1. Let $f(x, y) = e^x \sin y$. Expand $f(x+h, y+k)$ in power of h and k in Taylor's series up to second order term and write the remainder R_3.

2. Expand $f(x, y) = x^2y + \sin y + e^x$ in powers of $(x-1)$ and $(y-\pi)$ in Taylor's series up to second order term and and write the remainder R_3.

3. Show that for $0 < \theta < 1$,
$$e^{ax} \sin by = by + abxy + \frac{1}{6}[(a^3x^3 - 3ab^2xy^2)\sin(b\theta y) + (3a^2bx^2y - b^3y^3)\cos(b\theta y)]e^{a\theta x}.$$

4. Expand $e^x \tan^{-1} y$ about $(1, 1)$ upto second degree in $(x-1)$ and $(y-1)$, and find the remainder term.

5. Expand $f(x, y, z) = e^z \sin(x+y)$ in Taylor's series upto second order term about the point $(0, 0, 0)$.

6. Verify that the function $f(x, y) = x^3y^2(1-x-y)$ has a maximum at $(\frac{1}{2}, \frac{1}{3})$.

7. Find the stationary points of the following functions and classify them. Or in other words, find the local maximum or local minimum values of the following functions.
   (a) $f(x, y) = x^2y - 2xy^2 + 3xy + 4$
   (b) $f(x, y) = x^3 + y^3 - 3axy$
   (c) $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

8. Find a point within a triangle such that the sum of the square of its distances from the three vertices is minimum.

9. Show that the minimum value of $f(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$ is $3a^2$.

10. Of all the triangles with the same perimeter, determine the triangle with the greatest area using Lagrange's method of multiplier.

11. Express the number 27 into product of three parts x, y, z such that $2yz + 3zx + 4xy$ is maximum.

12. Find the greatest and least values of the function $f(x, y) = xy$ that the function takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

13. Prove that of all rectangular parallelopiped of the same volume, the cube has the least surface.

14. Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at a maximum distance from the point $P(2, 1, -2)$?

15. Prove that if $x + y + z = 1$, then the function $u(x, y, z) = ayz + bzx + cxy$ has an extreme value equal to

$$\frac{abc}{(2bc + 2ca + 2ab + a^2 - b^2 - c^2)}. \quad (1)$$

Prove also that if $a, b, c > 0$ and $c \in (a + b - 2\sqrt{(ab)}, a + b + 2\sqrt{(ab)})$ then (1) gives the true maximum and if $a, b, c < 0$ and $c \in (a + b - 2\sqrt{(ab)}, a + b + 2\sqrt{ab})$, then (1) gives the true minimum.

16. Determine the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the condition $x - 2y - 4z = 5$.

17. Show that the maximum and minimum value of $ax+by$ (where the constants $a, b > 0$) subject to the constraint $x^2 + y^2 = 1$ are $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$ respectively.

18. For the extremum values of $x^2+y^2+z^2$ subject to the constraints $ax^2+by^2+cz^2 = 1$ and $lx + my + nz = 0$, show that the stationary points satisfy the relation $\frac{l^2}{(1+a\lambda_1)} + \frac{m^2}{(1+b\lambda_1)} + \frac{n^2}{(1+c\lambda_1)} = 0$.

19. Show that the maximum and minimum of radius $r = \sqrt{x^2 + y^2 + z^2}$ of the section of the surface $(x^2 + y^2 + z^2)^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ by the plane $\lambda x + \mu y + \nu z = 0$ satisfies the equation

$$\frac{a^2\lambda^2}{1 - a^2r^2} + \frac{b^2\mu^2}{1 - b^2r^2} + \frac{c^2\nu^2}{1 - c^2r^2} = 0.$$

20. Find the maximum/minimum values of the following functions using Lagrange's method of multipliers.
    (a) $f(x, y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$.
    (b) $f(x, y) = 4x^2 + 10y^2$ on the disk: $x^2 + y^2 \leq 4$.
    (c) $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$.

21. Find the absolute maximum and absolute minimum of the following functions.
    (a) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate in the first quadrant bounded by $x = 0$, $y = 2$ and $y = 2x$.
    (b) $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by $x = 0$, $y = 4$ and $y = x$.

22. Expand $f(x, y) = \tan^{-1} \frac{y}{x}$ in the neighborhood of $(1, 1)$ upto third-degree terms. Hence compute $f(1.1, 0.9)$ approximately.

23. Use Taylor's formula to find quadratic approximation of $f(x, y) = e^x \sin y$ at the origin. Estimate the error in the approximation if $|x| < 0.1$ and $|y| < 0.1$.

24. Use Taylor's formula to find a quadratic approximation of $f(x, y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| < 0.1$ and $|y| < 0.1$.