Swirching Algebra

Assignments

1. Simplify the following switching expressions using algebraic method:
   a. $x' + y' + xyz'$
   b. $(x' + xyz') + (x' + xyz')(x + x'y'z)$
   c. $a + a'b + a'b'c + a'b'c'd + \ldots$
   d. $w'x' + x'y' + w'z' + yz$
   e. $((x + y'z')(y + x'z')(z + x'y'))'$

2. Given AB'+A'B=C, show that AC'+A'C=B.

3. Prove that if $w'x+yz'=0$, then
   $$wx + y'(w' + z') = wx + xz + x'z' + w'y'z$$

4. Determine the canonical sum-of-products expressions for the following functions:
   a. $f(x,y,z) = z + ((x' + y)(x + y'))$
   b. $f(x,y,z) = x + (x'y' + x'z)'$

5. Show that $f(A,B,C) = A'BC + AB' + B'C'$ is a universal operation (that is, functionally complete).

6. Assuming that a constant 1 is available, show that $f(A,B) = A'B$ is a universal operation.

7. A safe has five locks v, w, x, y and z, all of which must be unlocked for the safe to open. The keys to the locks are distributed among five executives in the following manner:

   > A has keys for locks v and x
   > B has keys for locks v and y
   > C has keys for locks w and y
   > D has keys for locks x and z
   > E has keys for locks v and z

   a. Determine the minimum number of executives required to open the safe.
   b. Find all the combinations of executives that can open the safe.
   c. Who is the "essential executive" without whom the safe cannot be opened?

8. Five soldiers A, B, C, D and E volunteer to perform an important military task if the following conditions are satisfied:
   - Either A or B or both must go
   - Either C or E, but not both, must go
   - Either both A and C go, or neither goes
   - If D goes then E must also go
   - If B goes then A and D must also go

   Define variables A, B, C, D, E such that an uncomplemented variable will mean that the corresponding soldier has been selected to go. Determine the expression that specifies the combinations of volunteers that can get the assignment.

9. You are presented with a set of requirements under which an insurance policy will be issued. The applicant must be:
   - A married female 25 years old or over, or
   - A female under 25, or
   - A married male under 25 who has not been involved in a car accident, or
   - A married male who has been involved in a car accident, or
   - A married male 25 years or over who has not been involved in a car accident.

   Define appropriate variables to capture the requirements.
   a. Find an algebraic expression that assumes the value 1 whenever the policy should be issued.
   b. Simplify algebraically the above expression and suggest a simpler set of requirements.