Minimization of Switching Functions

Assignments

1. Using Karnaugh map, derive minimal sum-of-products expressions for the following functions:
   a. $f(w,x,y,z) = \Sigma\ (0,1,2,3,4,6,8,9,10,11)$
   b. $f(w,x,y,z) = \Sigma\ (0,1,5,7,8,10,14,15)$
   c. $f(w,x,y,z) = \Sigma\ (0,2,4,5,6,8,10,12)$
   d. $f(w,x,y,z) = \Sigma\ (0,2,4,9,12,15) + \Sigma_{\phi}\ (1,5,7,10)$
   e. $f(w,x,y,z) = \Sigma\ (1,2,3,5,13) + \Sigma_{\phi}\ (6,7,8,9,11,15)$
   f. $f(v,w,x,y,z) = \Sigma(1,2,6,7,9,13,14,15,17,22,23,25,29,30,31)$
2. Using Karnaugh map, derive minimal product-of-sums expressions for the following functions:
   a. $f(w,x,y,z) = \Sigma\ (0,1,2,3,4,6,8,9,10,11)$
   b. $f(w,x,y,z) = \Sigma\ (0,1,5,7,8,10,14,15)$
   c. $f(w,x,y,z) = \Sigma\ (0,2,4,5,6,8,10,12)$
   d. $f(w,x,y,z) = \Sigma\ (0,2,4,9,12,15) + \Sigma_{\phi}\ (1,5,7,10)$
   e. $f(w,x,y,z) = \Sigma\ (1,2,3,5,13) + \Sigma_{\phi}\ (6,7,8,9,11,15)$
3. Let $f = \Sigma(5,6,13)$ and $f_1 = \Sigma(0,1,2,3,5,6,8,9,10,11,13)$. Find f_2 such that $f = f_1.f_2'$. Is f_2 unique? If not, indicate all possibilities.
4. A binary-coded-decimal (BCD) message appears in four input lines of a switching circuit. Design an AND, OR, NOT gate network that produces an output value 1 whenever the input combination is 0, 2, 3, 5 or 8.
5. Design an NAND gate network for a switching circuit that takes a 4-bit BCD message as input, and produces a 3-bit output which is modulo-7 of the input decimal digit.
6. For the following functions, (i) use the map to find all prime implicants, (ii) indicate which of the prime implicants are essential.
   a. $f(w,x,y,z) = \Sigma\ (0,1,2,3,4,6,7,8,9,11,15)$
   b. $f(w,x,y,z) = \Sigma\ (1,3,4,5,7,8.9.11.14,15)$
7. Use Quine-McCluskey method to generate the set of prime implicants, and obtain all minimal expressions for the following functions:
   a. $f(w,x,y,z) = \Sigma\ (0,1,5,7,8,10,14,15)$
   b. $f(w,x,y,z) = \Sigma\ (1,5,6,12,13,14) + \Sigma_{\phi}\ (2,4)$
   c. $f(w,x,y,z) = \Sigma\ (0,1,4,5,6,7,9,11,15) + \Sigma_{\phi}\ (10,14)$
   d. $f(v,w,x,y,z) = \Sigma(1,2,6,7,9,13,14,15,17,22,23,25,29,30,31)$
   e. $f(v,w,x,y,z) = \Sigma(0,1,3,8,9,13,14,15,16,17,19,24,25,27,31)$
8. Answer the following.
   a. Give the map of an irreducible 4-variable function whose sum-of-products representation consists of 2^3 minterms.
   b. Prove that there exists a function of *n* variables whose minimal sum-of-products form consists of 2^{n-1} minterms , and no functions when expressed in sum-of-products form requires more than 2^{n-1} product terms.
   c. Derive a bound on the number of literals needed to express any *n*-variable function.