Problem Sheet 3: Classical Plate Theory

In view of the COVID-19 situation, none of these problems will form part of any Assignment Sheet meant for submission. These are meant solely for practice and learning.

1. Determine the relation between σ_{nn}, σ_{ns}, σ_{ss} and σ_{xx}, σ_{xy}, σ_{yy} using the rotation matrix.
2. Determine the relation between σ_{nn}, σ_{ns} and σ_{xx}, σ_{xy}, σ_{yy} using the relation between traction vector and stress tensor.
3. Substituting the expressions $M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$, $M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$, and $M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$ in the relation $M_n = n_x^2 M_x + n_y^2 M_y + 2n_x n_y M_{xy}$, and transforming from the xy-coordinate system to the ns-coordinate system, show that (you can use SymPy in a Jupyter Notebook)
$$M_n = -D\left[\nu \frac{\partial^2 w}{\partial s^2} + \nu \frac{\partial \theta}{\partial s}\frac{\partial w}{\partial n} + \frac{\partial^2 w}{\partial n^2} + (1-\nu)\sin\theta\cos\theta \frac{\partial \theta}{\partial s}\frac{\partial w}{\partial s}\right].$$
4. Using the expression for M_n from the previous question, show that for a circular plate under axisymmetric conditions: $M_n \equiv M_r = -D\left(\frac{\mathrm{d}^2 w}{\mathrm{d}r^2} + \frac{\nu}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\right)$.
5. Using the expressions for the shear force per unit length $Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}$ and $Q_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y}$ in the relation $Q_n = n_x Q_x + n_y Q_y$ together with appropriate transformations between coordinate systems, show that for a circular plate under axisymmetric conditions $Q_r = D\frac{\mathrm{d}w}{\mathrm{d}r}\left\{\frac{1}{r}\frac{\mathrm{d}w}{\mathrm{d}r}\left(r\frac{\mathrm{d}w}{\mathrm{d}r}\right)\right\}$.
6. Starting from the general expressions for boundary conditions obtained from the variational formulation, show that the expression for the effective shear force per unit length for a circular plate under axisymmetric conditions reduces to just the shear force per unit length.
7. For an annular plate that is simply supported at the outer periphery and which is loaded by an applied moment at the inner periphery, write down the boundary conditions.