Projective Geometry-I
Week-02-L5

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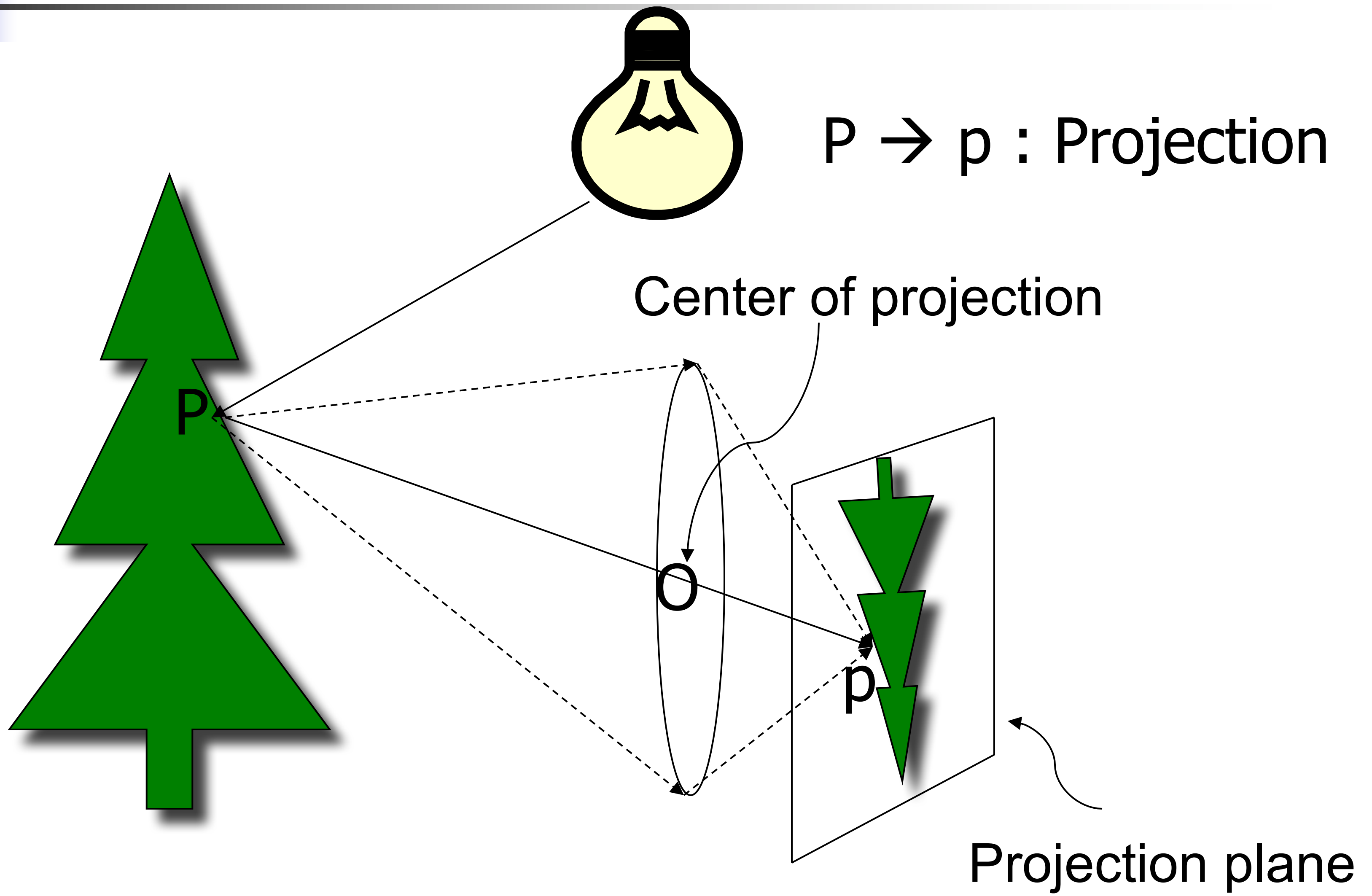
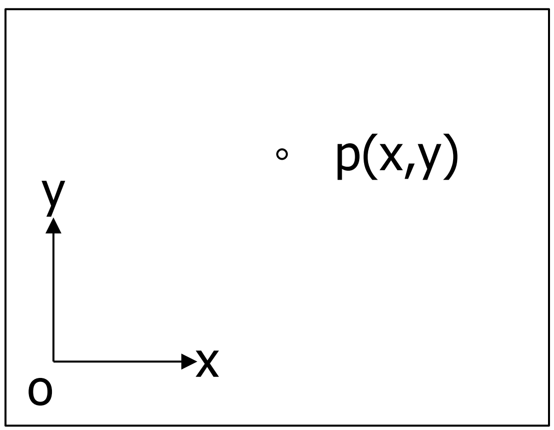
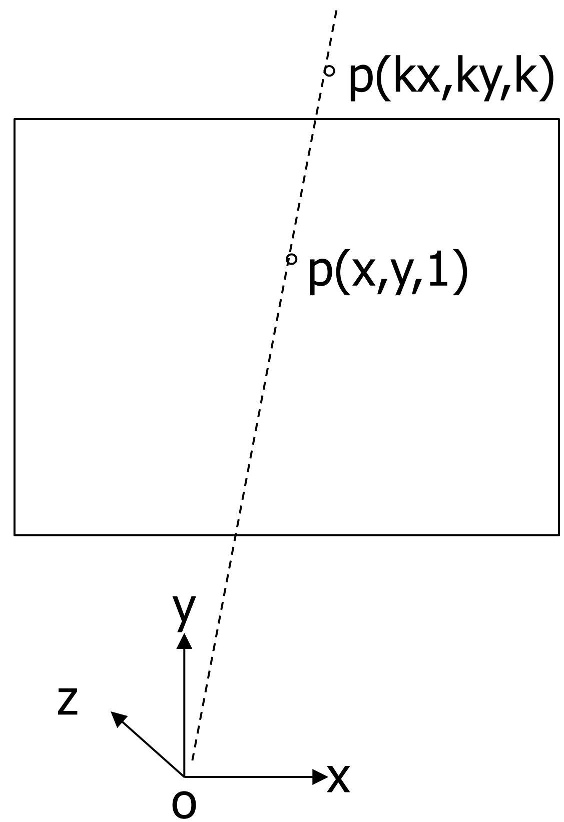
Image formation in optical camera



Real Space and Projective Space (2D)

$p \in R^2$





$p \in P^2$

p(x,y) $\Leftrightarrow$ p(kx,ky,k)

Homogeneous Coordinate system

Homogeneous Representation

A point in R^2: $\vec{x} \equiv \begin{bmatrix} x \\ y \end{bmatrix}$ $\Leftrightarrow$ A point in P^2: $\vec{X} \equiv \begin{bmatrix} kx \\ ky \\ k \end{bmatrix}$

$$P^2 = R^3 - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Singular point in the projective space.

Homogeneous Representation

In R^2: ? $\Leftrightarrow$ In $P^2 : \vec{X} \equiv \begin{bmatrix} 25 \\ 30 \\ 5 \end{bmatrix}$

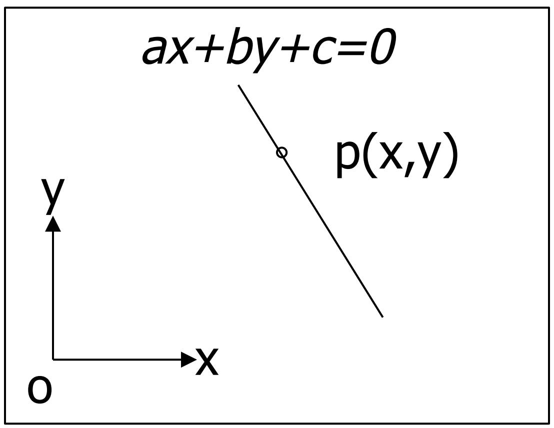
The point in R^2: $\vec{x} \equiv \begin{bmatrix} \frac{25}{5} \\ \frac{30}{5} \end{bmatrix} \equiv \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

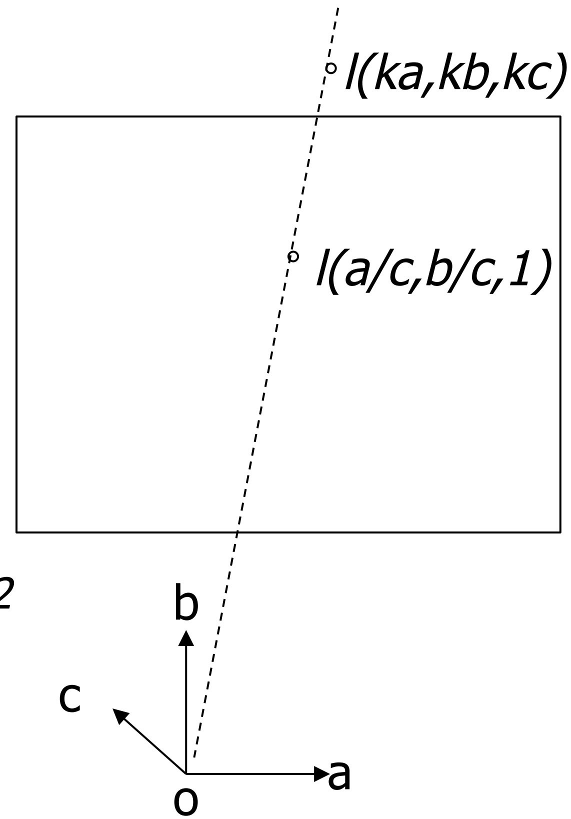
In $P^2 : \vec{X} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\Leftrightarrow$ In R^2: ?

Does not belong to P^2.

Homogeneous representation of a line in a plane

$p \in R^2$





$l \in P^2$

A point in P^2

A line in P^2

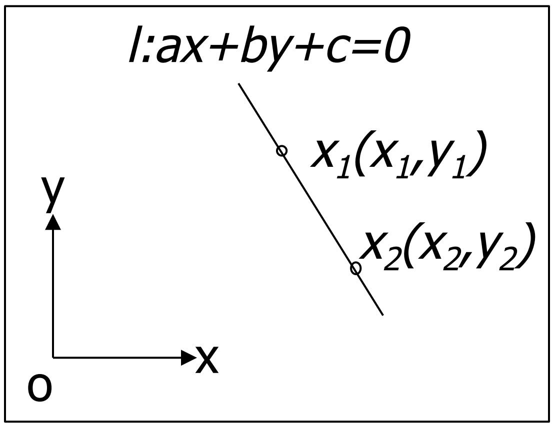
$$[x \quad y \quad 1]\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

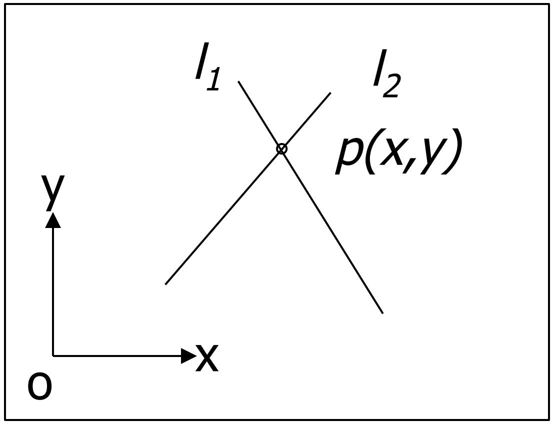
Point containment in P^2 $\Rightarrow$ $\vec{X}^T.\vec{l}=0 \Leftrightarrow \vec{l}^T.\vec{X}=0$

$$X_1 \times X_2 = \begin{vmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

Points and lines in P^2

$x_1 \in R^2$





$$\vec{l} = \vec{X}_1 \times \vec{X}_2$$

$$\vec{P} = \vec{l}_1 \times \vec{l}_2$$

Exactly one line through two points.

Exactly one point at intersection of two lines.

Examples

1. Compute the line passing through (3,5) and (5,0) in a plane.

$$\vec{l}=\begin{bmatrix}3\\5\\1\end{bmatrix}\times\begin{bmatrix}5\\0\\1\end{bmatrix}=\begin{bmatrix}5\\2\\-25\end{bmatrix}$$

2. Compute the point of intersection of the lines: 5x-2y+4=0 and 6x-7y-3=0.

$$\vec{P}=\begin{bmatrix}5\\-2\\4\end{bmatrix}\times\begin{bmatrix}6\\-7\\-3\end{bmatrix}=\begin{bmatrix}34\\35\\-23\end{bmatrix} \Rightarrow (-\frac{34}{23},-\frac{35}{23})$$

Duality

$$\mathbf{x} \longleftrightarrow \mathbf{l}$$

$$\mathbf{x}^{\mathsf{T}}\mathbf{l} = 0 \longleftrightarrow \mathbf{l}^{\mathsf{T}}\mathbf{x} = 0$$

$$\mathbf{x} = \mathbf{l} \times \mathbf{l}' \longleftrightarrow \mathbf{l} = \mathbf{x} \times \mathbf{x}'$$

Duality principle:

To any theorem of 2-dimensional projective geometry there corresponds a dual theorem, which may be derived by interchanging the role of points and lines in the original theorem.