# A Connectionist Approach for Gray Level Image Segmentation

V.V.Vinod† Santanu Chaudhury†† J.Mukherjee‡ S.Ghose†

†Dept. of Computer Sc. & Engg., Indian Institute of Technology, Kharagpur, India – 721302
‡Dept. of Electronics & Elect. Commn. Engg., Indian Institute of Technology, Kharagpur, India – 721302
††Dept. of Electrical Engg., Indian Institute of Technology, Delhi, India – 110016

## Abstract

*In this paper a connectionist network is presented for segmenting gray level images. The network detects the local peaks in the inverted histogram which will correspond to the bottoms of the valleys in the actual histogram. The neural network implementation successfully uses circumstantial evidence and detects multiple winners over the entire range of gray values such that these winners correspond to multiple thresholds for segmenting the image. The dynamics of the network has been analyzed and and the conditions for convergence have been established. Experimental results obtained by applying the network for segmenting two X-ray images are presented. The results obtained are promising.*

## 1 Introduction

Gray level images are composed of two or more regions wherein pixel values occupy different gray level ranges. A given gray level image can be segmented into the regions by selecting appropriate thresholds which clearly separate the peaks of the gray level histograms of the respective images [1]. These thresholds should correspond to the bottoms of the histogram valleys. But the lowest point between peaks can be identified reliably only after ascertaining the depth of the valley and separation of the valley from the peaks. The existing techniques for modal analysis of the histogram [2, 3, 4] cannot be applied to the intensity images whose histograms are characterized by extremely unequal peaks and broad or flat valleys [5].

Connectionist networks with their capabilities for local processing and integration of information over progressively larger neighbourhoods in an iterative fashion have the potential for providing reliable and robust techniques for finding bottom of the valleys in the gray level histogram. In this paper, we present a connectionist network which identifies bottom of the valleys of histogram. The network actually determines the peaks in the inverted histogram which correspond to the bottom points of the valleys in the actual histogram. It may be observed that known neural networks cannot be directly employed for this purpose. In Hopfield network [6] and its variants a cost function is to be specified. However, no such general cost function is avaliable for the peak detection problem. Simple winner take all networks cannot detect multiple peaks which will be present in the inverted histogram. $K$-winner networks proposed by Wolfe et.al. [7] are also unsuitable for the peak detection problem. These networks converge to the nearest stable state or the minimum energy state. This, however, leads to the detection of spurious peaks near the highest one while missing distinct but lower peaks.

## 2 Neural network implementation

The neural network has two neurons corresponding to every point in the inverted histogram. These neurons are arranged in two layers. The layers shall be referred to as accumulator layer and decision layer. Each layer is a linear array of neurons. The inverted histogram is the input to the neural network. Each accumulator layer neuron gathers support from its neighbourhood in favour of the point it represents being a peak. The activation of accumulator layer neurons are monitored by the corresponding decision layer neurons.

Let the $i^{th}$ point in the inverted histogram have a value $H_i$. Let $A_i$ and $D_i$ denote the neurons corresponding to the $i^{th}$ point in the accumulator layer and decision layer respectively. Denote the activation of $A_i$ by $\alpha_i$ and that of $D_i$ by $\beta_i$.

The accumulator layer neuron $A_i$ receives as external input the inverted histogram value $H_i$. In order to collect support from a neighbourhood, $A_i$ should receive inputs from other accumulator neurons nearby. $A_i$ receives inputs from $A_{i-1}$ and $A_{i+1}$ only so as to avoid flattening of the histogram. Further, since the purpose of evidence accumulation is to gather evidence in favour of a stronger peak from its neighbourhood, a self excitatory connection is given to $A_i$ with a weight $w_a$ larger than the weight $w_{aa}$ assigned to lateral interconnection between two accumulator neurons. The weight $W(A_i, A_j)$ between $A_i$ and $A_j$ may be specified by equation (1) with $w_a > w_{aa} > 0$.

$$W(A_i, A_j) = \begin{cases} w_{aa} & \text{if } |i-j| = 1 \\ w_a & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$



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Since the activation $\alpha_i$, of $A_i$ is expected to provide a measure of this accumulated strength a linear function $f_a$ saturating at +1 and −1 is employed as the activation function of accumulator neurons.

$$f_a(x) = \begin{cases} 1 & \text{if } x > 1 \\ -1 & \text{if } x < -1 \\ x & \text{otherwise} \end{cases} \quad (2)$$

The role of the decision layer is to accept as winner those accumulator layer neurons with high activation and to initiate a competition in the neighbourhood of the winners, whose inhibitory effect will be iteratively propagated through the network. Each neuron $D_i$ receives as input, the output of the corresponding accumulator neuron. The decision layer is constructed so that it accepts as peak an accumulator neuron having activation more than a specified $\phi$. A high gain sigmoid function $f_d$ with a threshold $\phi$ is used as the activation function of the decision layer neurons. With this construction a decision layer neuron $D_i$ will have a near one output when $\alpha_i > \phi$ and near zero when $\alpha_i < \phi$, with the transition occuring around $\alpha_i = \phi$. Once a neuron is identified as a winner it has to be strengthened wheras other neurons in its neighbourhood have to be inhibited. Since the gray level values denoting boundaries of the segments are not too close to each other we may inhibit neurons in a larger neighbourhood of a winner. Consider a neighbourhood $\mathcal{N}_i^D$ over which $D_i$ exerts its inhibition defined as
$\mathcal{N}_i^D = \{j | j \neq i \text{ and } |j - i| \leq r\}$ where $r > 1$.
For strengthening a winner and inhibiting other neurons near it, connections are established from the decision layer to the accumulator layer according to equation(3) with $w_{da} < 0$ and $w_d > 0$.

$$W(D_i, A_j) = \begin{cases} w_{da} & \text{if } j \in \mathcal{N}_i^D \\ w_d & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The above network is updated in a synchronous fashion. Let $\alpha_i^t$ and $\beta_i^t$ denote the activations of $A_i$ and $D_i$ at time $t$ respectively. The network updating equations may now be written as

$$\begin{aligned} \beta_i^0 &= \alpha_i^0 = 0 \quad \forall i \\ \alpha_i^t &= f_a(w_a \alpha_i^{t-1} + w_{aa}(\alpha_{i-1}^{t-1} + \alpha_{i+1}^{t-1}) \\ &\quad + H_i + w_d \beta_i^{t-1} + \sum_{j \in \mathcal{N}_i^D} w_{da} \beta_j^{t-1}) \quad (4) \\ \beta_i^t &= f_d(\alpha_i^t - \phi) \end{aligned}$$

## 3 Analysis of the network

In this section we state some of the properties of the peak detection network. Proofs may be constructed from the proofs given in [8]. Since the interconnections between neurons are resticted to a small neighbourhood only, the interactions between the neurons are delayed in time. The interactions between the neurons also change from cooperation to competition. In theorems 1 and 2 we derive the conditions on $w_{aa}$ and $w_a$ such that the network is cooperative as long as no peaks are detected and exhibits mixed behaviour once the peaks are detected. From theorem 3 it follows that two of the nearest neurons being detected as peaks compete with each other. The convergence conditions are established in theorem 4.

**Theorem 1** *If $w_{aa} > 0$, then starting at time $t = 0$, the network is cooperative till no peaks are detected.*

**Theorem 2** *When $\alpha_j \approx \phi$, then there are competitive interactions in the network provided $w_{aa} > 0$ and $w_{da} f_d'(0) < -w_{aa}$.*

**Theorem 3** *Let $\alpha_i^\tau \approx \phi$ and $A_j$ be such that $t(A_j, A_i) = r$. Further let there be no neurons $A_k$ such that $\alpha_k^{\tau_1} \approx \phi$, $\tau \leq \tau_1 \leq \tau + r$ and $t(A_j, A_k) + t(A_k, A_i) \leq r$, where $t(A_j, A_i)$ is the time taken by a change in $A_i$ to reach $A_j$. Then $A_i$ inhibits $A_j$ after $r$ steps.*

**Theorem 4** *All $\alpha_i$ in equation (4) are eventually monotone if the following conditions hold for all $i$.*
1. $w_d f_d(0) > \phi + 2w_{aa} - w_a\phi - H_i$
2. $w_{da} f_d(0) < \phi - 2w_{aa}\phi - w_a\phi - H_i - w_d f_d(0)$
3. $w_{aa}\phi + w_a\phi + H_i < \phi$

Theorem 4 states the conditions on the parameters such that the network converges with the correct results. It may be observed that the conditions are derived assuming extreme cases. Further since the conditions are inequalities, a range of values will satisfy them. The exact values assigned to be assigned will depend on the problem and will dictate the nature of the peaks which are detected.

## 4 Experimental results

In this section we present the results obtained by employing the neural network discussed in the previous section for segmenting two X-ray images. The inverted histogram values $H_i$ were computed from the intensity image with a bin size of 5 as follows.

$$\begin{aligned} h_i &= \text{number of pixels with intensity between } 5i \text{ and } 5(i+1), \text{ for } 0 \leq i < 50 \\ H_i &= \max_{0 \leq j < 50} h_j - h_i \end{aligned}$$

The values of $H_i$ scaled to lie within the range 0 to 0.1 were used as inputs to the accumulator layer of the network. The neighbourhood $\mathcal{N}_i^D$ over which the inhibition of $D_i$ exerts its influence was defined as $\mathcal{N}_i^D = \{j | j \neq i, |j - 1| \leq 4\}$ and $f_d()$ was chosen as

$$f_d(x) = \frac{1}{1 + \exp(-30x)}$$

The value of $\phi$ was fixed as 0.9. The weights $w_a$ and $w_{aa}$ were then determined from the third condition of theorem 4. Considering the maximum value of $H_i$, namely 0.1, this requires $w_{aa} + w_a < 0.89$. This along with $w_a > w_{aa}$ requires that $w_{aa} < 0.445$ and $w_a > 0.445$. $w_a = 0.5$ and $w_{aa} = 0.3$ were experimentally found to be satisfactory values.

Using the value of $w_a$ and $w_{aa}$ computed above and assuming $H_i$ to be zero, from condition 1 of theorem 4 we obtain $w_d f_d(0) > 0.9 + 0.6 - 0.45$ However, this will imply that even isolated points could be detected as peaks. In order to ensure that only those points receiving some amount of support from its neighbours were potential peaks a reduced value of $w_d = 1.5$ was used and found to be satisfactory.

The value of $w_{da}$ is obtained from the second condition of theorem 4. Using the values computed above and the maximum value of $H_i$, the inequality reduces to $w_{da} f_d(0) < 0.9 - 0.54 - 0.45 - 0.1 - 0.75 = -1.88$ A value of $w_{da} = -2.0$ was found to be satisfactory.

**Example 1** Figure 1 shows the results obtained for the X-ray image of human wrist. Figure 1(a) shows the intensity image and 1(b) shows the segmented image. The network with parameters computed as above converged in 50 iterations. In figure 1(c) the histogram is plotted. The black dots indicate the bottom points of the valleys of the histogram detected by the network. They were detected at bin numbers 9 (intensity between 45 and 50), 16 (intensity beween 80 and 85) and 25 (intensity between 125 and 130). The grey level values corresponding to bin numbers 9 and 25 clearly separate out the first and the last modes in the inverted histogram. The bottom point detected at bin number 16 separates out the smaller peak to its left. Pixels of the image have been classified into four distinct regions based on the bottom points detected. The different regions are shown in figure 1(b) with different shades. From the picture it is clear that the threshold determined by the first valley properly segment out the background. Pixels having grey values between 50 and 130 basically indicate the soft tissue. Pixels with grey values beyond 130 indicate the bones. The pixels corresponding to the soft tissue have been grouped in two different segments. Pixels with grey values greater than 50 and less than or equal to 85 are soft tissues in the transitory region.

It may be observed that since 16 had two equally lower points to either side of it, the network choose the middle point although it was not a minimum point of the histogram. This shows the ability of the network to utilize circumstantial information in deciding the valley points. □

**Example 2** In this case we consider a noisy X-ray image where the lighting is non uniform. The results obtained for the X-ray image of the human hand is given in figure 2. For this example the network took 40 iterations to reach a stable state. Figure 2(a) gives the

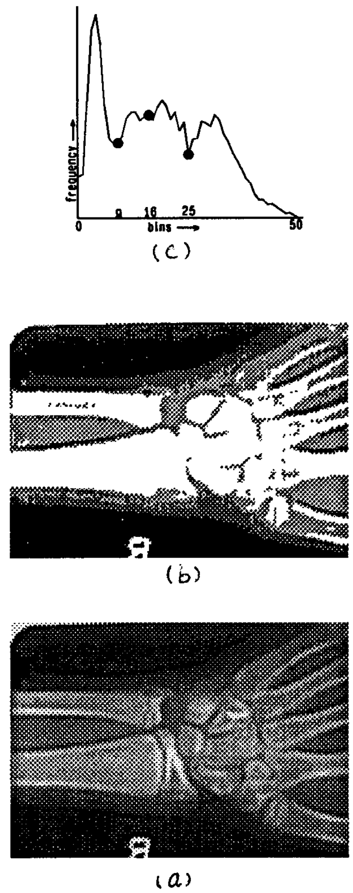


Figure 1: (a) The intensity image of the X-ray of the wrist. (b) The segmented image. (c) The histogram showing the points detected as grey level boundaries of the segments.

intensity image and 2(b) gives the segmented image. In figure 2(c) the histogram and the bottom points detected (black dots) are shown. The bottoms of the valleys were detected at bin numbers 9 (intensity between 45 and 50), 21 (intensity between 105 and 110) and 35 (intensity between 175 and 180). Pixels of the image have been classified into four distinct regions using the thresholds 50, 110 and 180. Due to the noise in the image bones have been partially merged with the soft tissues. These pixels belong to the regions lying between bin numbers 21 and 25 in the histogram. The pixels having grey level values upto 50 indicate the background. Pixels belonging to the region between bin numbers 9 and 21 belong to the transitory region. Since the peaks to either side of the relatively flat region between these points fall steeply the network did not club this region with either of the modes. It may be observed that the background has been clearly separeted out from the image because of the two valleys detected at 9 and 16. A valley inbetween these two would have partially merged background and soft tissue. This is seen from figure 2(d), where the different regions obtained by choosing the gray values corresponding to bin numbers 15, 26 and 35 (shown with squares in figure 2(c)) as thresholds are shown with different shades. □

## 5 Conclusion

In this paper a connectionist network has been proposed for segmenting gray level images by detecting the peaks in the inverted histogram. The network has the capability to integrate global information for identifying the correct winners. The most significant bottom among competing bottom points are chosen and separation induced by the peaks affect these decisions. Consequently, uneven undulations in the histograms, which is common in real images, does not affect the reliability of the thresholds determined by the network. Experimental study of applying the network for segmenting X-ray images has produced promising results. It was also observed that the network converges in a relatively small number of iterations.

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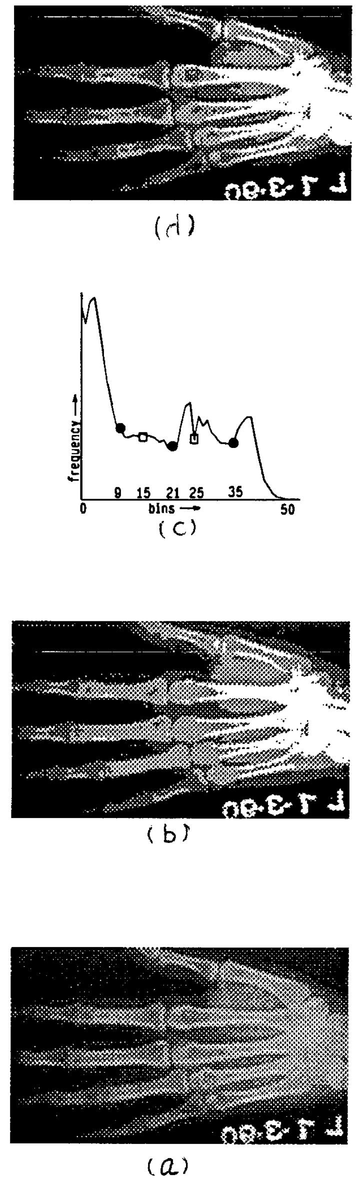


Figure 2: (a) The intensity image of the X-ray of the hand. (b) The segmented image. (c) The histogram showing the points detected as grey level boundaries of the segments. (d) The segemented image obtained by manually choosing bin numbers 15, 26 and 35