Tutorial Sheet-7

Autumm 2019

MATHEMATICS-I(MA10001) September, 2019

1. **Find the order and degree of the following differential equation:**

   (i) $\left(\frac{d^2x}{dt^2}\right)^2 + \frac{d^2x}{dt^2} + t\frac{dx}{dt} = 0.$

   (ii) $t\frac{d^2y}{dt^2} + t^2\frac{dy}{dt} - \sqrt{y}\cos t = 2t^2 - 3t + 4.$

   (iii) $\sqrt{y + \left(\frac{d^2y}{dx^2}\right)^2} = \left(\frac{dy}{dx}\right)^5.$

   (iv) $\frac{d^2y}{dx^2} = \frac{1}{4}\left(2 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{7}{2}}.$

   (v) $\left(1 + \frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \frac{dy}{dx}.$

2. **Form the ODE by eleminating the arbitary constants:**

   (i) $y = (A + Bx)e^{kx}$, where A, B are arbitary constant.

   (ii) $y = \ln(\sin(x + a)) + b.$

   (iii) $y = \alpha x + \alpha - \alpha^3.$

   (iv) $(x-h)^2 + (y-k)^2 = a^2$, where (h, k) is the center as parameter and a is the radius of the circle as constant.

   (v) Obtain the differential equation of all circles each of which touches the x-axis at the origin.

   (vi) Obtain the differential equation of the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, in which λ is the arbitary parameter and a, b are given constant.

3. **Solve the following Intial Value Problems:**

   (i) $x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx), \quad y(1) = \pi.$

   (ii) $(xy^2 - e^{\frac{1}{x^3}})dx - x^2ydy = 0, \quad y(1) = 0.$

   (iii) $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1, \quad y(0) = 1.$

4. **Check whether the differential equations are homogeneous or not (reduce it to homogeneous if not), then solve it:**

   (i) $x\sin\left(\frac{y}{x}\right)dy = (y\sin\frac{y}{x} - x)dx.$

   (ii) $2xydy = (y^2 - x^2)dx.$

   (iii) $(x + 2y - 3)dy = (2x - y + 1)dx.$

5. **Check whether the differential equations are exact or not (if not, reduce it to exact by using proper Integrating factor), then solve it:**

   (i) $x^2ydx - (x^3 + y^3)dy = 0.$

   (ii) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0.$

   (iii) $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0.$

   (iv) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}.$

   (v) $x\frac{dy}{dx} + (3x + 1)y = xe^{-2x}.$

6. **Solve the following ODEs by reducing them to linear differential equations:**

   (i) $\frac{dy}{dx} + \frac{1-2x}{x^2}y = 1.$

   (ii) $\sqrt{a^2 + x^2}\frac{dy}{dx} + y = \sqrt{a^2 + x^2} - x.$

   (iii) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)e^x\sec y.$

(iv) $y(2xy + e^x)dx - e^x dy = 0.$

(v) $y^2 + (x - \frac{1}{y})\frac{dy}{dx} = 0.$

(vi) $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2.$

(vii) $\cos x\frac{dy}{dx} - y\sin x + y^2 = 0.$

(viii) $ydy + by^2dx = a\cos x dx.$