Range Image Processing (Week-09: Lectures 41-45)

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Range data

- Range data is a 2-1/2 D or 3-D representation of the scene.
- An image $d(i, j)$, which records the distance *d* to the corresponding scene point (X, Y, Z) for each image pixel (i, j).
- It could be provided as a set of 3-D scene points (point cloud).

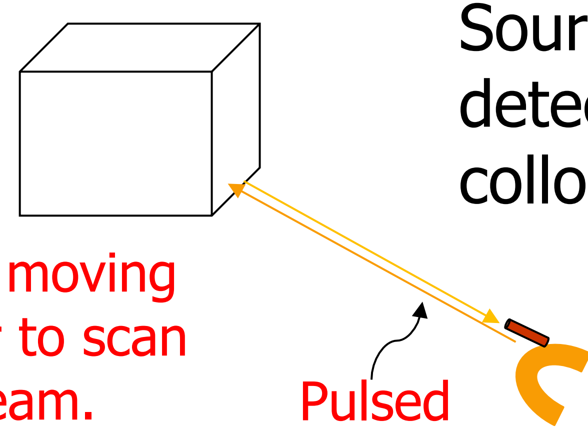
Imaging techniques

- Passive imaging.
  - Stereo imaging
- Active range sensing
  - Time-of-flight sensors
  - Triangulation-based sensors
  - Structured Light

Time-of-Flight Range Sensors

t: Time taken to travel the forward and return path.
v: speed of light in the given medium.

Distance:
d: $(v \times t)/2$



Laser-based time-of-flight range sensors: light detection and ranging (LIDAR) or laser radar (LADAR) sensors.

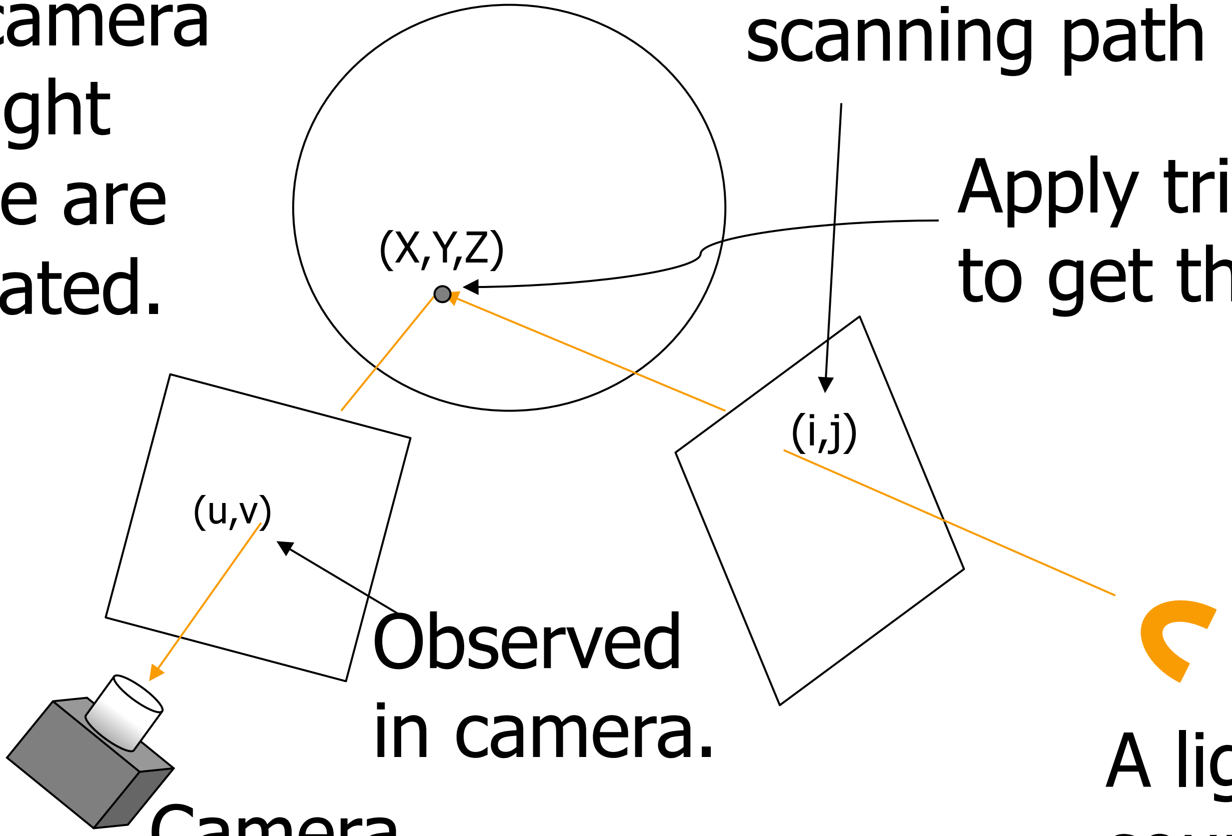
Limitation:
- the minimum observation time thus limited by the minimum distance observable.

Triangulation based Sensors

The camera and light source are calibrated.

Known for a predetermined scanning path of the beam.

Apply triangulation to get the 3D point.



Observed in camera.

Camera

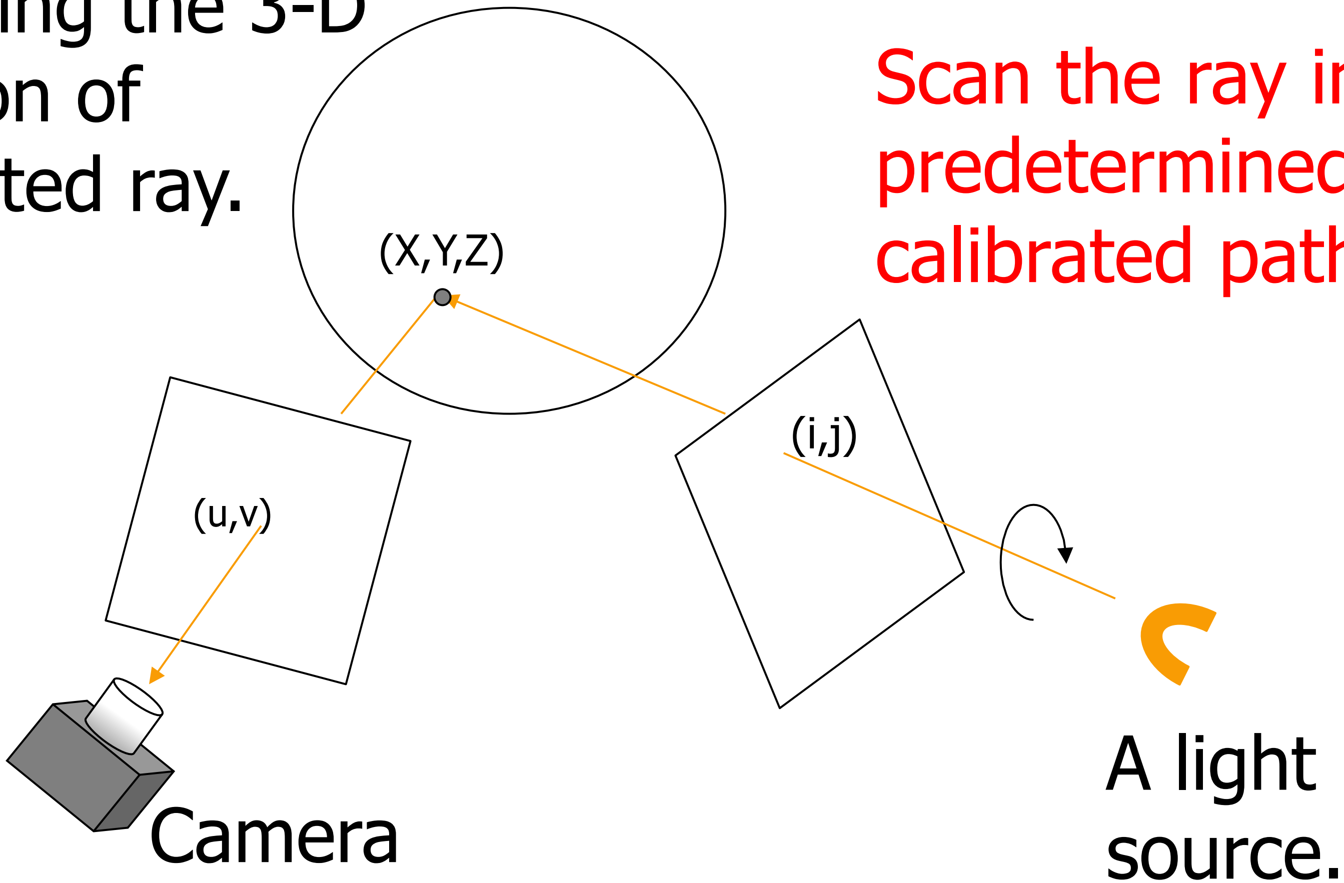
A light source.

Yi-Chih Hsieh, Decoding structured light patterns for three-dimensional imaging systems, Pattern Recognition 34 (2001) 343-349 .

Imaging principle

Encoding the 3-D position of projected ray.

Scan the ray in a predetermined calibrated path.



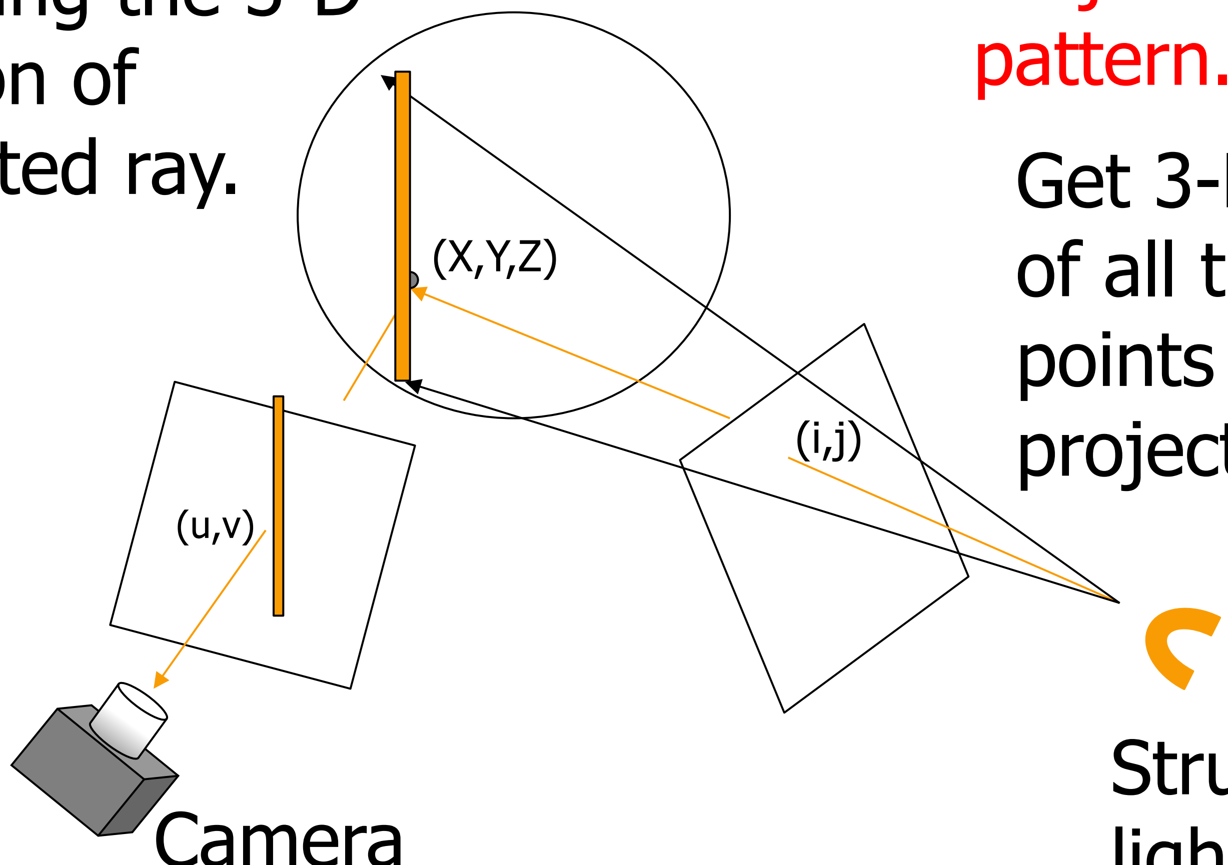
Yi-Chih Hsieh, Decoding structured light patterns for three-dimensional imaging systems, Pattern Recognition 34 (2001) 343-349 .

Structured Light

Encoding the 3-D position of projected ray.

Project a strip or pattern.

Get 3-D positions of all the scene points lying on the projected strip.



Structured light

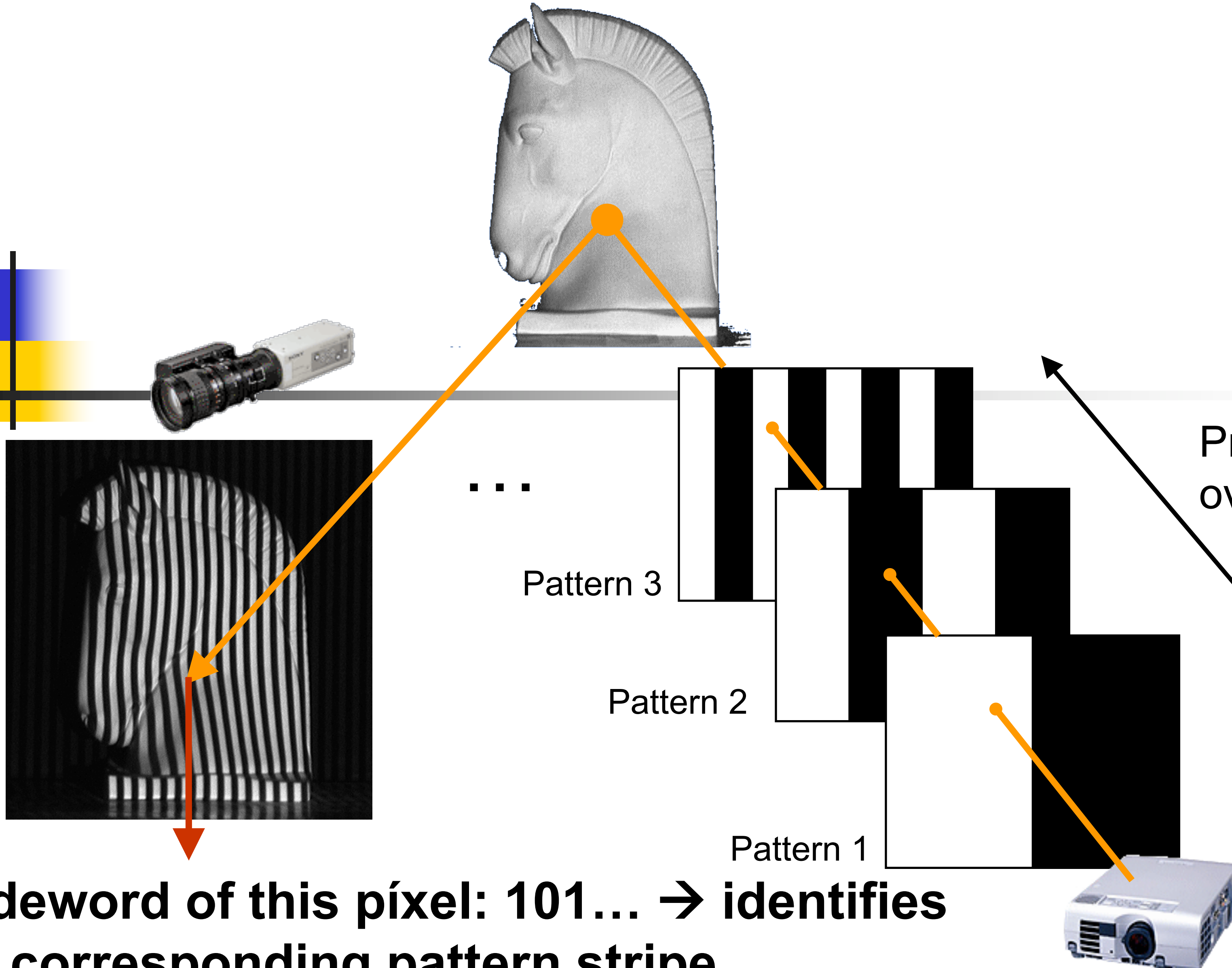
Yi-Chih Hsieh, Decoding structured light patterns for three-dimensional imaging systems, Pattern Recognition 34 (2001) 343-349 .

Space Encoding

- Projection of a sequence of *m* patterns to encode 2^m stripes using a plain binary code.
- Number of stripes get doubled at every new pattern.
- Each light point (i,j) is associated with an *m*-bit binary code and its image is observed.

Binary Coding

Courtesy: Prof. Guido Gerig
http://www.cs.cmu.edu/afs/cs/academic/class/15385-s06/lectures/ppts/lec-17.ppt



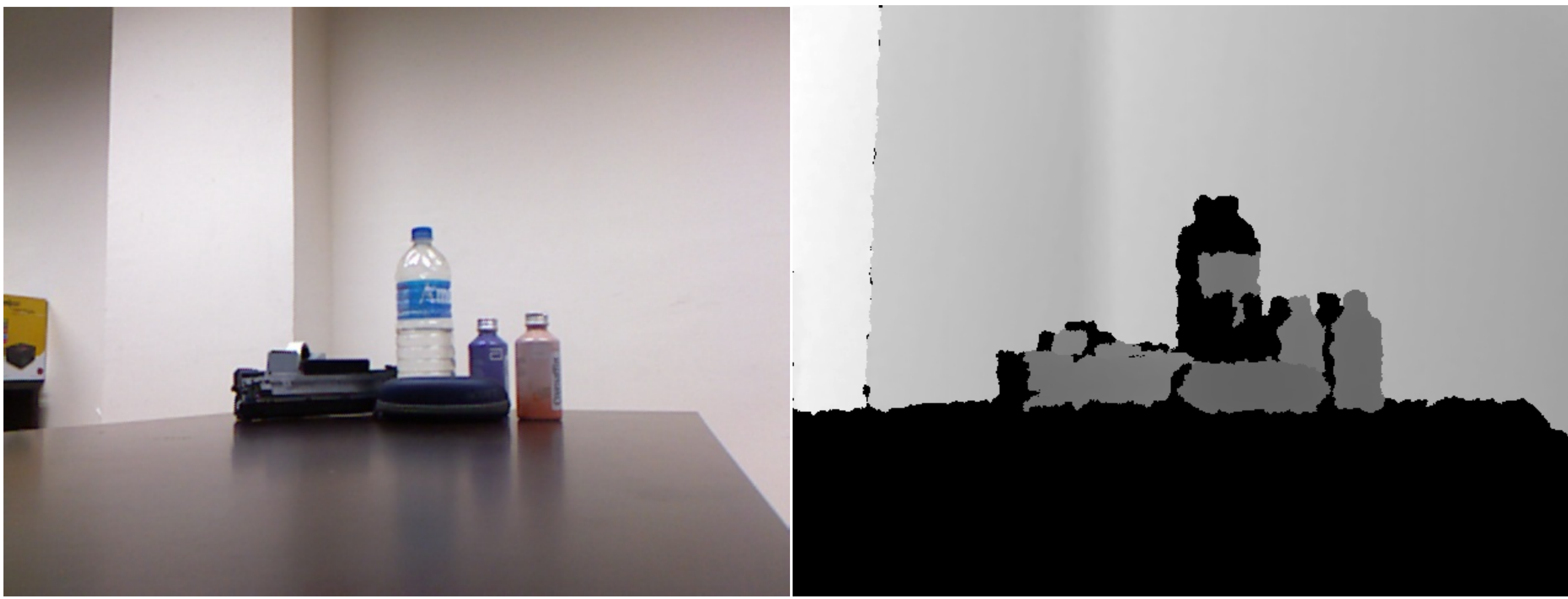
Codeword of this píxel: 101… → identifies the corresponding pattern stripe

Courtesy: Guido Gerig: http://www.cs.cmu.edu/afs/cs/academic/class/15385-s06/lectures/ppts/lec-17.ppt (adapted)

Spatial Codification

- Project a certain kind of spatial pattern uniquely appearing in the image as a set of neighborhood points.
  - Could be spatially arranged dots with varying in size and colors.
- The pattern (providing a codeword) is associated with a calibrated light point.
  - does not require multiple projection over the object.
- The codeword obtained from a neighborhood of the point around it.

RGBD images from Microsoft Kinect



Uses infrared laser light with speckle pattern.

Why lasers?

Light Amplification by Stimulated Emission of Radiation (LASER)

- easily generate bright beams with lightweight sources.
- infrared beams used unobtrusively.
- focus well to give narrow beams.
- single-frequency sources easier to detect.
  - do not disperse from refraction as much as full-spectrum sources.
- semiconductor devices easily generate short pulses.

Parametric curve (2D)

- Parametric curve: $X : I \subset \mathrm{R} \to \mathrm{R}^2$

$$X(t) = (u(t), v(t))$$

- Tangent at a point: $T(t) = (u'(t), v'(t))$

•Curvature at a point: $k(t) = \dfrac{|X' \times X''|}{|X'|^3}$

Determinant in 2D

$$\begin{vmatrix} \dfrac{\partial u}{\partial t} & \dfrac{\partial v}{\partial t} \\ \dfrac{\partial^2 u}{\partial t^2} & \dfrac{\partial^2 v}{\partial t^2} \end{vmatrix}$$

Parametric curve (3D)

- Parametric curve: $X : I \subset \mathrm{R} \to \mathrm{R}^3$

$$X(t) = (u(t), v(t), w(t))$$

- Tangent at a point: $T(t) = (u'(t), v'(t), w'(t))$

•Curvature at a point: $k(t) = \dfrac{|X' \times X''|}{|X'|^3}$

Cross Product of 3-vectors

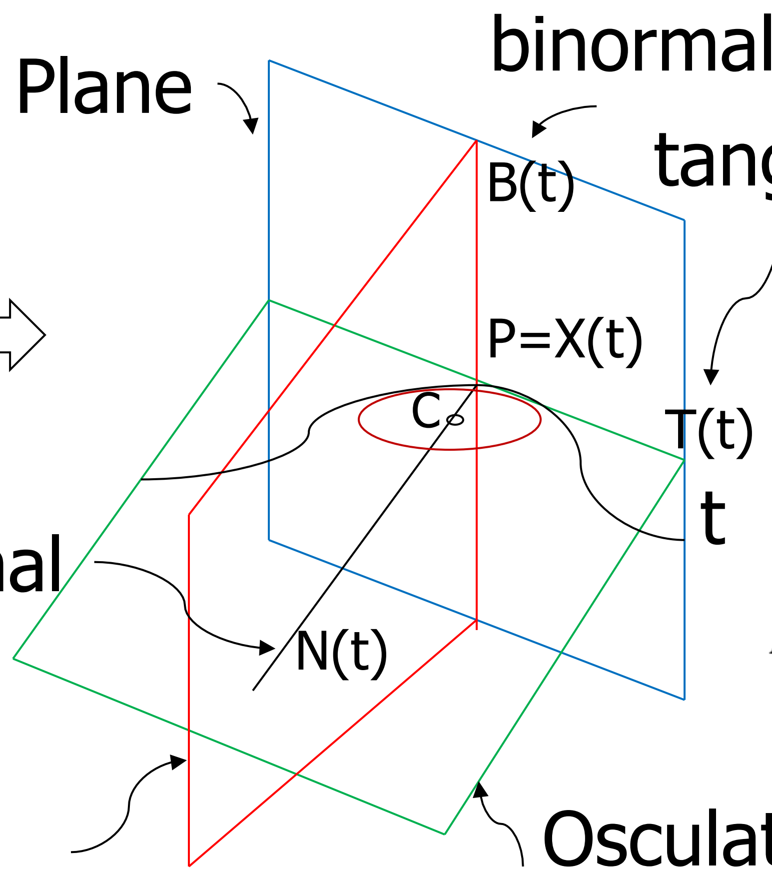
$$X(t) = (u(t), v(t), w(t))$$

$$T(t) = (u'(t), v'(t), w'(t))$$

Parametric curve (3D)

$$k(t) = \frac{|X' \times X''|}{|X'|^3}$$

Moving trihedron
Or
Frenet Frame



Unit vectors

$$\widehat{T'(t)} = k(t)\widehat{N(t)}$$

$$\widehat{B'(t)} = -\tau(t)\widehat{N(t)}$$

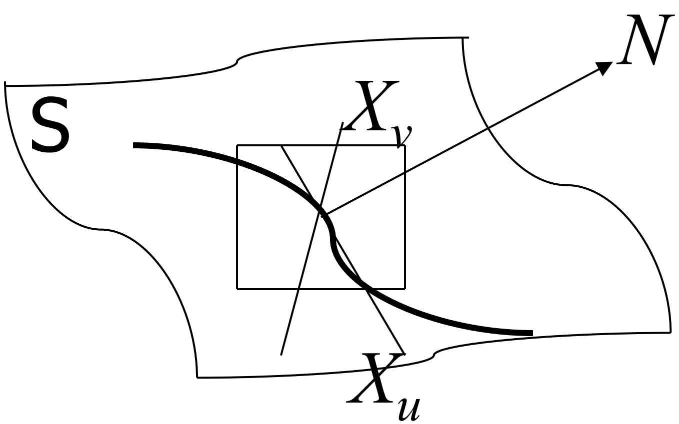
$$\widehat{N'(t)} = -k(t)\widehat{T(t)} + \tau(t)\widehat{B(t)}$$

Formed by T(t) and a point near P=X(t).

Parametric surfaces

- Parametric surface: $x : U \subset \mathbb{R}^2 \to \mathbb{R}^3$

$$X(u,v) = (x(u,v), y(u,v), z(u,v))$$



- Surface Normal:

$$\vec{N} = \frac{X_u \times X_v}{|X_u \times X_v|}$$

Parametric surfaces

- Parametric surface: $x : U \subset \mathrm{R}^2 \rightarrow \mathrm{R}^3$

$$X(u,v) = (x(u,v), y(u,v), z(u,v))$$

- Surface Normal: $\vec{N} = \dfrac{X_u \times X_v}{|X_u \times X_v|}$

- Parameterized curve on the surface: $\beta(t) = (u(t), v(t))$

$$\beta(t) \equiv (x(u(t), v(t), y(u(t), v(t), z(u(t), v(t))$$

- Tangent vector to the curve $\beta(t)$: $\vec{t} = u'(t)X_u + v'(t)X_v$

First Fundamental Form

First fundamental form: The bilinear form that associates two vectors in the tangent plane in the form of dot product.

$$I(\vec{u}.\vec{v}) = \vec{u}.\vec{v}$$

w.r.t. $\beta(t)$

$$I(\vec{t}.\vec{t}) = \vec{t}.\vec{t} = (u'X_u + v'X_v).(u'X_u + v'X_v)$$
$$= (X_u.X_u)u'^2 + 2(X_u.X_v)u'v' + (X_v.X_v)v'^2$$
$$= Eu'^2 + 2Fv' + Gv'^2$$

Magnitude of the tangent vector

$$\begin{cases} E = x_u \,.\, x_u \\ F = x_u \,.\, x_v \\ G = x_v \,.\, x_v \end{cases}$$

Second Fundamental Form

$$X(u,v)=(x(u,v),y(u,v),z(u,v))$$

$$\vec{N}=\frac{X_u\times X_v}{|X_u\times X_v|}$$

$$II(\vec{u}.\vec{v})=\vec{u}.\overrightarrow{dN(v)}$$

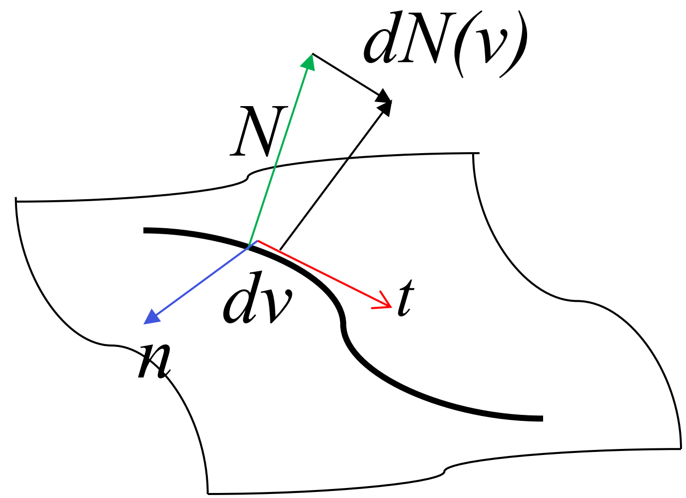
$$\vec{t}.\vec{N}=0$$

$$\frac{d\vec{t}}{dv}.\vec{N}+\vec{t}.\overrightarrow{dN(v)}=0$$

$$k\hat{n}.\vec{N}+\vec{t}.\overrightarrow{dN(v)}=0$$

When they are unit vectors

$$II(\vec{t}.\vec{t})=-k.\cos(\varphi)$$



Angle between curve normal and surface normal

For normal section, φ =zero.➔ $II(\vec{t}.\vec{t})=-k_t$

Normal Curvature

Second Fundamental Form

$$X(u,v) = (x(u,v), y(u,v), z(u,v))$$

$$\vec{t} = u'(t)X_u + v'(t)X_v$$

$$II(\vec{t}.\vec{t}) = \vec{t}.d\overrightarrow{N(t)}$$

- Second fundamental form:

$$II(\vec{t}.\vec{t}) = eu'^2 + 2fu'v' + gv'^2 \qquad \begin{cases} e = -N \, . \, x_{uu} \\ f = -N \, . \, x_{uv} \\ g = -N \, . \, x_{vv} \end{cases}$$

- Normal Curvature: Second fundamental form to be normalized by magnitude of tangent (first normal form)

Second Fundamental Form

$$\vec{t} = u'(t)X_u + v'(t)X_v$$

$$II(\vec{t}.\vec{t}) = \vec{t}.d\overrightarrow{N(t)}$$

- Second fundamental form:

$$\begin{cases} e = -N \,.\, x_{uu} \\ f = -N \,.\, x_{uv} \\ g = -N \,.\, x_{vv} \end{cases}$$

$$II(\vec{t}.\vec{t}) = eu'^2 + 2fu'v' + gv'^2$$

- Normal Curvature: Second fundamental form to be normalized by magnitude of tangent (first normal form)

$$k_t = \frac{II(\vec{t},\vec{t})}{I(\vec{t},\vec{t})} = \frac{eu'^2 + 2fu'v' + gv'^2}{Eu'^2 + 2Fu'v' + Gv'^2}$$

Linear map, Gaussian and Mean curvatures

$e = -N \cdot x_{uu}$
$f = -N \cdot x_{uv}$
$g = -N \cdot x_{vv}$

$E = x_u \cdot x_u$
$F = x_u \cdot x_v$
$G = x_v \cdot x_v$

Linear map: $\begin{bmatrix} e & f \\ f & g \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1}$

Eigen values and eigen vectors of linear map provide principal curvatures (k_1,k_2) associated with principal directions.

Gaussian curvature (K): Determinant of linear map.

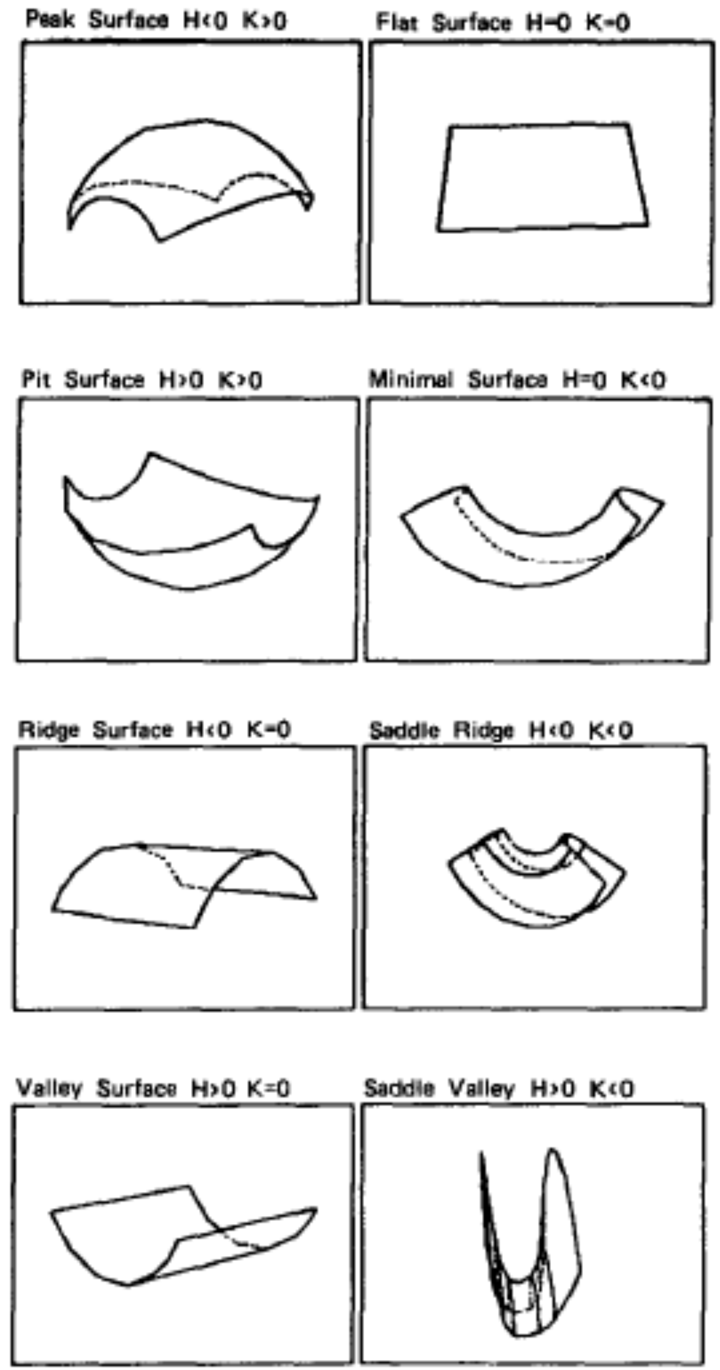
Mean curvature (H): Half of trace of linear map.

$$H = \frac{Eg + Ge - 2Ff}{2(EG - F^2)} = \frac{k_1 + k_2}{2} \qquad K = \frac{eg - f^2}{EG - F^2} = k_1 k_2$$

Principal curvatures are roots of following equation:

$$k^2 - 2Hk + K = 0 \qquad k_{1,2} = H \pm \sqrt{H^2 - K}$$

Eight visible Surface types from signs of curvatures



Invariant Surface Characteristics for 3D Object Recognition in Range Images, PAUL J. BESL AND RAMESH C. JAIN, COMPUTER VISION, GRAPHICS, AND IMAGE PROCESSING 33, 33-80 (1986)

Surface types from signs of curvatures

k_1 (columns), k_2 (rows)

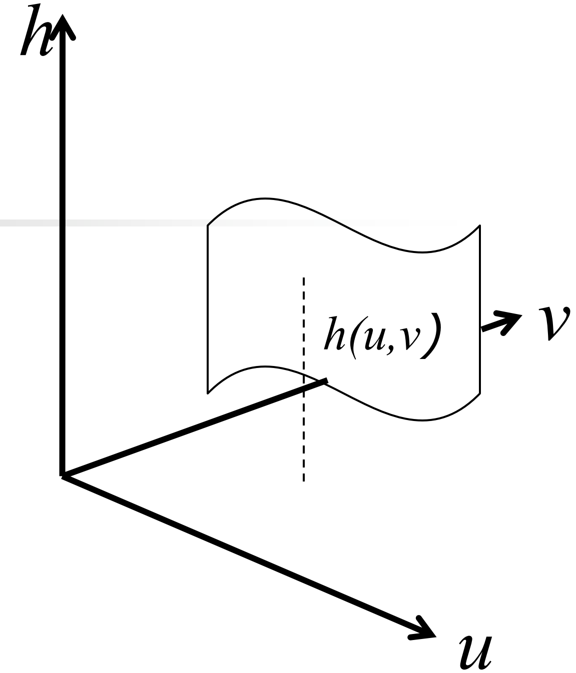
	-	0	+
-	peak	ridge	saddle
0	ridge	flat	valley
+	saddle	valley	pit

K (columns), H (rows)

	-	0	+
-	peak	ridge	Saddle ridge
0	none	flat	Minimal surface
+	pit	valley	Saddle valley

Example: Monge Patches

$$\boxed{X(u,v) = (u, v, h(u,v))}$$



$$\vec{N} = \frac{X_u \times X_v}{|X_u \times X_v|}$$

$$e = -N \,.\, x_{uu} \qquad E = x_u \,.\, x_u$$
$$f = -N \,.\, x_{uv} \qquad F = x_u \,.\, x_v$$
$$g = -N \,.\, x_{vv} \qquad G = x_v \,.\, x_v$$

In this case

- $\mathbf{N} = \dfrac{1}{(1+h_u^2+h_v^2)^{1/2}} (-h_u, -h_v, 1)^T$
- $E = 1+h_u^2; \;\; F = h_u h_v; \; G = 1+h_v^2$
- $e = \dfrac{-h_{uu}}{(1+h_u^2+h_v^2)^{1/2}}; \;\; f = \dfrac{-h_{uv}}{(1+h_u^2+h_v^2)^{1/2}}; \;\; g = \dfrac{-h_{vv}}{(1+h_u^2+h_v^2)^{1/2}}$

Monge Patches: Mean and Gaussian Curvatures

Mean Curvature

$$H = \frac{-h_{uu}(1 + h_u^2) \quad - h_{vv}(1 + h_v^2) \quad + 2h_{uv}h_u h_v}{2(1 + h_u^2 + h_v^2)^{\frac{3}{2}}}$$

Gaussian Curvature

$$K = \frac{h_{uu}h_{vv} - h_{uv}^2}{(1 + h_u^2 + h_v^2)^2}$$

Ex. 1

Consider the following representation of a surface ($x(u,v)$, $y(u,v)$, $z(u,v)$), 0 <= u,v <=1.

$$\begin{bmatrix} f_1(u) \\ f_2(u) \\ f_3(u) \end{bmatrix} = \begin{bmatrix} 3 & 2 & 4 \\ -5 & 6 & 2 \\ 1 & 4 & -3 \end{bmatrix} \begin{bmatrix} u^2 \\ u \\ 1 \end{bmatrix} \qquad \begin{bmatrix} g_1(v) \\ g_2(v) \\ g_3(v) \end{bmatrix} = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 7 & 8 \\ -5 & 6 & 3 \end{bmatrix} \begin{bmatrix} v^2 \\ v \\ 1 \end{bmatrix}$$

$$x(u,v) = f_1(u)g_1(v) \quad y(u,v) = f_2(u)g_2(v) \quad z(u,v) = f_3(u)g_3(v)$$

Compute the surface normal, Gaussian and mean curvatures at (u,v)=(0.5,0.5).

Ans.1

$$F(u) = \begin{bmatrix} 3 & 2 & 4 \\ -5 & 6 & 2 \\ 1 & 4 & -3 \end{bmatrix} \begin{bmatrix} u^2 \\ u \\ 1 \end{bmatrix} \qquad G(v) = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 7 & 8 \\ -5 & 6 & 3 \end{bmatrix} \begin{bmatrix} v^2 \\ v \\ 1 \end{bmatrix}$$

$$F'(u) = \begin{bmatrix} 3 & 2 & 4 \\ -5 & 6 & 2 \\ 1 & 4 & -3 \end{bmatrix} \begin{bmatrix} 2u \\ 1 \\ 0 \end{bmatrix} \qquad G'(v) = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 7 & 8 \\ -5 & 6 & 3 \end{bmatrix} \begin{bmatrix} 2v \\ 1 \\ 0 \end{bmatrix}$$

At $u=v=0.5$,

$$F(0.5) = \begin{bmatrix} 5.75 \\ 3.75 \\ -0.75 \end{bmatrix} \quad G(0.5) = \begin{bmatrix} 4.5 \\ 11.75 \\ 4.75 \end{bmatrix} \quad F'(0.5) = \begin{bmatrix} 5 \\ 1 \\ 5 \end{bmatrix} \quad G'(0.5) = \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix}$$

$$X_u = F'(u) \odot G(v) = \begin{bmatrix} 22.5 \\ 11.75 \\ 23.75 \end{bmatrix} \qquad X_v = F(u) \odot G'(v) = \begin{bmatrix} 40.25 \\ 30 \\ -0.75 \end{bmatrix}$$

Element wise multiplication

Ans.1 (contd.)

Surface normal as (u,v): $\hat{n} = \dfrac{X_u \times X_v}{\|X_u \times X_v\|}$ $\xrightarrow{\text{at u=v=0.5}}$ $\begin{bmatrix} -0.587 \\ 0.792 \\ 0.164 \end{bmatrix}$

$$\begin{aligned} E &= X_u . X_u & e &= X_{uu} . \hat{n} \\ F &= X_u . X_v & f &= X_{uv} . \hat{n} \\ G &= X_v . X_v & g &= X_{vv} . \hat{n} \end{aligned} \qquad F''(u) = \begin{bmatrix} 6 \\ -10 \\ 2 \end{bmatrix} \qquad G''(v) = \begin{bmatrix} 8 \\ 2 \\ -10 \end{bmatrix}$$

$$X_{uu} = F''(u) \odot G(v) \quad X_{uv} = F'(u) \odot G'(v) \quad X_{vv} = F(u) \odot G''(v)$$

Linear map: $\begin{bmatrix} e & f \\ f & g \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1}$ Principal Curvatures (k_1, k_2): Eigen values of linear map.

Gaussian curvature: $k_1.k_2$ $\Rightarrow$ $\mathrm{K} = \dfrac{eg - f^2}{EG - F^2}$

Mean curvature: $(k_1+k_2)/2$ $\Rightarrow$ $H = \dfrac{Eg + Ge - 2Ff}{2(EG - F^2)}$

Ans.1 (contd.)

At u=v=0.5

e=107.352 E=1208.375

f= 13.389 F= 1240.312

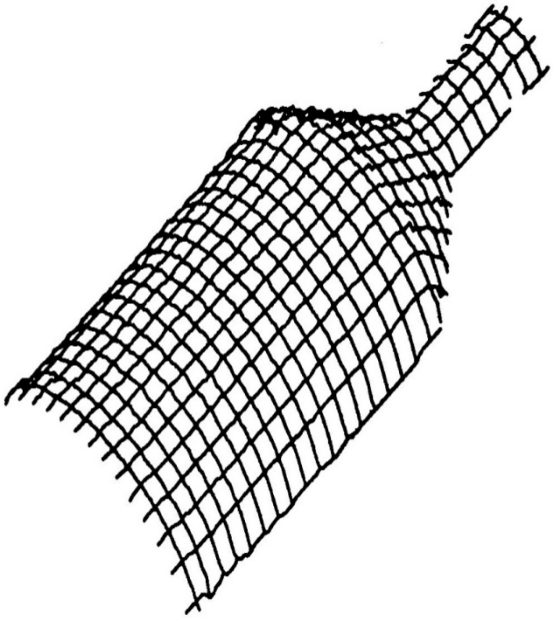
g=19.832 g=2520.625

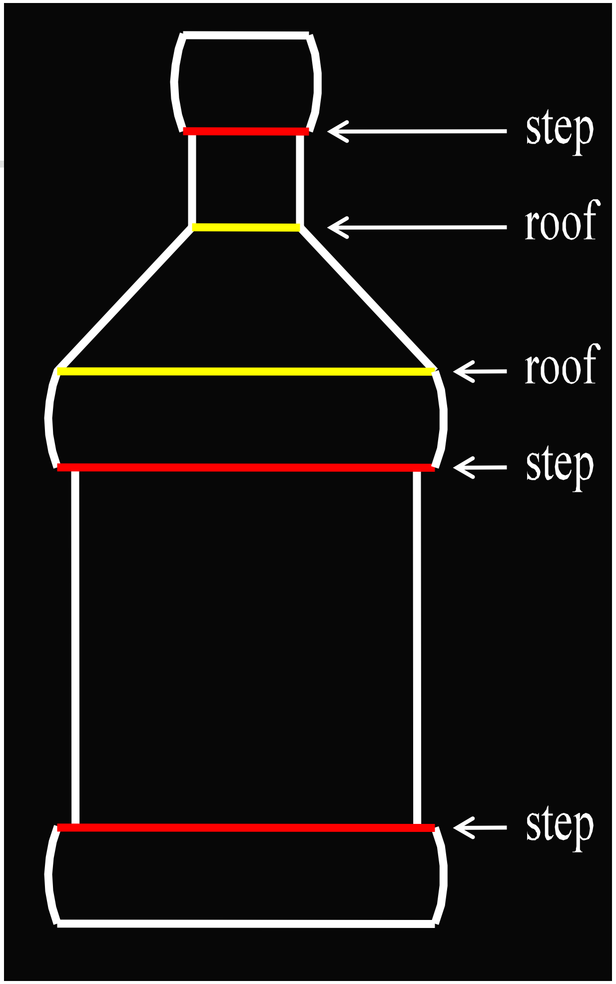
Linear map (L): $\begin{bmatrix} 0.168 & -0.077 \\ 0.006 & .005 \end{bmatrix}$

Gaussian Curvature: Det.(L)=0.0013

Mean Curvature: Trace (L) /2 =0.086

Finding Step and Roof Edges in Range Images

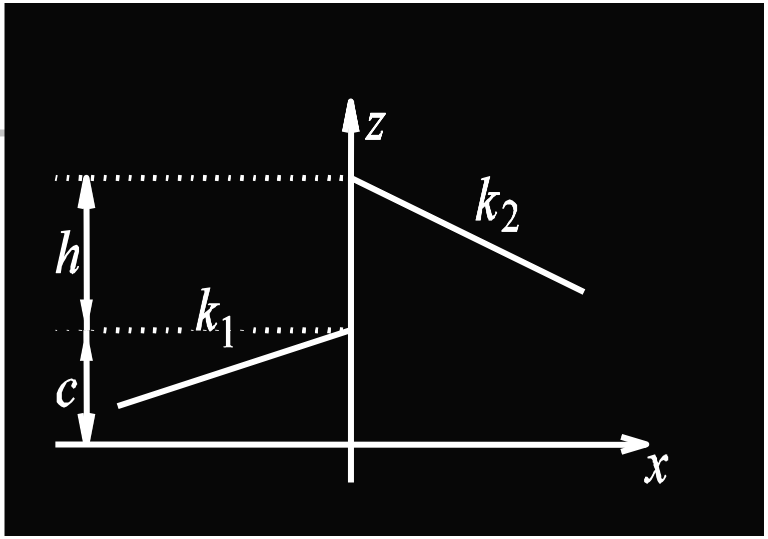




“Describing Surfaces,” by J.M. Brady, J. Ponce, A. Yuille and H. Asada, Proc. International Symposium on Robotics Research, H. Hanafusa and H. Inoue (eds.), MIT Press (1985)

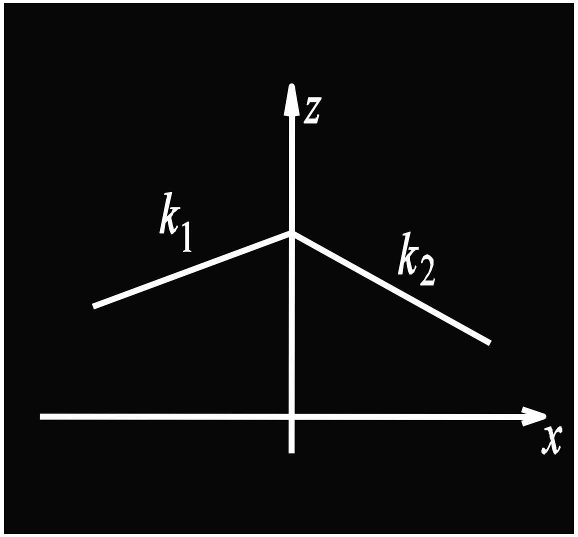
Courtesy : www.cs.unc.edu/Research/vision/comp256/vision23.ppt

Edge Models



Step edge:

$$z = \begin{cases} k_1 x + h \text{ when } x < 0, \\ k_2 x + c + h \text{ when } x > 0. \end{cases}$$

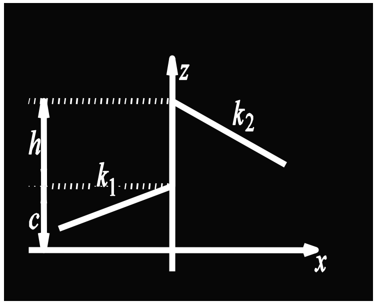


Roof edge:

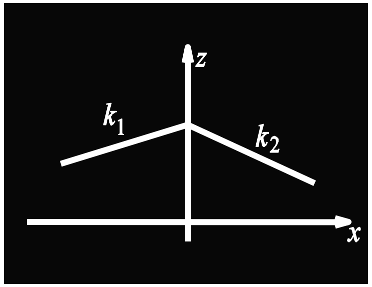
$$z = \begin{cases} k_1 x + h \text{ when } x < 0, \\ k_2 x + h \text{ when } x > 0. \end{cases}$$

Adapted from: www.cs.unc.edu/Research/vision/comp256/vision23.ppt

Characterizing Edges



Step edge



Roof edge

$G(x;\sigma)$=Gaussian mask of scale σ.

$$z''_\sigma(x) \equiv \frac{\partial^2 G(x;\sigma)}{\partial \sigma^2} * z(x)$$

Curvature at Gaussian smoothened $z(x)$: $k_\sigma(x) = \dfrac{z''_\sigma}{\left(1 + z'_\sigma\right)^{\frac{3}{2}}}$

Ratios of 2nd and 1st derivatives of curvatures:

Step edges: Roughly remains constant across scales.

$$\frac{k''_\sigma(x)}{k'_\sigma(x)} = \frac{z''''_\sigma(x)}{z'''_\sigma(x)}$$

Roof edges: Inversely proportional to scale.

Adapted from: www.cs.unc.edu/Research/vision/comp256/vision23.ppt

Characterization of edges

Step edges are zero crossings of one of principal curvatures (or Gaussian curvature), whose position changes with scale.

$$\frac{k''_\sigma}{k'_\sigma}$$ roughly remains constant across scales.

Roof edge is characterized by local maxima of curvature and to be sought in the direction of dominant principal curvature.

$$\sigma \frac{k''_\sigma}{k'_\sigma}$$ roughly remains constant across scales.

"Describing Surfaces," by J.M. Brady, J. Ponce, A. Yuille and H. Asada, Proc. International Symposium on Robotics Research, H. Hanafusa and H. Inoue (eds.), MIT Press (1985)

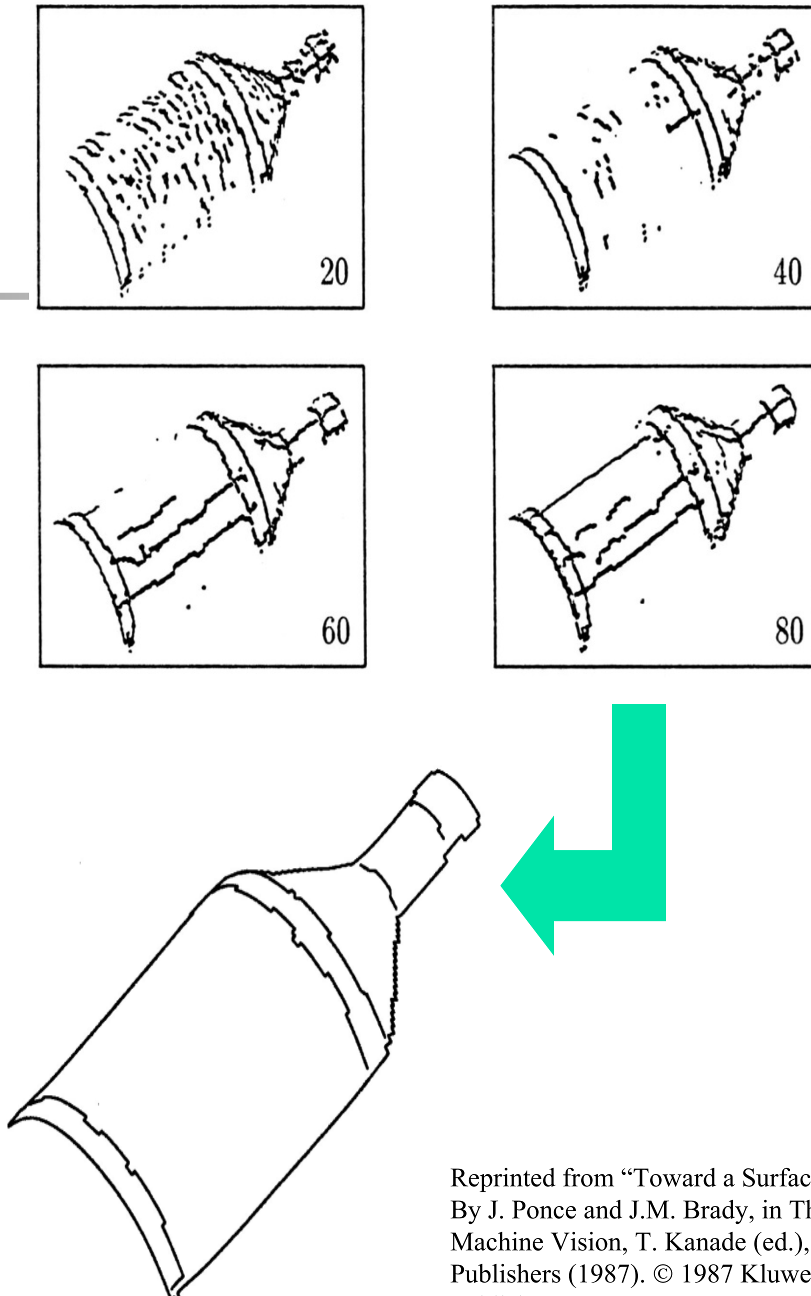
The Algorithm

- Compute a set of Gaussian smoothed images at multiple scales.
- Compute the principal directions and curvatures at each point of the smoothed images.
- Compute zero crossings of the Gaussian curvature and the extrema of the dominant principal curvature in the corresponding principal direction.
  - Zero-crossings: Candidate step edge points.
  - Extrema: Candidate roof edge points.
- Use the analytical models across scales to select candidate points for respective edges.

Multi-scale edge tracking

- Features are tracked from course to fine scales.
- All features at a given scale having no ancestor in coarse scale are eliminated.
- If $\frac{k_\sigma^{''}}{k_\sigma^{'}}$ remains roughly constant across scales output step edge point.
- If $\sigma\frac{k_\sigma^{''}}{k_\sigma^{'}}$ remains roughly constant across scales output roof edge point.
- Retain the points at finest scale as shift from the point is more for larger scale.

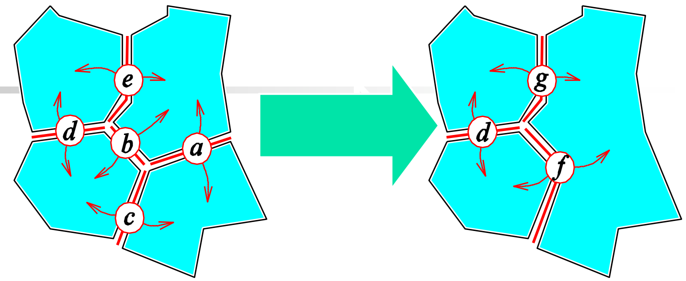
Scale-Space Matching



Reprinted from “Toward a Surface Primal Sketch,” By J. Ponce and J.M. Brady, in Three-Dimensional Machine Vision, T. Kanade (ed.), Kluwer Academic Publishers (1987). © 1987 Kluwer Academic Publishers.

Courtesy: www.cs.unc.edu/Research/vision/comp256/vision23.ppt

Segmentation into Planes via Region Growing



Idea: Iteratively merge the pair of planar regions minimizing the average distance to the plane best fitting them.

Nodes: Planar Patches
Edges: Between adjacent patches.
Arc cost: Avg. distance to the plane best fitting them.

Greedy approach:
Select the best (minimum cost) arc,
and merge the nodes.

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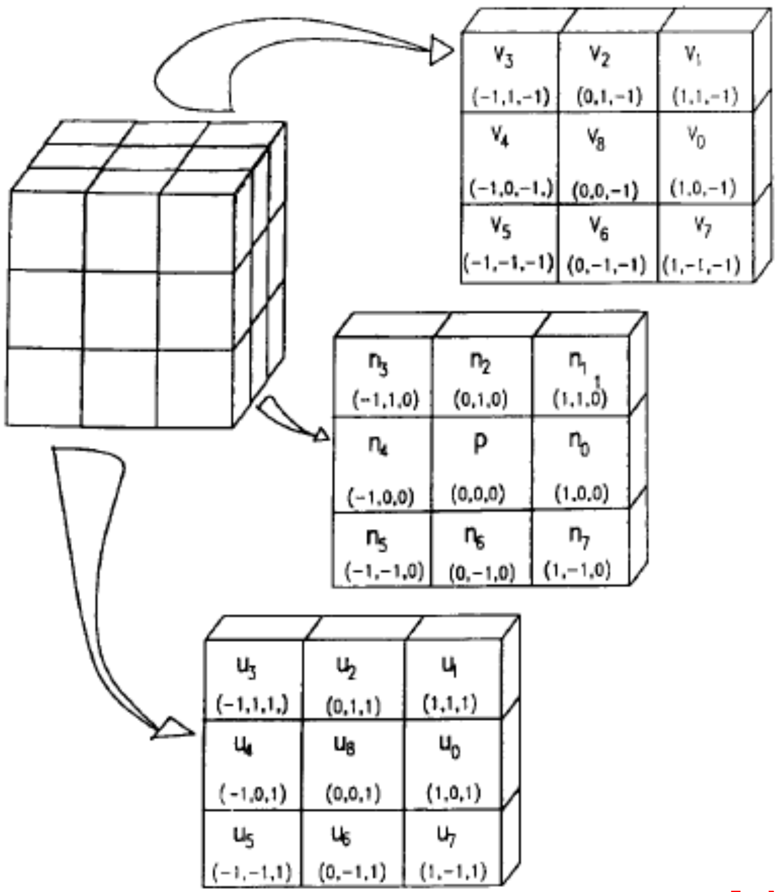
Courtesy: www.cs.unc.edu/Research/vision/comp256/vision23.ppt

Range image segmentation: Morphological approach

- Uses information of local orientation at a point.
- Introduces concepts of digital neighborhood plane (DNP) and neighborhood plane set (NPS).
- Fast and easy computation on checking the arrangement of neighboring points in the 3D discrete space.
- The NPS at each point computed, which induces a unique partition under the equivalence relation on equality of NPS.
- Approximate planar segments formed by region growing.

J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

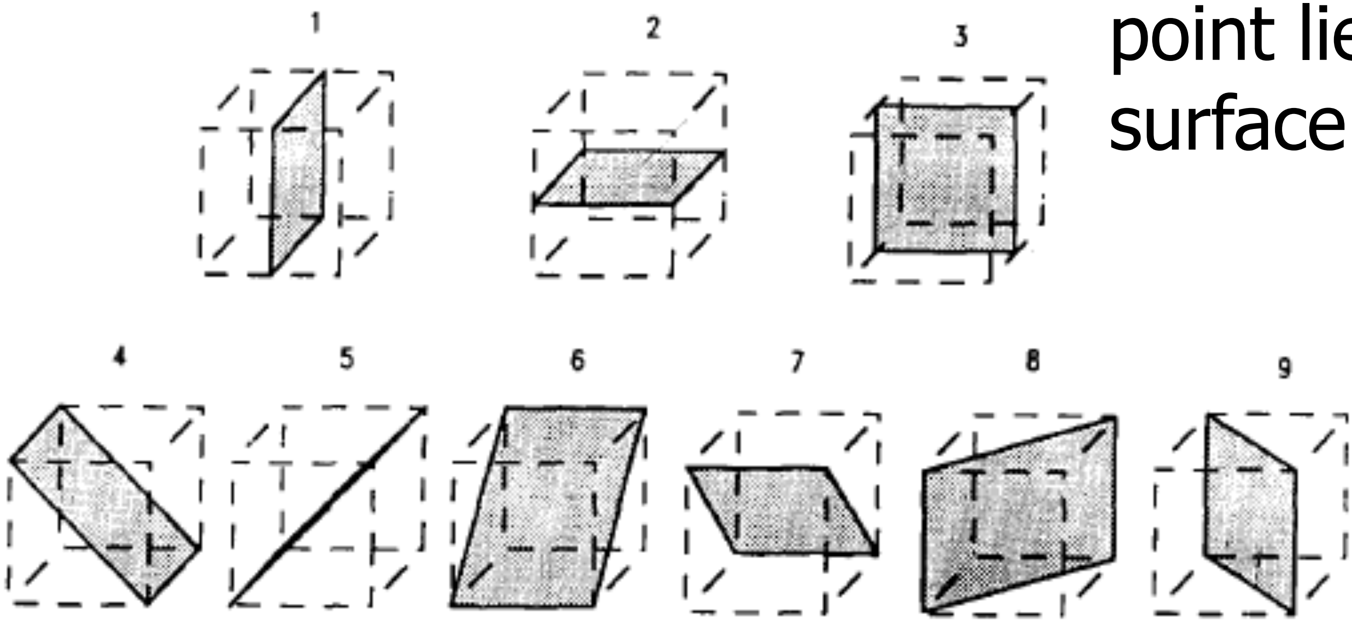
3-D Neighborhood of a point p



J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Digital Neighborhood planes

Assume the point lies on a surface.



Digital Neighborhood planes

J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Digital neighborhood planes

u_2	n_2	v_2
u_8	p	v_8
u_6	n_6	v_6

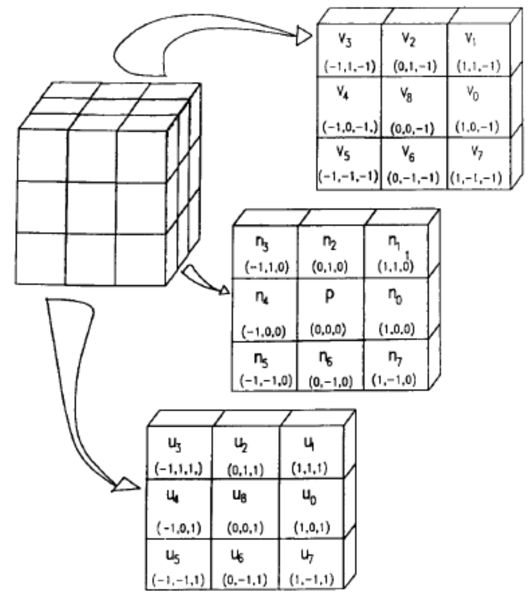
(1)

v_4	v_8	v_0
n_4	p	n_0
u_4	u_8	u_0

(2)

n_3	n_2	n_1
n_4	p	n_0
n_5	n_6	n_7

(3)



Set of points defining DNP's.

J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Digital neighborhood planes

u_3	n_3	v_3
u_8	p	v_8
u_7	n_7	v_7

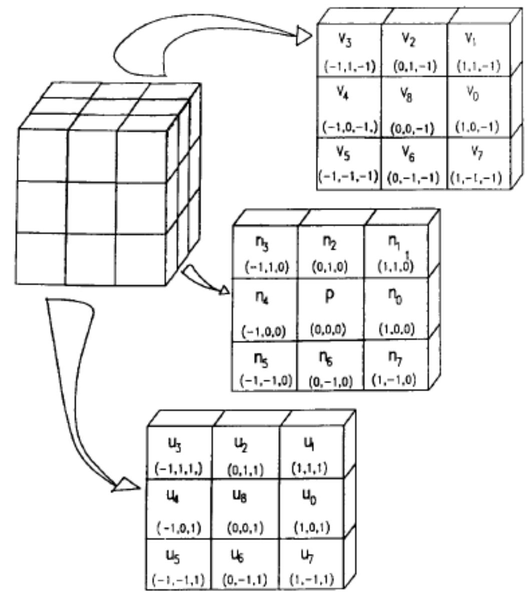
(4)

u_1	n_1	v_1
u_8	p	v_8
u_5	n_5	v_5

(5)

u_3	n_4	v_5
u_2	p	v_6
u_1	n_0	v_7

(6)



Set of points defining DNP’s.

J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Digital neighborhood planes

u_5	n_4	v_3
u_6	p	v_2
u_7	n_0	v_1

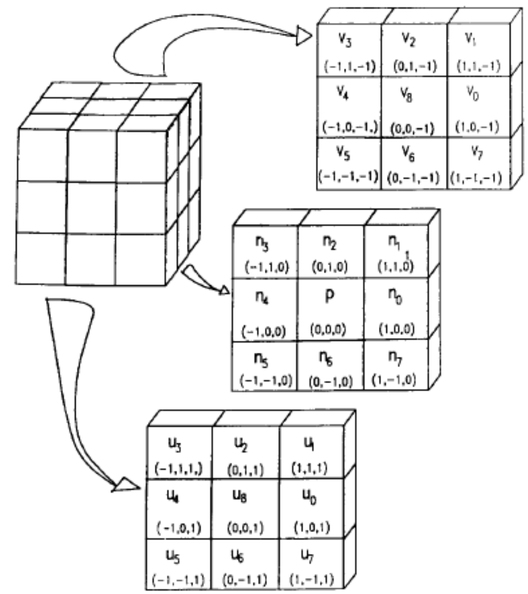
(7)

u_1	n_2	v_3
u_0	p	v_4
u_7	n_6	v_5

(8)

u_3	n_2	v_1
u_4	p	v_0
u_5	n_6	v_7

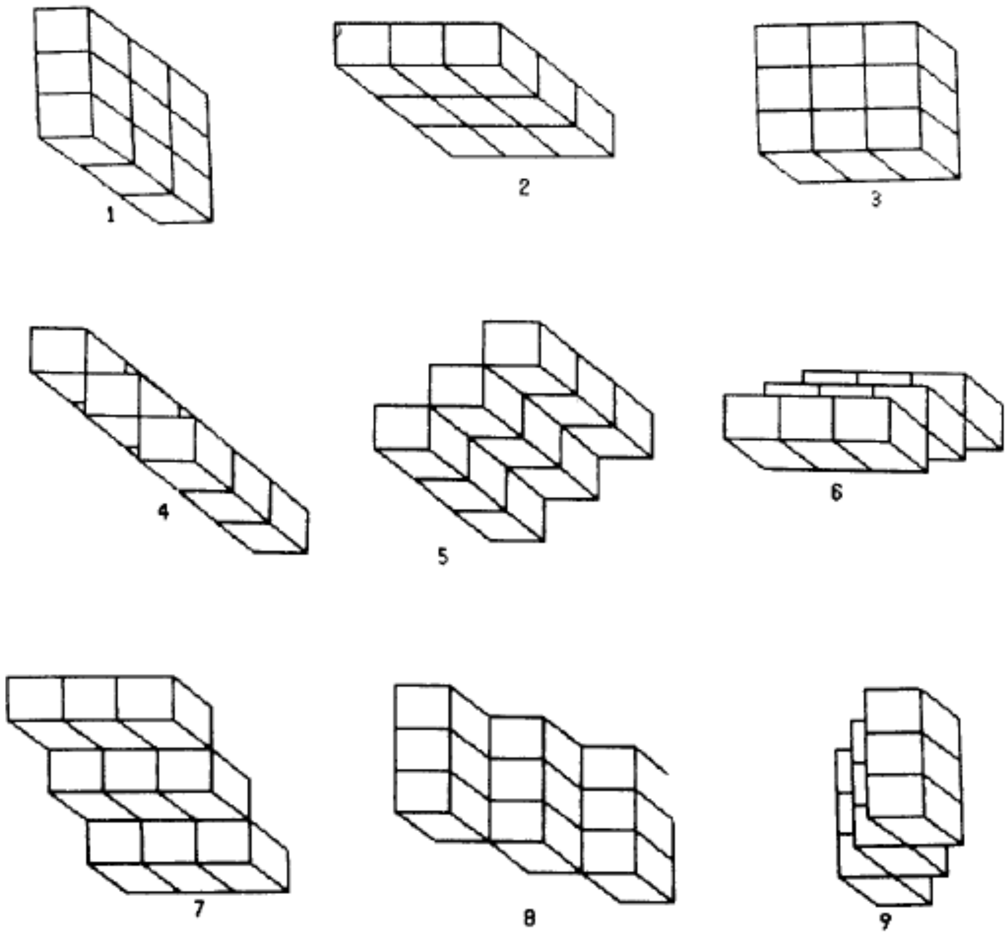
(9)



Set of points defining DNP's.

J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

DNPs: Voxel Sets



J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Neighborhood Plane Set (NPS)

- Let P_i be the set of points assigned to the ith DNP.
- Neighborhood plane set (NPS) of a point p in an image A in Z^3 :

$$[p]_k = \{i : |N(p) \cap A \cap P_i| > k\}, k \geq 3$$

k is a parameter

- threshold number of points required for accepting a DNP in the neighborhood.

Handling noise in range image

NPS at p

$$[p]_k = \{i: |N(p) \cap A \cap P_i| > k\}, k \geq 3$$

Range image: D(x,y) $\Rightarrow$ 3D point: p (x,y, D(x,y))

$N_{abc}(p)$: An extended neighborhood around p of size axbxc.

Map points from $N_{abc}(p)$ to $N(p)$, and use the same definitions of DNP and NPS.

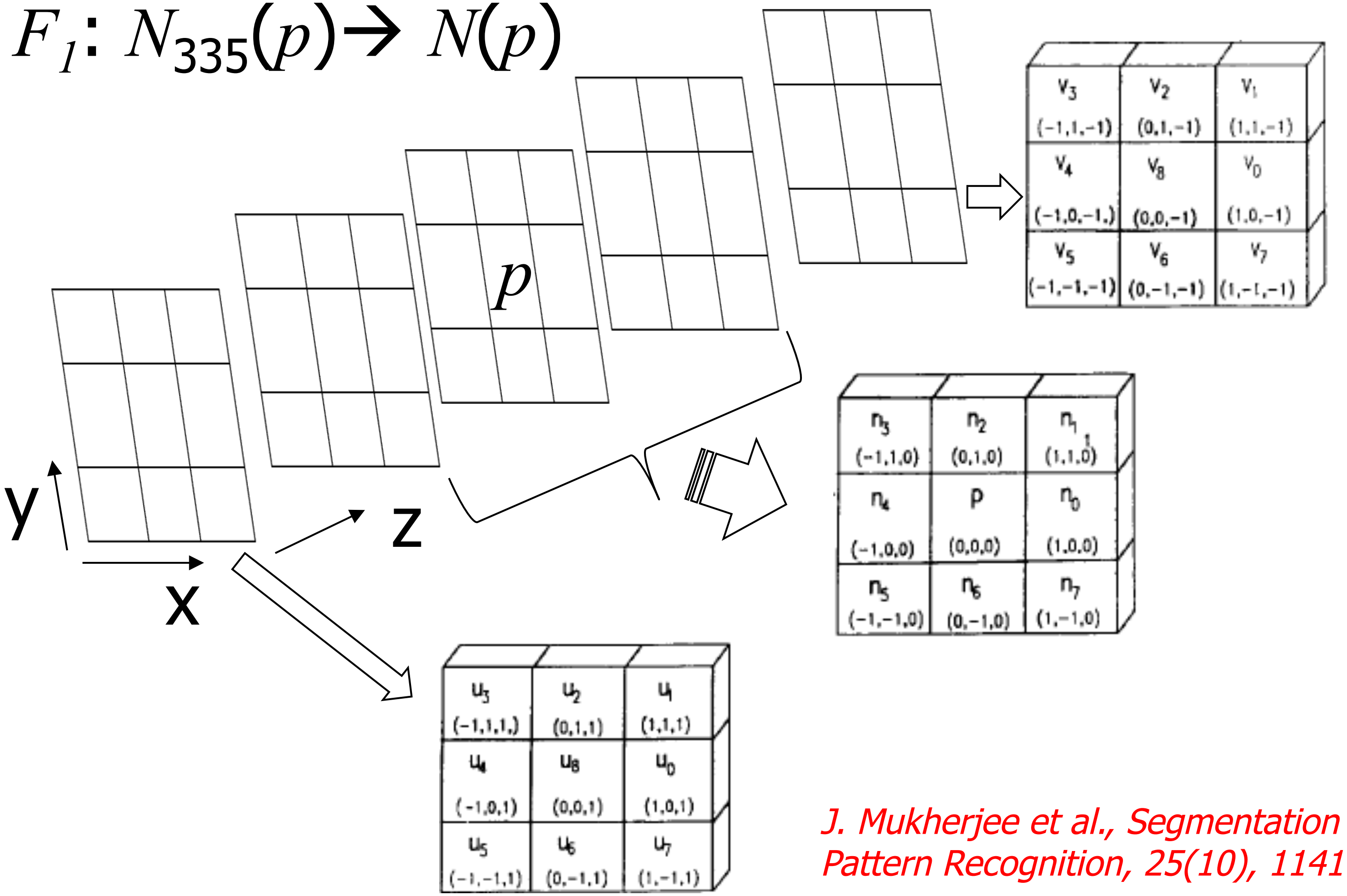
Neighborhood mapping function (NMF)

$F: N_{abc}(p) \rightarrow N(p)$.

1. The NMF F should be total and onto.
   i. For every point in $N_{abc}(p)$ there exists a unique point in $N(p)$.
   ii. For every point in $N(p)$ at least there exists a point in $N_{abc}(p)$ mapped to it.
2. The NMF F should induce connected partition in $N_{abc}(p)$.
3. Induced DNP's should be connected.
4. Induced DNP's should have strong structural similarity with the respective DNP defined in $N(p)$.

Neighborhood mapping function (NMF): An example

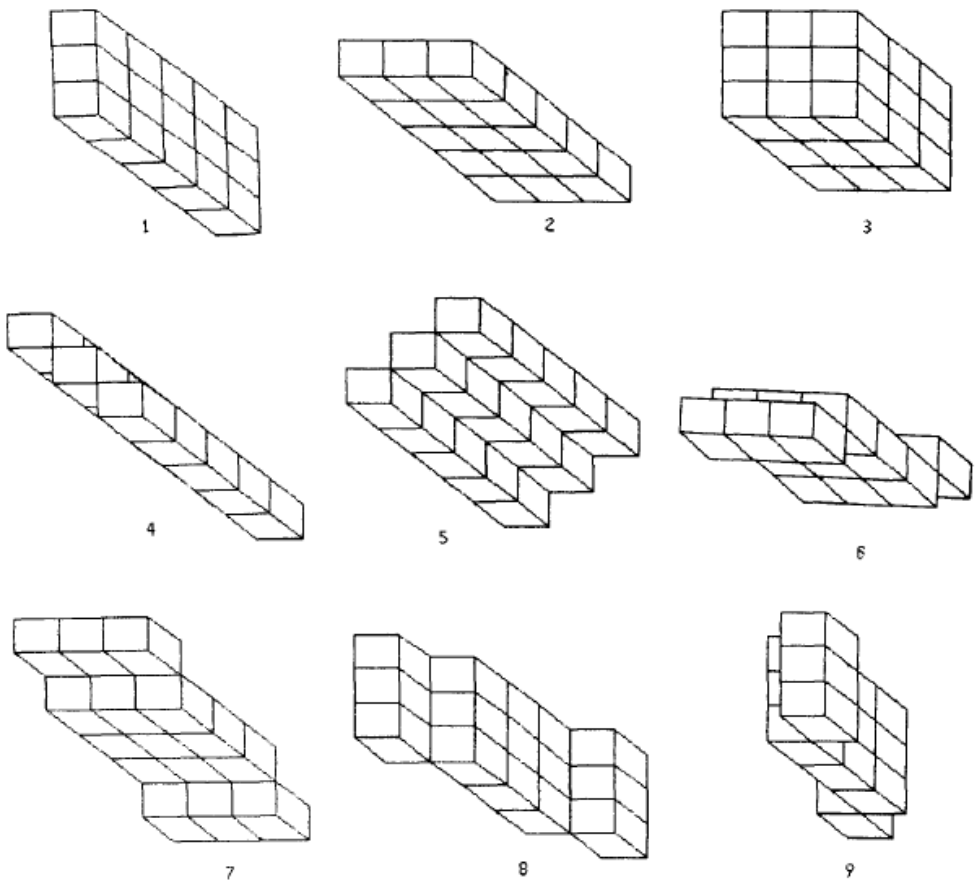
$F_1 : N_{335}(p) \rightarrow N(p)$

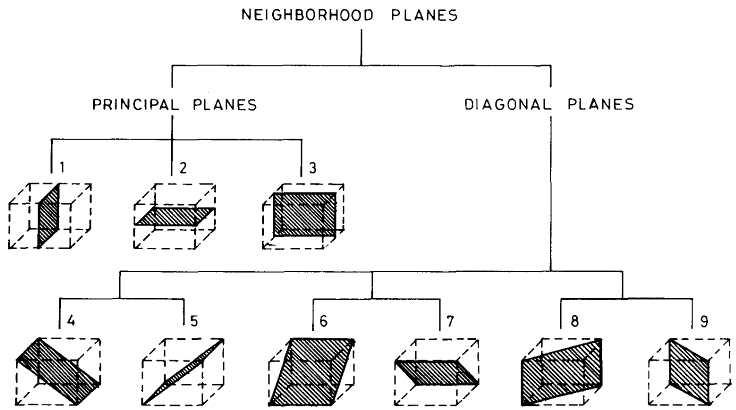


J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Modified DNPs induced by F_1

- Induced DNPs are connected.
- Structurally similar.





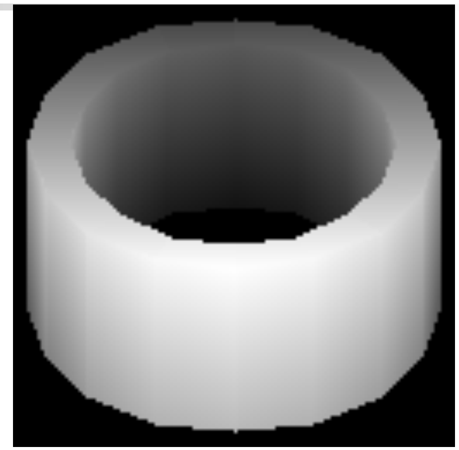
DNPs in base configuration

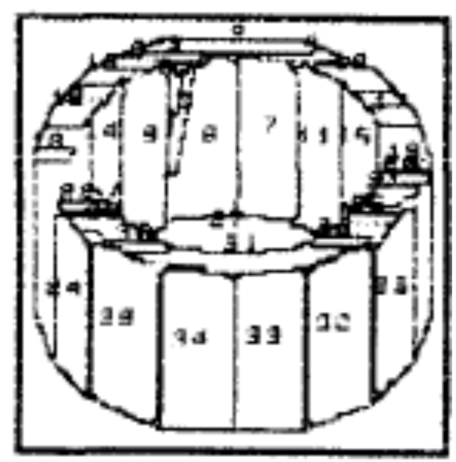
J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

J. Mukherjee et al., Segmentation of 3D surfaces, Pattern Recognition Letters, 11, 215-223, 1990.

The algorithm

1. Compute NPS at each pixel.
2. Compute connected components having the same NPS.
3. Remove small connected components.
4. Smooth a region by assigning its label to spurious unlabeled pixels within it.





J. Mukherjee et al., Segmentation of range images, Pattern Recognition, 25(10), 1141-1156, 1992.

Registration of range data

- Assumptions
  - Surface belongs to the same object.
  - Captured from different viewing direction.
  - Coordinates of corresponding points are related by rigid body transformation.
  - Assume same scales for the coordinate axes.
- Computational problem
  - Estimation of rotation and translation parameters.

Estimation of rigid body transformation

- Two corresponding point sets $\{m_i\}$ and $\{d_i\}$, $i = 1, .. \mathrm{N}$

$$d_i = Rm_i + T + v_i$$

noise

Solve for R and T to minimize

$$E = \sum_{i=1}^{N} \left\| d_i - \hat{R}m_i - \hat{T} \right\|^2$$

$$\frac{\partial E}{\partial \hat{T}} = 0 \quad \Rightarrow \quad -\sum_{i=1}^{N} \left(d_i - \hat{R}m_i - \hat{T} \right) = 0$$

$$\Rightarrow \quad \hat{T} = \bar{d} - \hat{R}\bar{m}$$

Means of d_i and m_i's

A solution for R and T

$$d_{c_i} = d_i - \bar{d} \qquad m_{c_i} = m_i - \bar{m}$$

$$E = \sum_{i=1}^{N} \left\| d_{c_i} - \hat{R} m_{c_i} \right\|^2 = \sum_{i=1}^{N} \Big(\overbrace{d_{c_i}^T d_{c_i} + m_{c_i}^T m_{c_i} - 2 \underbrace{d_{c_i}^T \hat{R} m_{c_i}}_{\text{Maximize}}}^{\text{Minimize}} \Big)$$

Solution of R and T

$$H = UDV^T$$

SVD of H

$$\hat{R} = VU^T$$

$$\hat{T} = \bar{d} - \hat{R}\bar{m}$$

Maximize trace of $\hat{R}H$

Where, $H = \sum_{i=1}^{N} m_{c_i} d_{c_i}^T$

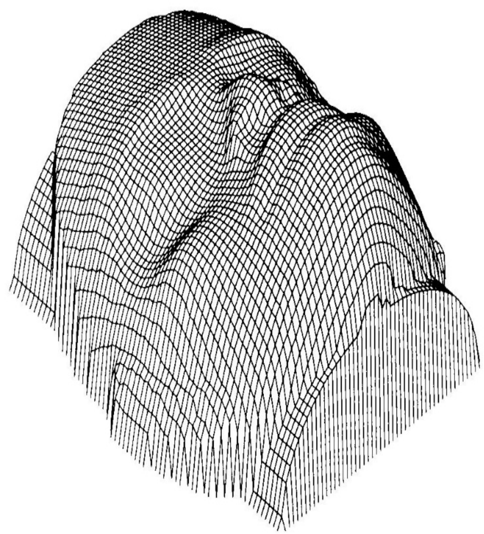
Correlation matrix

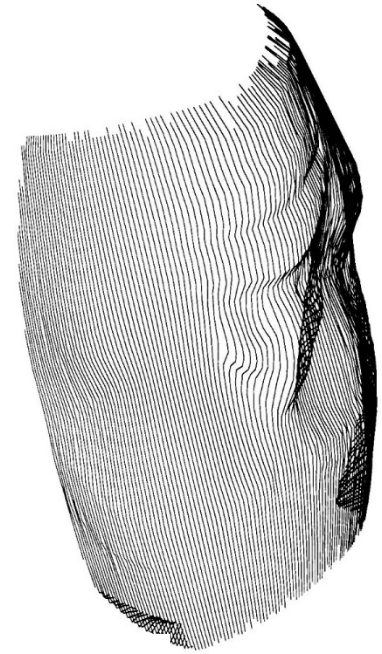
The Iterative Closest Point (ICP) Registration Algorithm

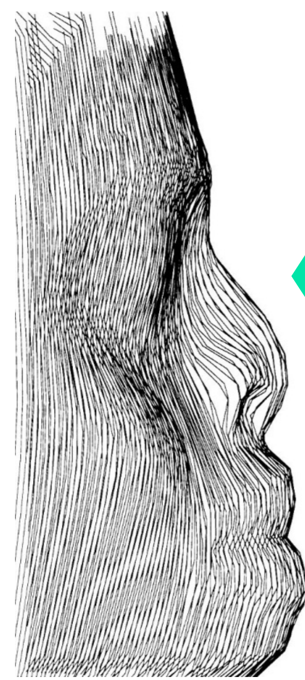
1. Compute initial registration parameters R_0 and T_0. Initialize error E to E_0 (error of model fitting).
2. Repeat following steps till error converges or becomes small.
   i. Apply transformation to the source scene (or point clouds).
   ii. Compute closest pairs between source and target scenes.
   iii. Re-compute registration parameters and error of fitting.

“A Method for Registration of 3D Shapes,” by P.J. Besl and N.D. McKay, IEEE Trans. on Pattern Analysis and Machine Intelligence, 14(2):238-256

ICP Registration Results







Reprinted from “A Method for Registration
of 3D Shapes,” by P.J. Besl and N.D. McKay,
IEEE Trans. on Pattern Analysis and Machine
Intelligence, 14(2):238-256 (1992). © 1992 IEEE.

Summary

1. Different types of range sensors.
   - Stereo imaging
   - Time of flight
   - Triangulation through scanning beams
   - Structured light
2. Use of differential geometry in extracting local features of a pixel (point) in a range image.
   - Surface normal, Principal Curvatures, Gaussian curvature, mean curvature.
   - Signs of curvatures characterize the local topology of surface.
3. Characterization of step and roof edges.
   - Step edge: Zero crossings of Gaussian curvature.
   - Roof edge: Extrema of dominant curvature along its direction.
   - Multiscale tracking of edge points.

Summary (contd.)

4. Segmenting range images into planar patches.
   - A greedy approach by fitting local surface patches and merging them.
   - A morphological processing based approach by computing neighborhood planes and local orientation.
5. Registration of range images.
   - Rigid body registration.
   - Least square error estimation for rotation and translation transformation matrices.
   - Iterative refinement from initial estimates by computing nearest neighboring pairs in two images.