

## Number Systems

Decimal number:  $123.45 = 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0 + 4 \cdot 10^{-1} + 5 \cdot 10^{-2}$ Base *b* number:  $N = a_{q-1}b^{q-1} + \cdots + a_0b^0 + \cdots + a_{-p}b^p$   $b > 1, \quad 0 <= a_i <= b \cdot 1$ Integer part:  $a_{q-1}a_{q-2} \cdots a_0$ Fractional part:  $a_{-1}a_{-2} \cdots a_{-p}$ Most significant digit:  $a_{q-1}$ Least significant digit:  $a_{-p}$ Binary number (*b*=2): 1101.01 =  $1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2}$ Representing number *N* in base *b*: (*N*)<sub>*b*</sub> Complement of digit *a*: *a*' = (*b*-1)-*a* Decimal system: 9's complement of  $3 = 9 \cdot 3 = 6$ Binary system: 1's complement of  $1 = 1 \cdot 1 = 0$ <sup>2</sup>

	l	Base				
2	4	8	10	12		
0000	0	0	0	0		
0001	1	1	1	1		
0010	2	2	2	2		
0011	3	3	3	3		
0100	10		4	4		
0101	11			5 6		
0110	12			7		
1000	20	10	8	8		
1000	21	11	9	9		
1010	22	12	10	α		
1011	23	13	11	β		
1100	30	14	12	10		
1101	31	15	13	11		
1110	32	16	14	12		
1111	33	17	15	13		

 $\begin{aligned} \text{Conversion of Bases} \\ \text{Example: Base 8 to base 10} \\ (432.2)_8 &= 4 \cdot 8^2 + 3 \cdot 8^1 + 2 \cdot 8^0 + 2 \cdot 8^{-1} = (282.25)_{10} \\ \text{Example: Base 2 to base 10} \\ (1101.01)_2 &= 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2} = (13.25)_{10} \\ \text{Base } b_1 \text{ to } b_2, \text{ where } b_1 > b_2 \end{aligned} \\ \\ \text{(N)}_{b_1} &= a_{q-1} b_2^{q-1} + a_{q-2} b_2^{q-2} + \dots + a_1 b_2^1 + a_0 b_2^0 \\ \\ \frac{(N)_{b_1}}{b_2} &= \underbrace{a_{q-1} b_2^{q-2} + a_{q-2} b_2^{q-3} + \dots + a_1}_{Q_0} + \frac{a_0}{b_2} \\ \\ \left( \underbrace{\frac{Q_0}{b_2}}_{b_1} \right)_{b_1} &= \underbrace{a_{q-1} b_2^{q-3} + a_{q-2} b_2^{q-4} + \dots}_{Q_1} + \frac{a_1}{b_2} \end{aligned}$ 









Bina	ry	γ Δ	rith	meti	С			
	$\frac{B}{a}$	$\frac{its}{b}$	Sum $a$	$\overline{Carry}$ + b	Difference a -	Borrow	$\frac{Product}{a \cdot b}$	
	0 0 1	0 1 0	0 1 1	0 0 0	0 1 1	0 1 0	0 0 0	
	1	1	0	1	0	0	1	
								9





Decimal					1	$v_4 w$	${}_{3}w_{2}$	$w_1$				
digit	8	4	2	1	2	4	2	1	6	4	2	-3
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1	0	1	0	1
2	0	0	1	0	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1	1	0	0	1
4	0	1	0	0	0	1	0	0	0	1	0	0
5	0	1	0	1	1	0	1	1	1	0	1	1
6	0	1	1	0	1	1	0	0	0	1	1	0
7	0	1	1	1	1	1	0	1	1	1	0	1
8	1	0	0	0	1	1	1	0	1	0	1	0
9	1	0	0	1	1	1	1	1	1	1	1	1
	L											
		BC	D			Sel	f-co	mpler	nentin	a C	ode	 S



Decimo	l	G	ray			Bir	nary		-
numbe	$r \mid g_3$	$g_2$	$g_1$	$g_0$	$b_3$	$b_2$	$b_1$	$b_0$	
0	0	0	0	0	0	0	0	0	-
1	0	0	0	1	0	0	0	1	
2	0	0	1	1	0	0	1	0	
3	0	0	1	0	0	0	1	1	
4	0	1	1	0	0	1	0	0	
5	0	1	1	1	0	1	0	1	
6	0	1	0	1	0	1	1	0	
7	0	1	0	0	0	1	1	1	
8	1	1	0	0	1	0	0	0	
9	1	1	0	1	1	0	0	1	
10	1	1	1	1	1	0	1	0	
11	1	1	1	0	1	0	1	1	
12	1	0	1	0	1	1	0	0	
13	1	0	1	1	1	1	0	1	
14	1	0	0	1	1	1	1	0	
15	1	0	0	0	1	1	1	1	



00	0	00	0	000	
01	0	01	0	001	
11	0	11	0	011	
10	0	10	0	010	
	1	10	0	110	
	1	11	0	111	
	1	01	0	101	
	1	00	0	100	
			1	100	
			1	101	
			1	111	
			1	110	
			1	010	
			1	011	
			1	001	
			1	000	

Decimal	Eu	ven-	para	ity E	BCD		2-0	out-o	of-5	
digit	8	4	<b>2</b>	1	p	0	1	<b>2</b>	4	7
0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	1	1	1	0	0	0
2	0	0	1	0	1	1	0	1	0	0
3	0	0	1	1	0	0	1	1	0	0
4	0	1	0	0	1	1	0	0	1	0
5	0	1	0	1	0	0	1	0	1	0
6	0	1	1	0	0	0	0	1	1	0
7	0	1	1	1	1	1	0	0	0	1
8	1	0	0	0	1	0	1	0	0	1
9	1	0	0	1	0	0	0	1	0	1
p: parity bit: eve	n pa	aritv	use	d in	above c	odes				
Distance betwee	an c	ode	word	ds: n	o of hit	s they (	liffe	r in		



## Hamming Codes (Contd.)

No. of parity check bits, k, must satisfy:  $2^k \ge m+k+1$ Example: if m = 4 then k = 3

Place check bits at the following locations: 1, 2, 4, ...,  $2^{k-1}$ Example code word: **1100**110

- Check bits:  $p_1 = 1$ ,  $p_2 = 1$ ,  $p_3 = 0$
- Information bits: 0, 1, 1, 0

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	Error position	Po	sition	number	
		$c_1$	$c_2$	$c_3$	
	0 (no error)	0	0	0	
	1	0	0	1	
	2	0	1	0	
	3	0	1	1	
	4	1	0	0	
	5	1	0	1	
	6	1	1	0	
	7	1	1	1	
Select $p_1$ to est Select $p_2$ to est	ablish even parity in p ablish even parity in p	1     1	1 ons: 1 ons: 2	1 , 3, 5, 7 , 3, 6, 7	



Hamming	С	00	de	f	or	B	CD					
	_											
Decimal	Po	sitie	n	1	2	3	4	5	6	7		
digit				$p_1$	$p_2$	$m_1$	$p_3$	$m_2$	$m_3$	$m_4$		
0				0	0	0	0	0	0	0		
1				1	1	0	1	0	0	1		
2				0	1	0	1	0	1	0		
3				1	0	0	1	1	1	1		
4				1	1	0	1	1	0	1		
о 6				1	1	0	0	1	1	1		
7				0	0	0	1	1	1	1		
8				1	1	1	0	0	0	0		
9				0	0	1	1	0	0	1		
Position	: 1	2	3	4	5	6 7						
Intended message	e: 1	1	0	1	0	0 1						
Message received	l: 1	1	0	1	1	0 1						
4-5-6-7 parity check	:			1	1	0 1	<i>C</i> ₁ =	1 sin	ce pa	ritv is	odd	
2-3-6-7 parity check		1	0		-	0 1	$C_0 =$	0 sin	ce pa	ritv is	even	
1-3-5-7 parity check	: 1		0		1	1	$c_{3} =$	1 sin	ce pa	rity is	odd	

## SEC/DED Code

Add another parity bit such that all eight bits have even parity

- Two errors occur: overall parity check satisfied, but position number indicates error → double error (cannot be corrected)
- Single error occurs: overall parity check not satisfied
  - Position no. is 0: error in last parity bit
  - Else, position no. indicates erroneous bit
- No error occurs: all parity checks indicate even parities

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