Problem Set - 11

Autumn 2019

MATHEMATICS-I (MA10001)

1. (a) Evaluate $\int\limits_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along

   (i) the parabola $x = 2t, y = t^2 + 3$.

   (ii) the straight line segments from $(0,3)$ to $(2,3)$ and then from $(2,3)$ to $(2,4)$.

   (iii) a straight line from $(0,3)$ to $(2,4)$.

   (b) Evaluate $\int\limits_C f(z)dz$, where $f(z) = y - x - i3x^2$ and C is the line segment from $z = 0$ to $1 + i$.

   (c) Evaluate $\int\limits_C \overline{z}dz$ from $z = 0$ to $4 + 2i$ along the curve C consisting of line segment from $z = 0$ to $2i$ followed by a line segment from $z = 2i$ to $4 + 2i$.

   (d) Evaluate $\int\limits_C |z|\overline{z}dz$, where C is the closed curve consisting of the upper semi circle $|z| = 1$ and the segment $-1 \leq x \leq 1$.

   (e) Evaluate $\int\limits_C \frac{z+2}{z}dz$, where C is the semi circle $z = 2e^{i\theta}$ where $0 \leq \theta \leq \pi$.

2. (a) Verify that the value of integral $\int\limits_C (z^2 + 1)dz$ is same in all the case:

   (i) C is the straight line joining the point $A(0,0)$ and $B(1,1)$.

   (ii) C is the straight line joining the point $A(0,0)$ to $P(1,0)$ followed by a straight line path from $P(1,0)$ to $B(1,1)$.

   (iii) C be the parabolic path $y = x^2$ joining the point $A(0,0)$ and $B(1,1)$.

   (b) Show that $\int\limits_C |z|^2dz = -1 + i$, where C is the square with vertices $O(0,0)$, $A(1,0)$, $B(1,1)$ and $C(0,1)$.

3. Evaluate the following integrals.

   (i) $\int\limits_C \frac{dz}{2z+3}$, where C is the circle $|z| = 2$.

(ii) $\int\limits_C \frac{3z-4}{z(z-1)}dz$, where C is the circle $|z| = \frac{3}{2}$.

(iii) $\int\limits_C \frac{e^z}{z(z-1)(z-2)}dz$, where C is the circle $|z| = \frac{3}{2}$.

(iv) $\int\limits_C \frac{z^2+3}{z(2-\overline{z})}dz$, where C is the circle $|z| = 1$.

(v) $\int\limits_C \frac{1}{(z-a)^n}dz$, where C is the circle wih centre a and radius r and $n \in \mathbb{Z}$.

4. Without evaluating the integral show that

(i) $|\int\limits_C \frac{dz}{z^2+1}| \leq \frac{\pi}{3}$, where C is the arc of circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

(ii) $|\int\limits_C \frac{dz}{z^4}| \leq 4\sqrt{2}$, where C is the line segment from $z = i$ to $z = 1$.

5. Evaluate the following integrals.

(i) $\int\limits_C \frac{e^z + z\sinh z}{(z-\pi i)^2}dz$, where C is the circle $|z| = 4$.

(ii) $\int\limits_C \frac{\sin 2z}{(z-\frac{\pi}{4})^4}dz$, where C is the circle $|z| = 1$.

(iii) $\int\limits_C \frac{dz}{(z^2+4)^2}dz$, where C is the circle $|z-i| = 2$.

(iv) $\int\limits_C \frac{\cos z}{z(z^2+8)}dz$, where C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the positive sense.

(v) $\int\limits_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$, where C is the circle $|z| = 3$.

6. Evaluate the integral $\int\limits_C \frac{e^z}{z(z-1)^2}dz$, where C is a closed curve in the following cases

(i) the point 0 lies inside and the point 1 is outside C.

(ii) the point 1 lies inside and the point 0 is outside C.

(iii) the point 0 and 1 both lie inside C.

7. Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$,
if C is (i) the circle $|z| = 1$.
(ii) the circle $|z - 1| = 3$.
(iii)if C is the square with vertices $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$.

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