Discrete Logarithms and Diffie Hellman Key Exchange

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Diffie-Hellman Key Exchange

- first public-key type scheme proposed
- by Diffie & Hellman in 1976 along with the exposition of public key concepts
  - note: now know that Williamson (UK CESG) secretly proposed the concept in 1970
- is a practical method for public exchange of a secret key
- used in a number of commercial products

Diffie-Hellman Key Exchange

- a public-key distribution scheme
  - cannot be used to exchange an arbitrary message
  - rather it can establish a common key
  - known only to the two participants
- value of key depends on the participants (and their private and public key information)
- based on exponentiation in a finite (Galois) field (modulo a prime or a polynomial) - easy
- security relies on the difficulty of computing discrete logarithms (similar to factoring) – hard

Diffie-Hellman Setup

- all users agree on global parameters:
  - large prime integer or polynomial q
  - a being a primitive root mod q
- each user (eg. A) generates their key
  - chooses a secret key (number): $x_A < q$
  - compute their **public key**: $y_A = a^{x_A} \bmod q$
- each user makes public that key y_A

Message Authentication and Hash Function

Message Authentication

- message authentication is concerned with:
  - protecting the integrity of a message
  - validating identity of originator
  - non-repudiation of origin (dispute resolution)
- will consider the security requirements
- then three alternative functions used:
  - message encryption
  - message authentication code (MAC)
  - hash function

Security Requirements

- disclosure
- traffic analysis
- masquerade
- content modification
- sequence modification
- timing modification
- source repudiation
- destination repudiation

Message Encryption

- message encryption by itself also provides a measure of authentication
- if symmetric encryption is used then:
  - receiver know sender must have created it
  - since only sender and receiver now key used
  - know content cannot of been altered
  - if message has suitable structure, redundancy or a checksum to detect any changes

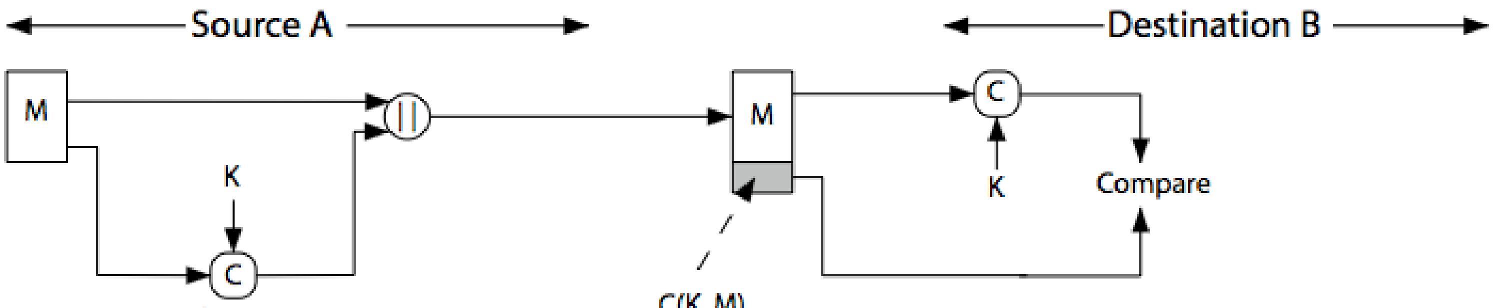
Message Encryption

- if public-key encryption is used:
  - encryption provides no confidence of sender
  - since anyone potentially knows public-key
  - however if
    - sender **signs** message using their private-key
    - then encrypts with recipients public key
    - have both secrecy and authentication
  - again need to recognize corrupted messages
  - but at cost of two public-key uses on message

Message Authentication Code (MAC)

- generated by an algorithm that creates a small fixed-sized block
  - depending on both message and some key
  - like encryption though need not be reversible
- appended to message as a **signature**
- receiver performs same computation on message and checks it matches the MAC
- provides assurance that message is unaltered and comes from sender

Message Authentication Code



(a) Message authentication

Message Authentication Codes

- as shown the MAC provides authentication
- can also use encryption for secrecy
  - generally use separate keys for each
  - can compute MAC either before or after encryption
  - is generally regarded as better done before
- why use a MAC?
  - sometimes only authentication is needed
  - sometimes need authentication to persist longer than the encryption (eg. archival use)
- note that a MAC is not a digital signature

MAC Properties

- a MAC is a cryptographic checksum

  $MAC = C_K(M)$

  - condenses a variable-length message M
  - using a secret key K
  - to a fixed-sized authenticator
- is a many-to-one function
  - potentially many messages have same MAC
  - but finding these needs to be very difficult

Requirements for MACs

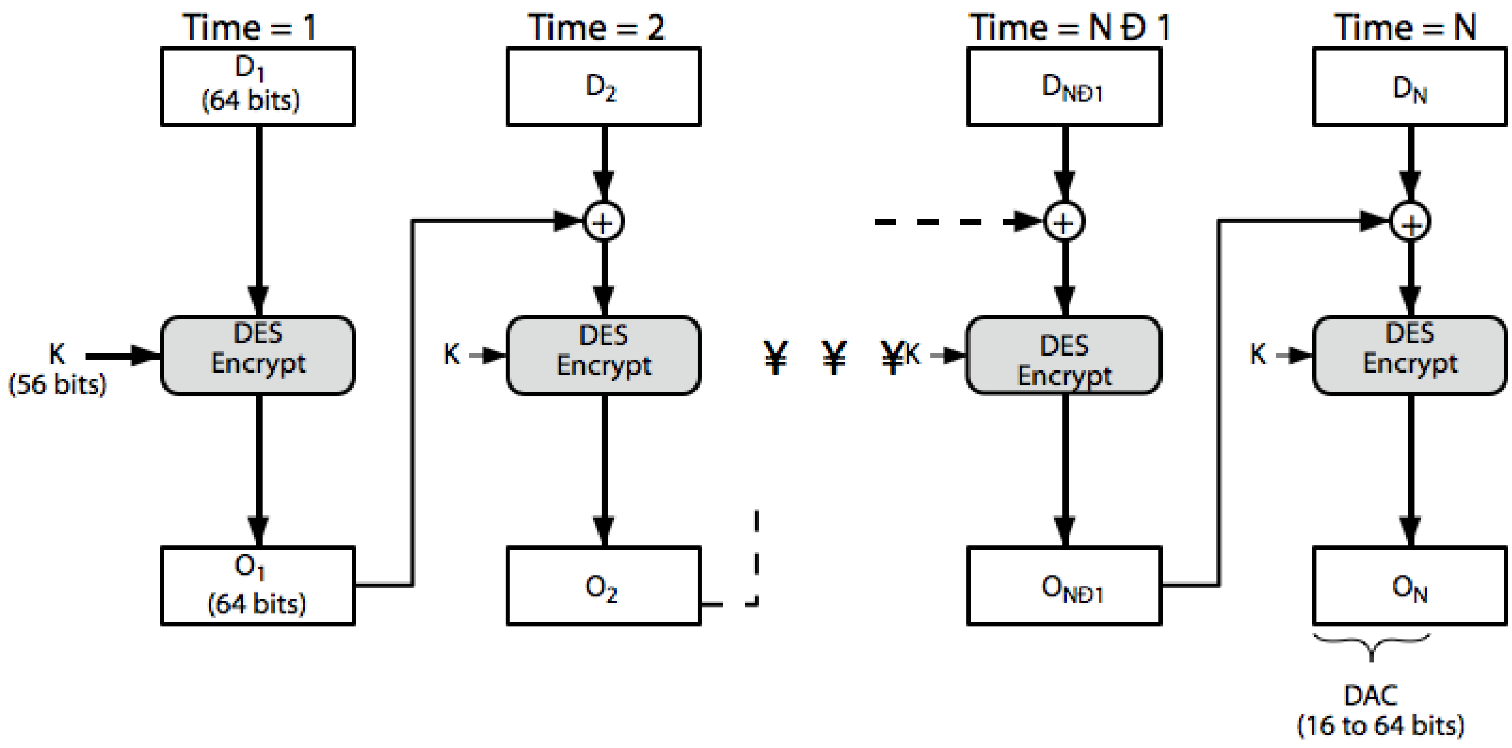
- taking into account the types of attacks
- need the MAC to satisfy the following:

1. knowing a message and MAC, is infeasible to find another message with same MAC
2. MACs should be uniformly distributed
3. MAC should depend equally on all bits of the message

Using Symmetric Ciphers for MACs

- can use any block cipher chaining mode and use final block as a MAC
- **Data Authentication Algorithm (DAA)** is a widely used MAC based on DES-CBC
  - using IV=0 and zero-pad of final block
  - encrypt message using DES in CBC mode
  - and send just the final block as the MAC
    - or the leftmost M bits ($16 \leq M \leq 64$) of final block
- but final MAC is now too small for security

Data Authentication Algorithm



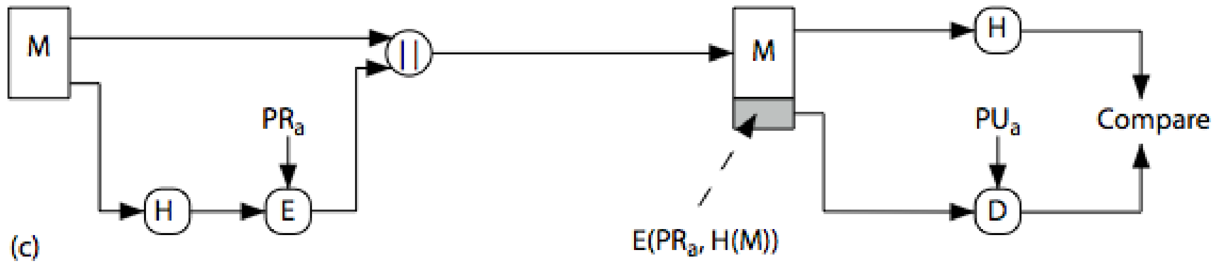
Hash Functions

- condenses arbitrary message to fixed size

  `h = H(M)`

- usually assume that the hash function is public and not keyed
  - cf. MAC which is keyed
- hash used to detect changes to message
- can use in various ways with message
- most often to create a digital signature

Hash Functions & Digital Signatures



Requirements for Hash Functions

1. can be applied to any sized message M
2. produces fixed-length output h
3. is easy to compute h=H(M) for any message M
4. given h is infeasible to find x s.t. H(x)=h
   - one-way property
5. given x is infeasible to find y s.t. H(y)=H(x)
   - weak collision resistance
6. is infeasible to find any x,y s.t. H(y)=H(x)
   - strong collision resistance

Simple Hash Functions

- are several proposals for simple functions
- based on XOR of message blocks
- not secure since can manipulate any message and either not change hash or change hash also
- need a stronger cryptographic function (next chapter)

Birthday Attacks

- might think a 64-bit hash is secure
- but by **Birthday Paradox** is not
- **birthday attack** works thus:
  - opponent generates $2^{m/2}$ variations of a valid message all with essentially the same meaning
  - opponent also generates $2^{m/2}$ variations of a desired fraudulent message
  - two sets of messages are compared to find pair with same hash (probability > 0.5 by birthday paradox)
  - have user sign the valid message, then substitute the forgery which will have a valid signature
- conclusion is that need to use larger MAC/hash

Block Ciphers as Hash Functions

- can use block ciphers as hash functions
  - using $H_0=0$ and zero-pad of final block
  - compute: $H_i = E_{M_i} [H_{i-1}]$
  - and use final block as the hash value
  - similar to CBC but without a key
- resulting hash is too small (64-bit)
  - both due to direct birthday attack
  - and to "meet-in-the-middle" attack
- other variants also susceptible to attack

Hash Functions & MAC Security

- like block ciphers have:
- **brute-force** attacks exploiting
  - strong collision resistance hash have cost $2^{m/2}$
    - have proposal for h/w MD5 cracker
    - 128-bit hash looks vulnerable, 160-bits better
  - MACs with known message-MAC pairs
    - can either attack keyspace (cf key search) or MAC
    - at least 128-bit MAC is needed for security

Hash Functions & MAC Security

- **cryptanalytic attacks** exploit structure
  - like block ciphers want brute-force attacks to be the best alternative
- have a number of analytic attacks on iterated hash functions
  - $CV_i = f[CV_{i-1}, M_i]$; $H(M)=CV_N$
  - typically focus on collisions in function f
  - like block ciphers is often composed of rounds
  - attacks exploit properties of round functions

Summary

- have considered:
  - message authentication using
  - message encryption
  - MACs
  - hash functions
  - general approach & security