Tutorial Sheet 8

MATHEMATICS-1(MA10001) AUTUMN-2019

1 Solve the following homogeneous differential equations:

(a) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 0$

(b) $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 3y = 0$

(c) $\frac{d^4y}{dx^4} - 64y = 0$

(d) $\frac{d^4y}{dx^4} - 4\frac{d^2y}{dx^2} + 4y = 0$

(e) $\big(\frac{d^2y}{dx^2} + 4y\big)\big(\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y\big) = 0$

(f) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$

(g) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$

(h) $\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 0$

(i) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 2y = 0$

(j) $\frac{d^3y}{dx^3} = 8y$

2 Solve the following initial value problems:

(a) $\frac{d^3y}{dx^3} + 3\frac{dy}{dx} - 4y = 0; \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 6$

(b) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0; \quad y(1) = e,\ y'(1) = -2$

(c) $\frac{d^3y}{dx^3} = 0; \quad y(0) = 3,\ y'(0) = -5,\ y''(0) = 1$

(d) $\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} = 0; \quad y(0) = 3,\ y'(0) = -5,\ y''(0) = 1$

(e) $\frac{d^2y}{dx^2} - k^2y = 0\ (k \neq 0); \quad y(0) = 1,\ y'(0) = 1$

(f) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 0; \quad y(-1) = 2,\ y'(-1) = 5$

3 Solve the following differential equations:

(a) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = 5\sin 2x$

(b) $\dfrac{d^2y}{dx^2} - 16y = e^x + \sin 3x$

(c) $\dfrac{d^2y}{dx^2} + y = 6\cos x + 2$

(d) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = e^{-2x}\sin 2x$

(e) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 2e^{-2x}$

(f) $6\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + y = x^2$

(g) $\dfrac{d^2y}{dx^2} - 4y = 5(\cosh 2x - x)$

(h) $\dfrac{d^2y}{dx^2} + y = 3\sin 2x - 5 + 2x^2$

(i) $\dfrac{dy}{dx} + y = 5 - e^{-x}$

(j) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$

4 Solve the following problems:

(a) If the roots of the characteristic equation are as follows then obtain the differential equation and also find the general solution of it:

(i) $2,\ 6$

(ii) $2i,\ -2i$

(iii) $4-2i,\ 4+2i$

(b) Construct a linear homogeneous second order differential equation such that the given functions are solutions of differential equation:

(i) $u(x) = \cos 5x,\ v(x) = \sin 5x$

(ii) $u(x) = x^2,\ v(x) = x^2 \ln x$

(iii) $u(x) = \dfrac{1}{x},\ v(x) = e^{-x}$

(c) Show that the substitution $z = \sinh^{-1} x$ transforms the equation $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 4y$ into $\dfrac{d^2y}{dz^2} = 4y$.