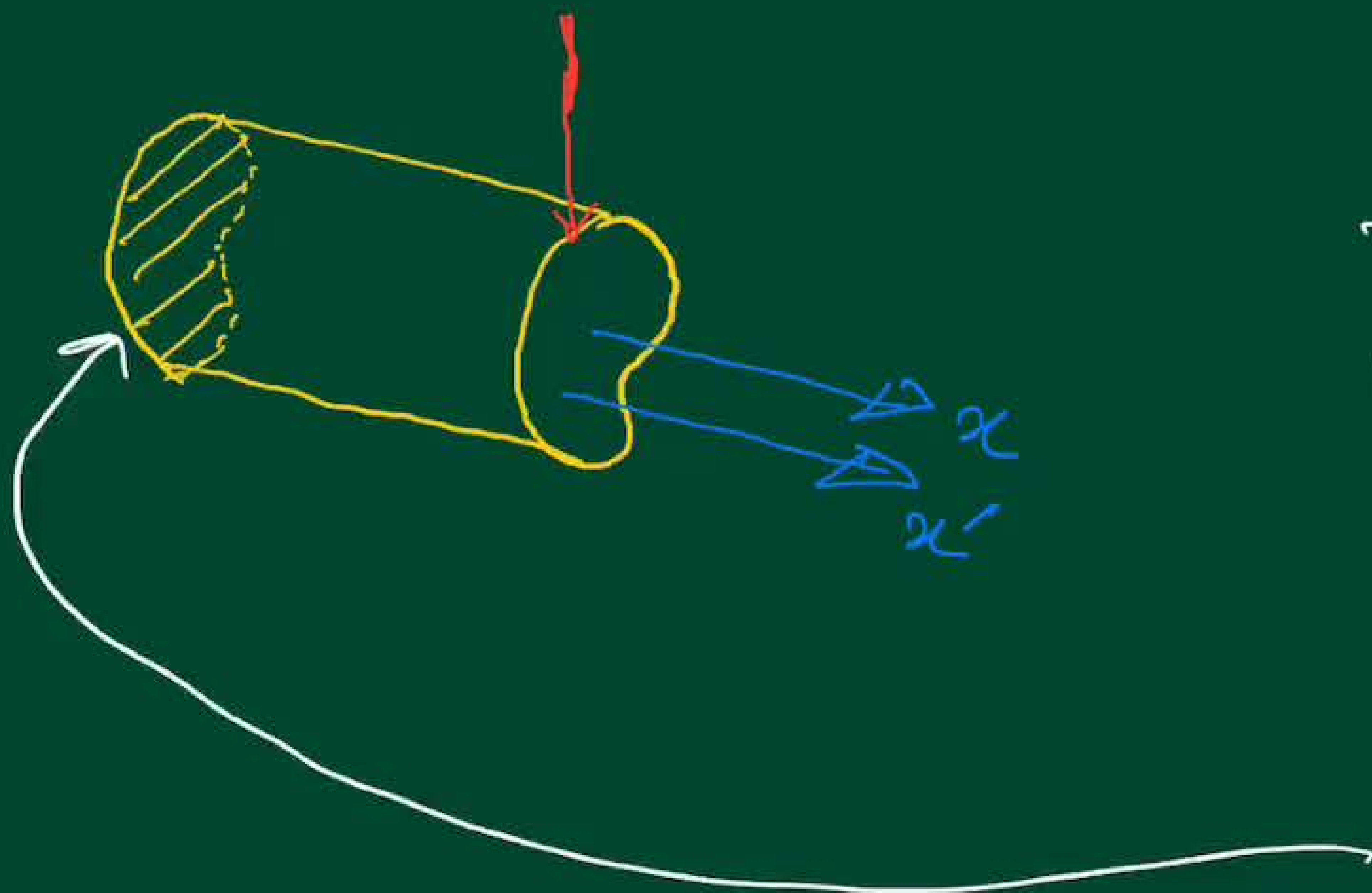
SHEAR CENTRE is a purely geometric entity

— a particular c/s will have a particular SHEAR CENTRE that does <u>not</u> change with loading

SHEAR CENTRE need <u>not</u> lie within the c/s







If the transverse force passes through the SHEAR CENTRE, then the torsional moment of that force about any <u>axis</u> is going to be balanced by the torsional moment contributed by the shear stresses on the <u>hatched part</u> about the same axis.

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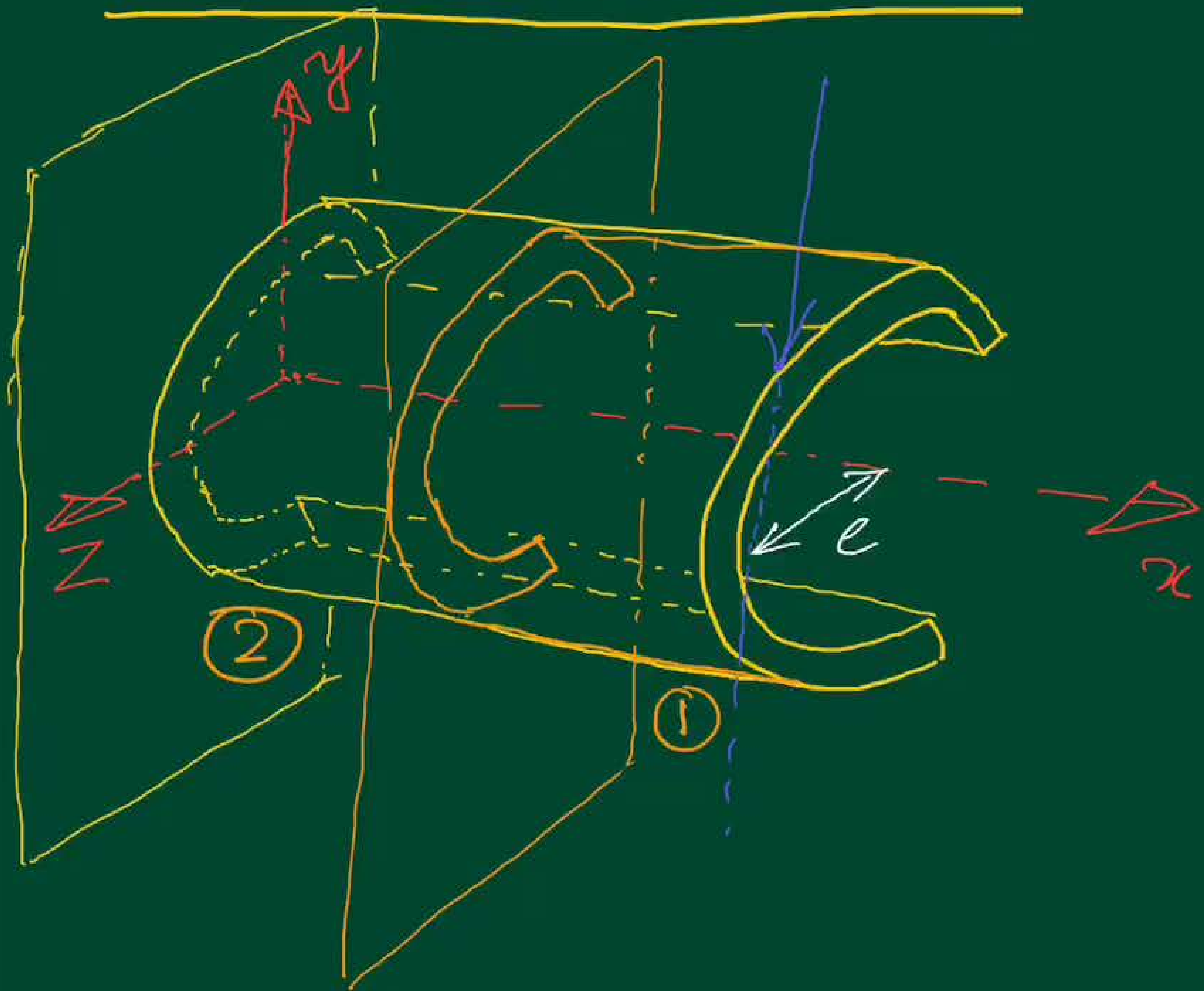
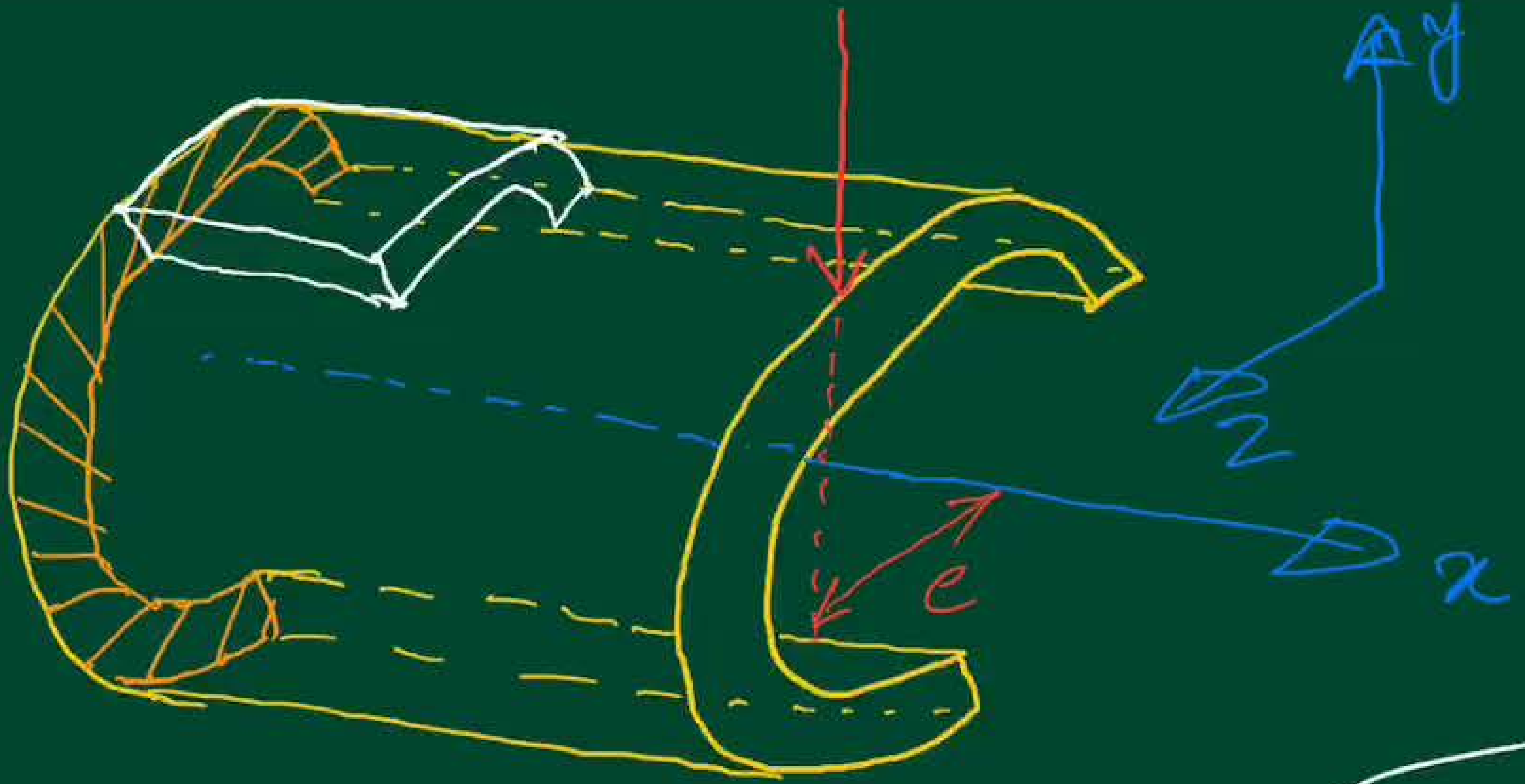
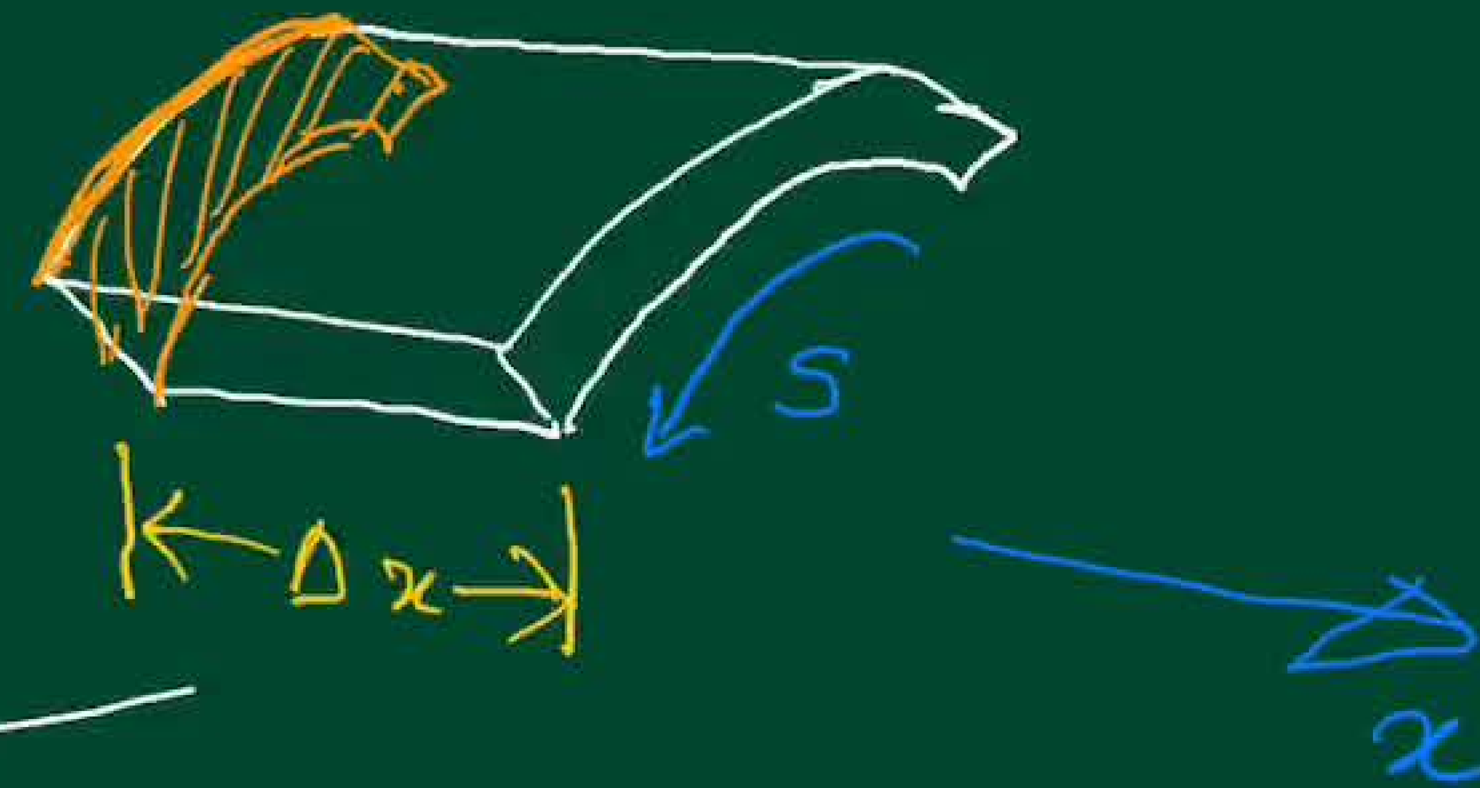
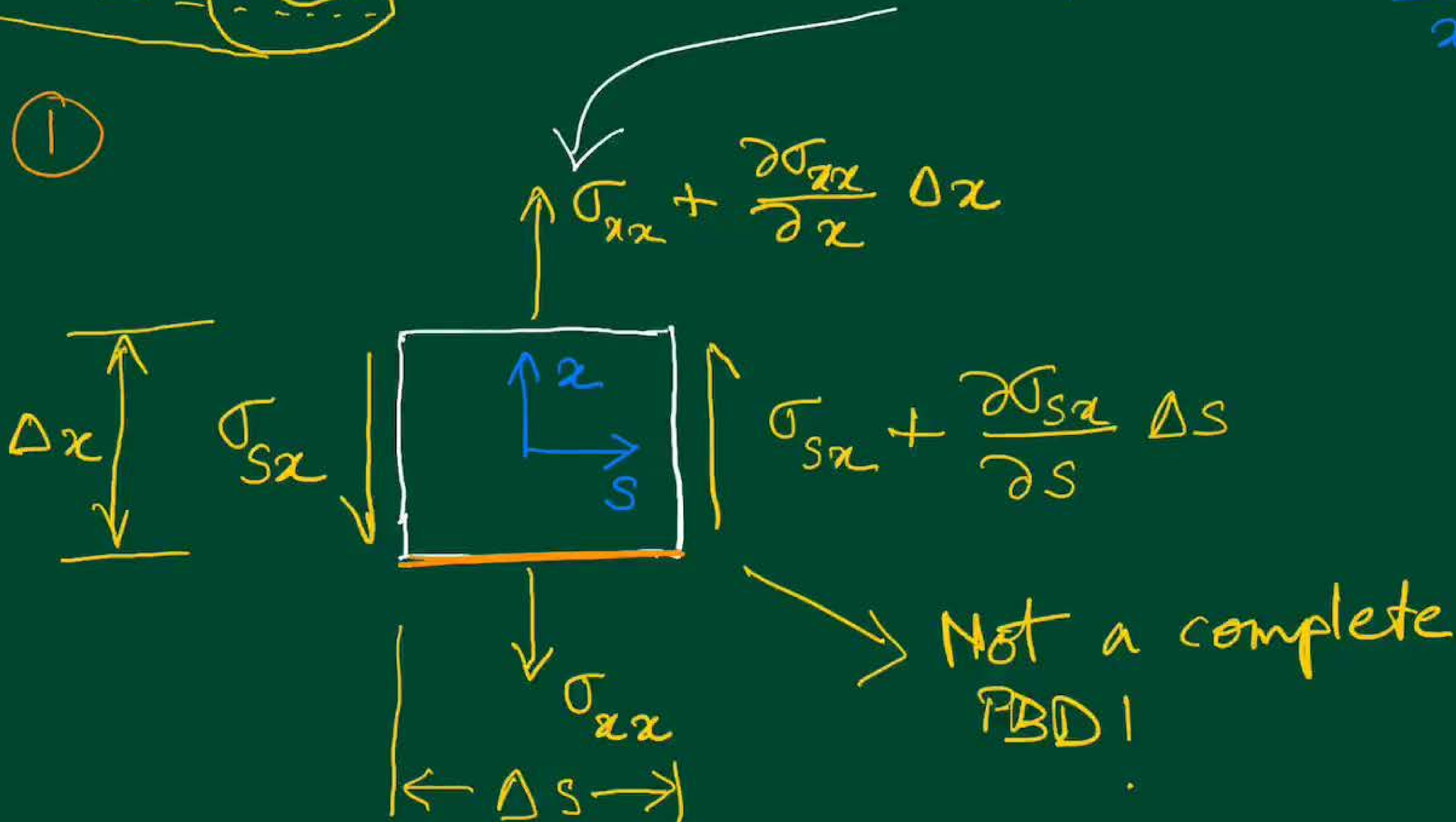



y
z
x
e

1


s
$\leftarrow \Delta x \rightarrow$
x


$\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} \Delta x$
Δx
σ_{sx}
x
s
$\sigma_{sx} + \frac{\partial \sigma_{sx}}{\partial s} \Delta s$
σ_{xx}
$\leftarrow \Delta s \rightarrow$
Not a complete FBD!

For force equilibrium along x-direction:

$$-\sigma_{xx}\, t\, \Delta s + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x\right) t\, \Delta s$$

$$-\sigma_{sx}\, t\, \Delta x + \left(\sigma_{sx} + \frac{\partial \sigma_{sx}}{\partial s}\Delta s\right) t\, \Delta x = 0$$

$$\Rightarrow \frac{\partial \sigma_{sx}}{\partial s}\cancel{t} + \frac{\partial \sigma_{xx}}{\partial x}\cancel{t} = 0$$

$$\Rightarrow \sigma_{sx}\Big|_{s} - \cancel{\sigma_{sx}\Big|_{0}}^{\,0} = -\int_0^s \frac{\partial \sigma_{xx}}{\partial x}\, ds$$

$$\Rightarrow \sigma_{sx} = -\int_0^s \frac{\partial \sigma_{xx}}{\partial x}\, ds$$

But $$\sigma_{xx} = \frac{M_z\left(y I_y - z I_{yz}\right)}{I_{yz}^2 - I_y I_z}$$

$$\therefore \frac{\partial \sigma_{xx}}{\partial x} = \frac{\left(y I_y - z I_{yz}\right)}{I_{yz}^2 - I_y I_z} \frac{\partial M_z}{\partial x}$$

But $$V_y = -\frac{\partial M_z}{\partial x}$$

$$\therefore \sigma_{sx} = -\int_0^s \frac{\partial \sigma_{xx}}{\partial x} ds = \int_0^s V_y \frac{\left(y I_y - z I_{yz}\right)}{I_{yz}^2 - I_y I_z} ds$$

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$$\tau_{xs} = \tau_{sx} = \int_0^s V_y \frac{y I_y - z I_{yz}}{I_{yz}^2 - I_y I_z} \, ds$$

$$= \frac{V_y}{I_{yz}^2 - I_y I_z} \left[I_y \int_0^s y \, ds - I_{yz} \int_0^s z \, ds \right]$$

$$= \frac{V_y}{\left(I_{yz}^2 - I_y I_z\right) t} \left[I_y \int_0^s y t \, ds - I_{yz} \int_0^s z t \, ds \right]$$

$$= \frac{\cancel{V_y} (-P)}{\left(I_{yz}^2 - I_y I_z\right) t} \left[I_y Q_z - I_{yz} Q_y \right]$$

EasyTeach

Next in an actual problem:

Calculate the torsional moment contributed by σ_{xs} ($= \sigma_{sx}$)

and equate it to the torsional moment contributed by the given transverse load P

$$\text{LHS} = \text{RHS} = Pe$$

RHS ↑ contribution from P directly

LHS ↑ contribution from σ_{xs} ($= \sigma_{sx}$) ↳ involve P

[Ultimately, P will cancel from LHS & RHS and e will be independent of P]

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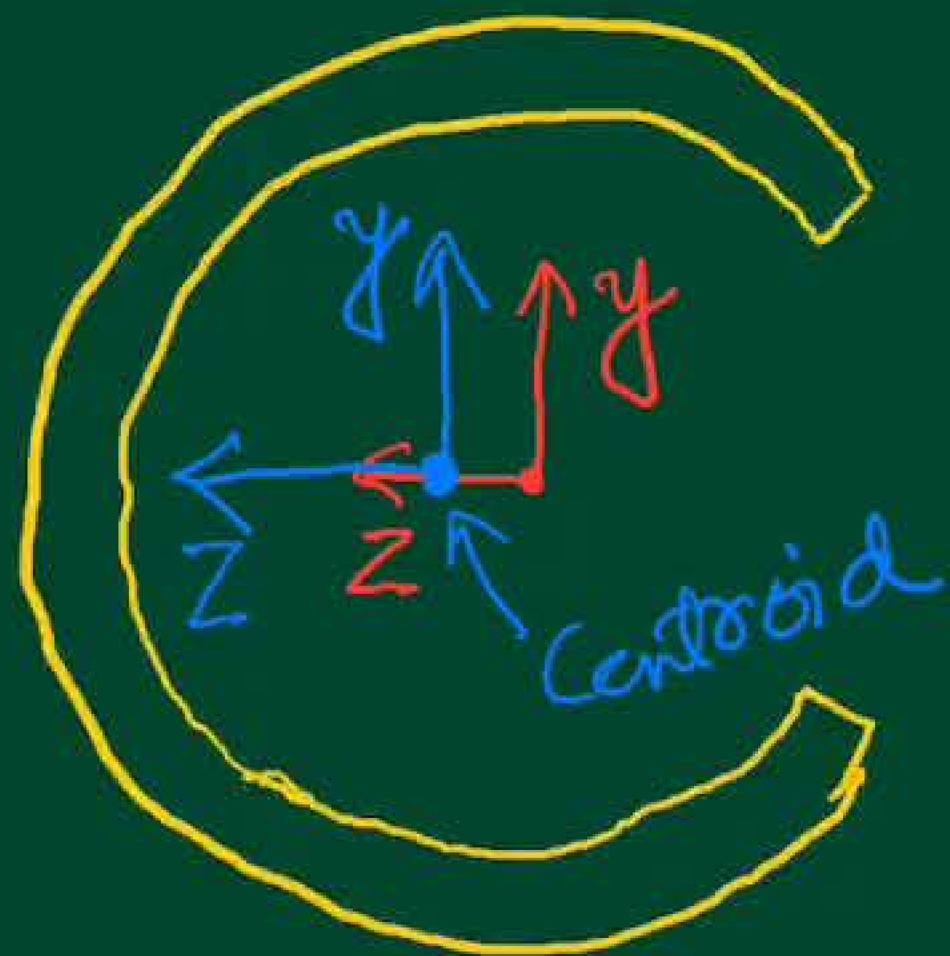


$$\sigma_{xs} = \sigma_{sx} = \frac{-P}{t\left(I_{yz}^2 - I_y I_z\right)}\left[I_y Q_z - I_{yz} Q_y\right]$$

Because of the choice of our axes,

$$I_{yz} = 0$$

$$\therefore \sigma_{xs} = \sigma_{sx} = \frac{P}{\cancel{I_y} I_z} \cancel{I_y} Q_z = \frac{P Q_z}{t I_z}$$

To find the location of the SHEAR CENTRE

✓# Find Q_z all along the c/s

✓# Find I_z for the c/s

\# Find $\sigma_{xs} = \sigma_{sx}$ distribution

\# Find torsional moment contribution and balance

Find Q_z all along the c/s

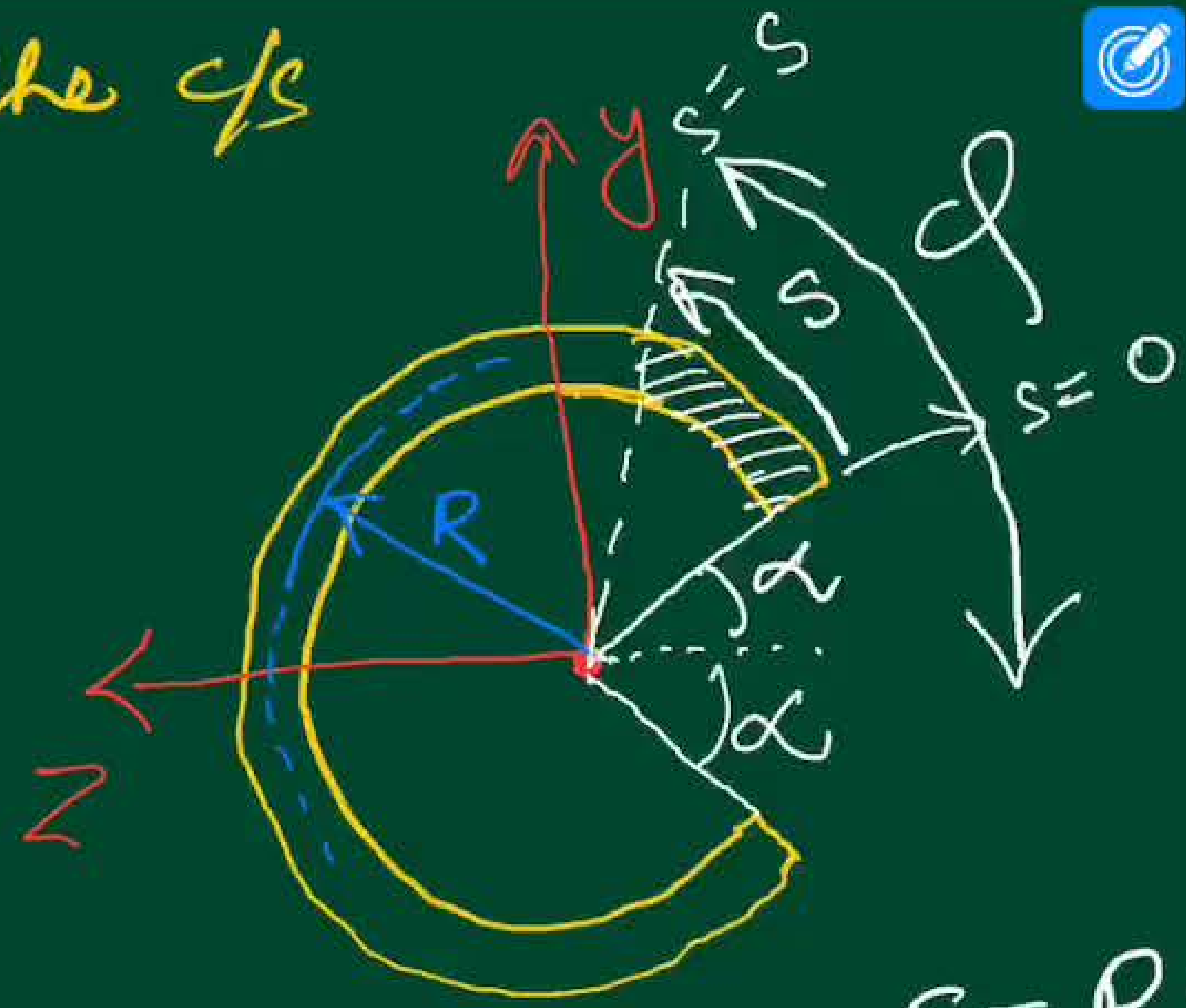
$$Q_z = \int_0^s y\, t\, ds$$

$$= \int_\alpha^\varphi (R\sin\varphi)\, t\, R\, d\varphi$$

$$= R^2 t \int_\alpha^\varphi \sin\varphi\, d\varphi$$

$$= R^2 t \left[-\cos\varphi\right]_\alpha^\varphi$$

$$= R^2 t \left(\cos\alpha - \cos\varphi\right)$$



$$s = R(\varphi - \alpha)$$

$$ds = R\, d\varphi$$

$$y = R\sin\varphi$$

Find I_z for the c/s

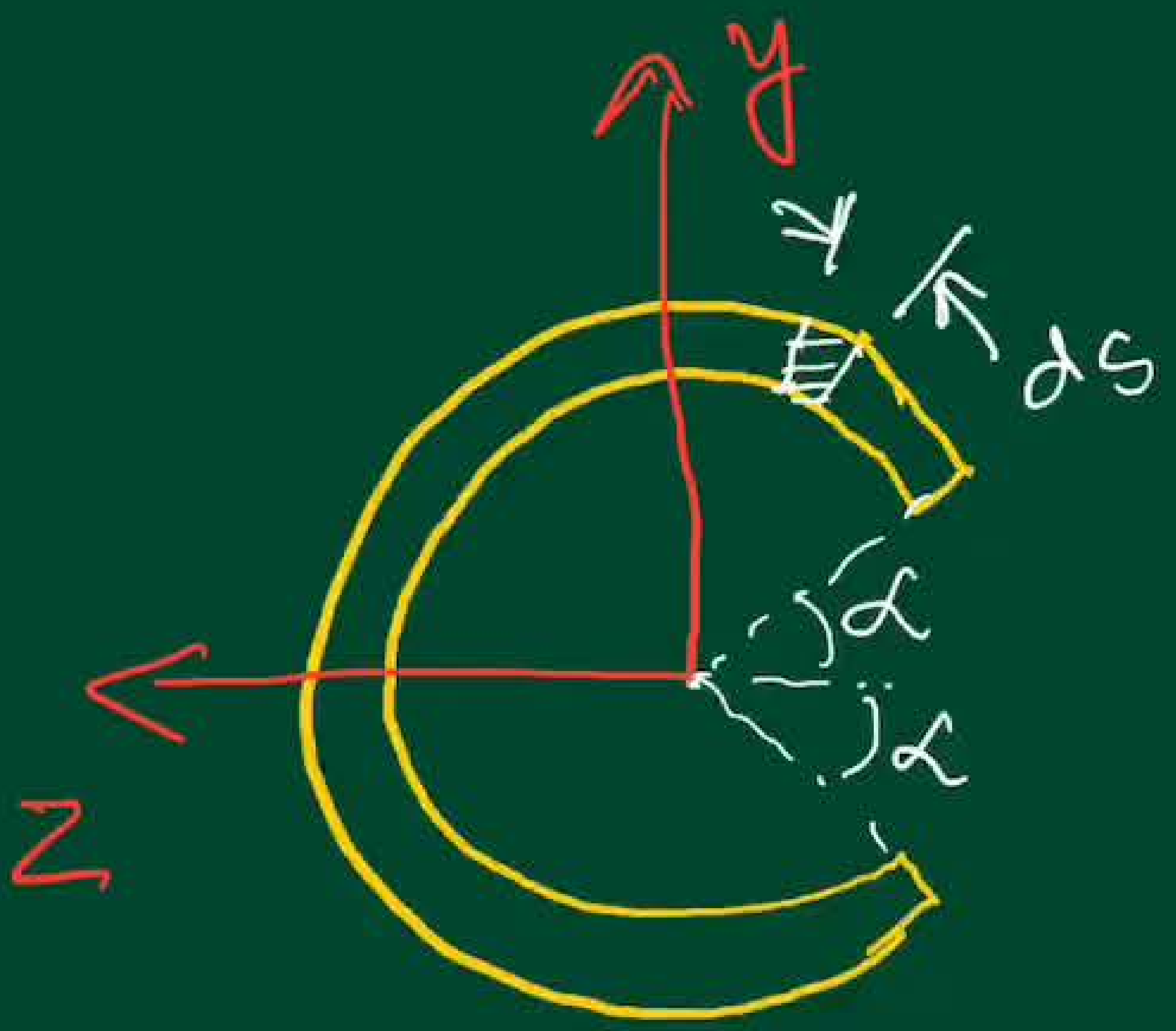
$$I_z = \int_A y^2\, dA$$

$$= \int_{\alpha}^{2\pi-\alpha} (R\sin\varphi)^2\, t R\, d\varphi$$

$$= R^3 t \int_{\alpha}^{2\pi-\alpha} \sin^2\varphi\, d\varphi$$

$$= \frac{1}{2} R^3 t \int_{\alpha}^{2\pi-\alpha} (1-\cos 2\varphi)\, d\varphi$$

$$= \frac{1}{2} R^3 t \left[(2\pi - 2\alpha) + \sin 2\alpha \right]$$

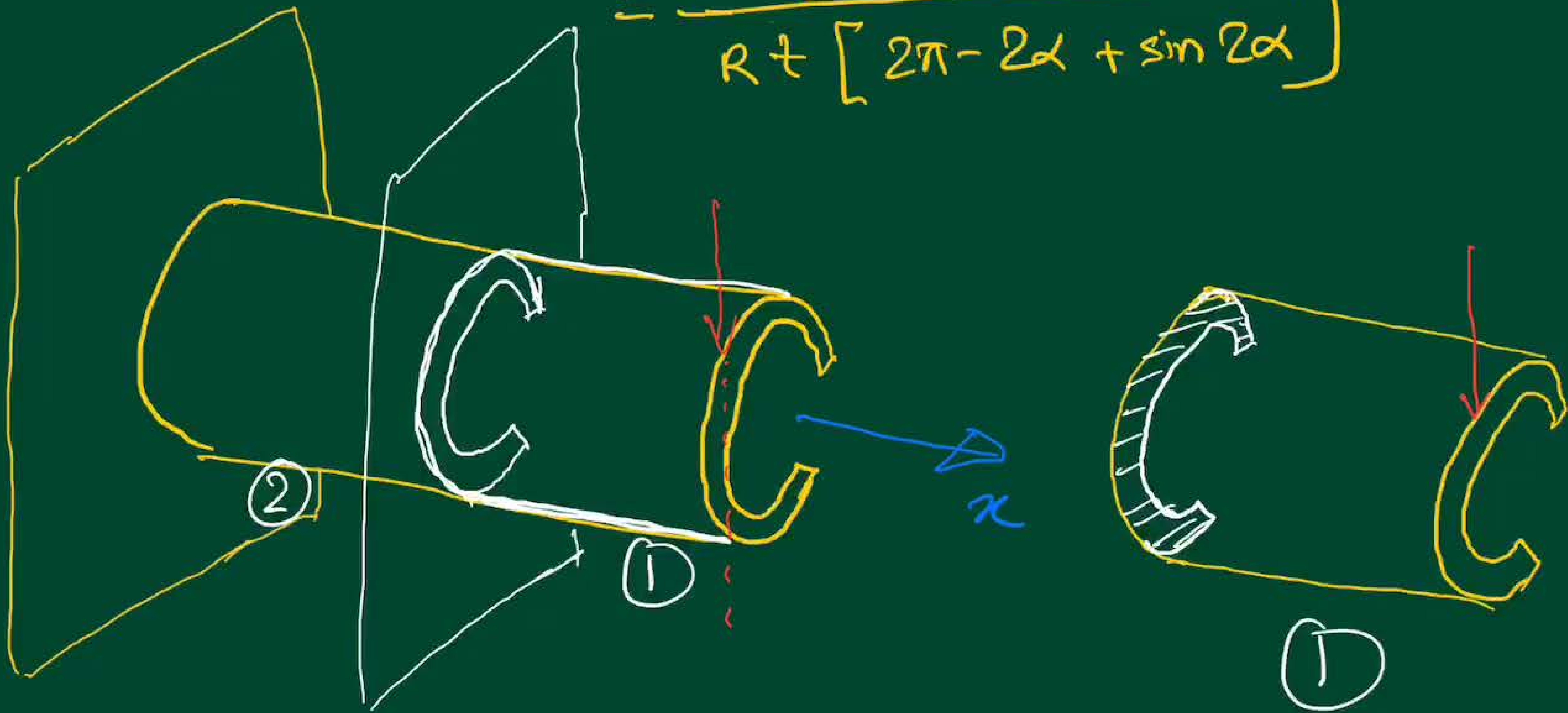


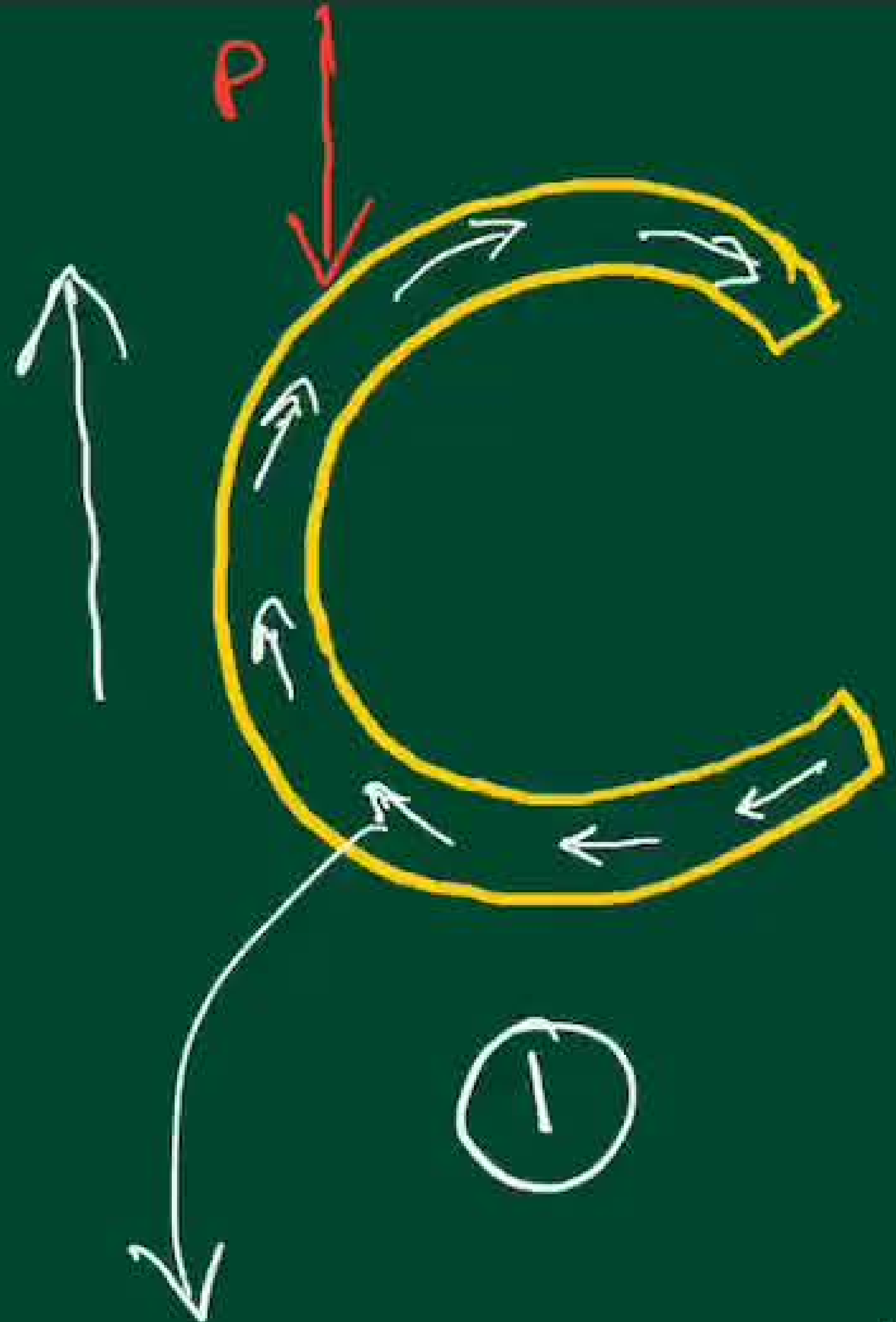
$$dA = t\, ds = t R\, d\varphi$$

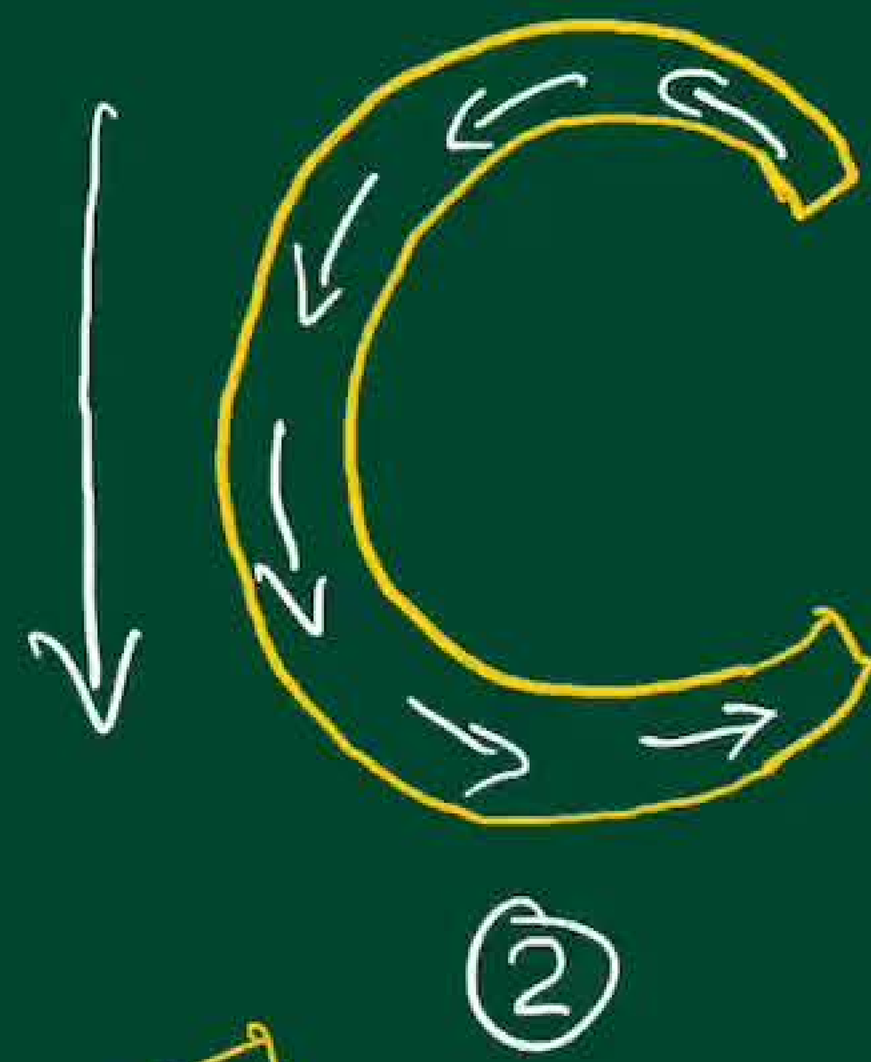
Find $\tau_{xs} = \tau_{sx}$ distribution

$$\tau_{xs} = \frac{P Q_z}{t I_z} = \frac{P R^2 t (\cos\alpha - \cos\phi)}{t \frac{1}{2} R^3 t \left[2\pi - 2\alpha + \sin 2\alpha \right]}$$

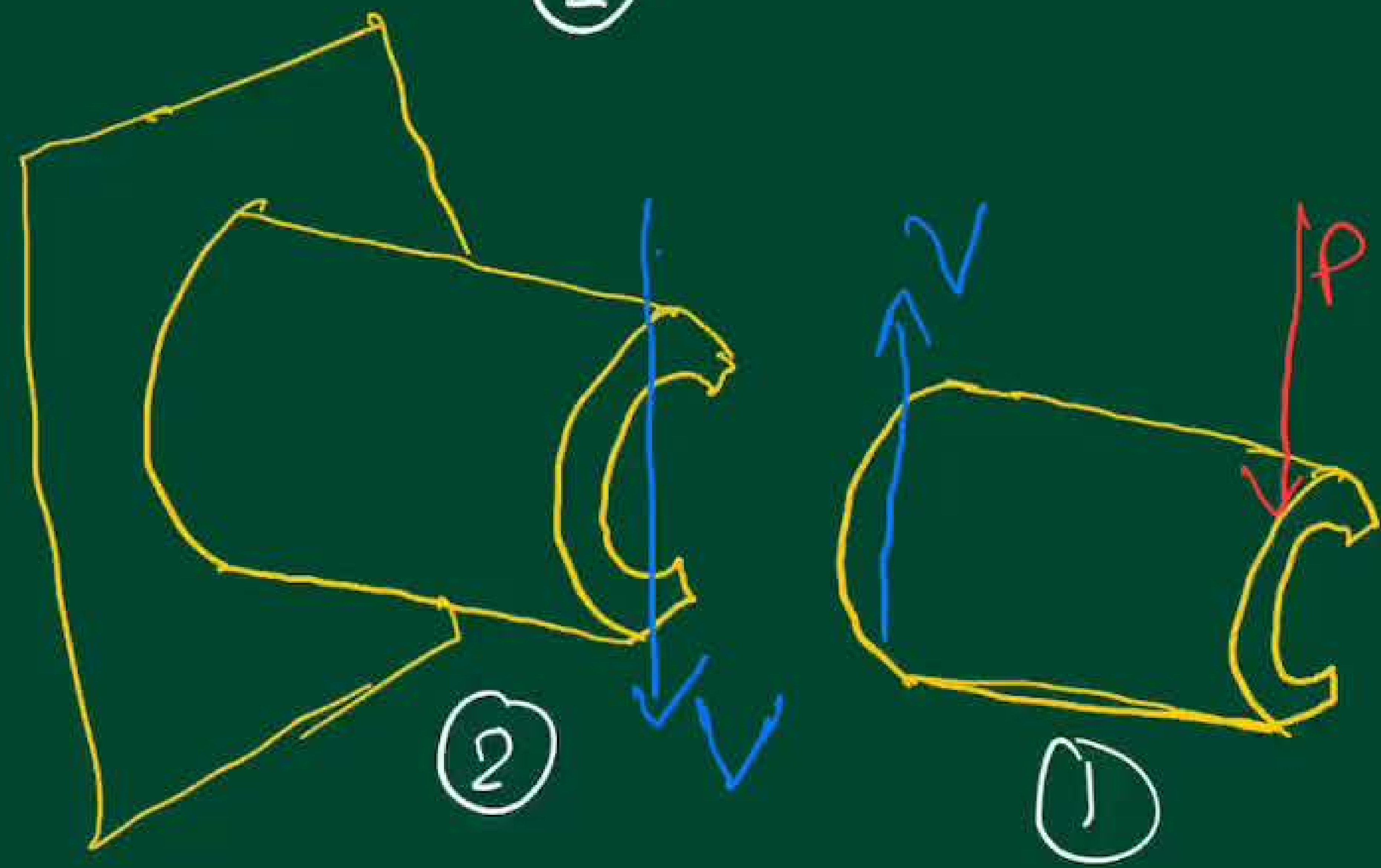
$$= \frac{2P (\cos\alpha - \cos\phi)}{R t \left[2\pi - 2\alpha + \sin 2\alpha \right]}$$

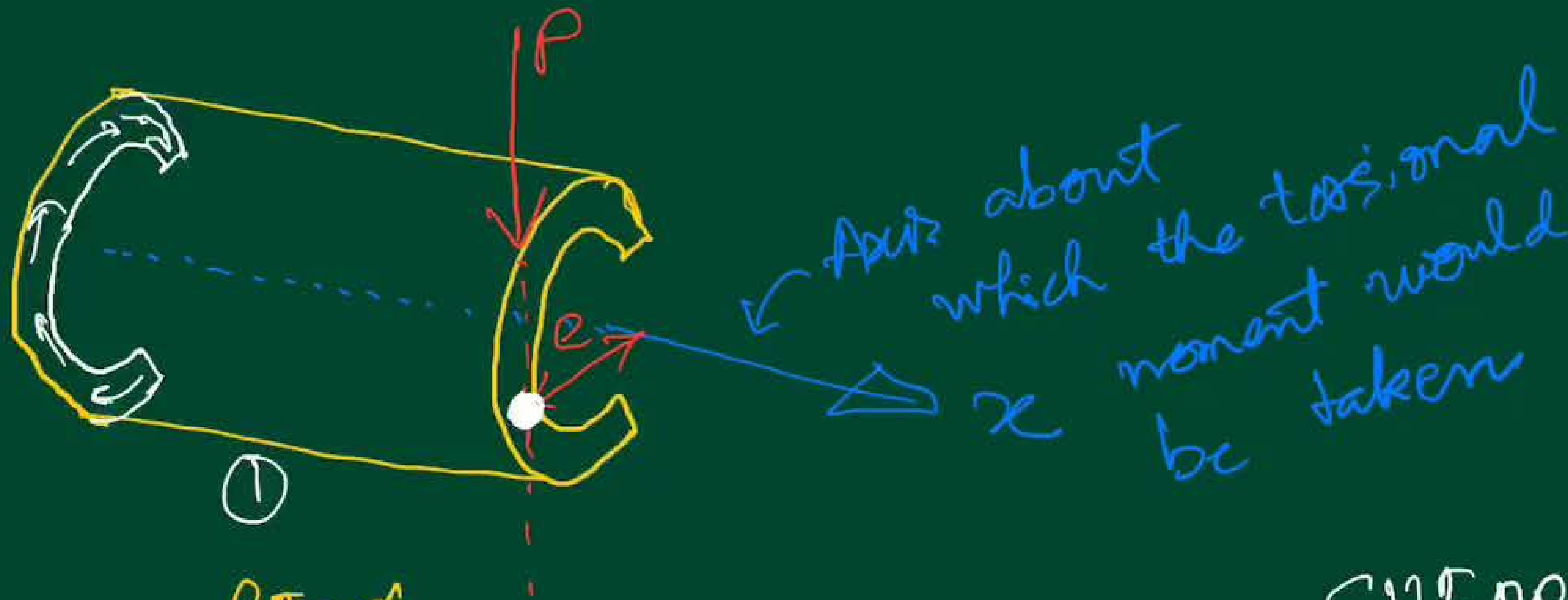






Shear stress distribution is shown on the back face of cut piece ①, i.e. the face flush with the cutting plane and in front of it.





SHEAR CENTRE will be located at $z = e$ and $y = 0$

$$Pe = \int_{\alpha}^{2\pi-\alpha} R\,\tau_{xs}\,(\underbrace{t\,R\,d\varphi}_{dA})$$

$$\Rightarrow \cancel{P}e = \frac{2\cancel{P}R\left[(2\pi-2\alpha)\cos\alpha + 2\sin\alpha\right]}{\left[2\pi - 2\alpha + \sin 2\alpha\right]}$$

e is the distance ∥ to z-axis from the x-axis

The y-coordinate for the SHEAR CENTRE is 0.