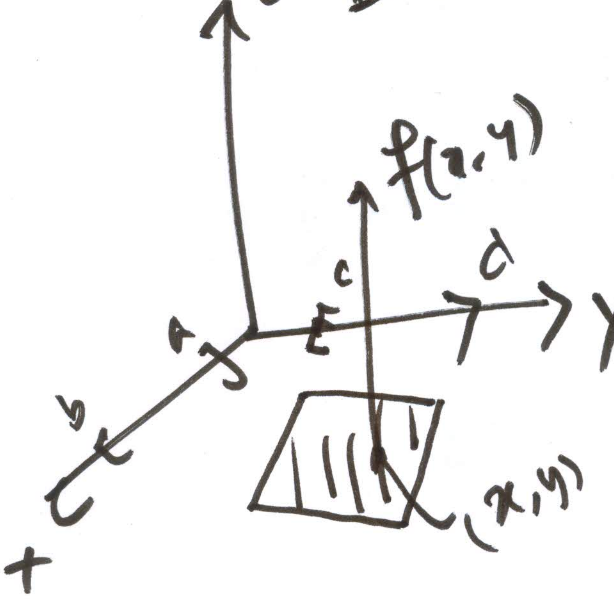
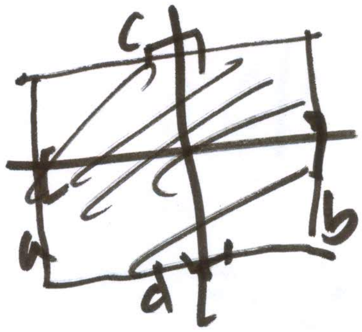
Lecture 5

$$f : D \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$$

In particular.

$D = \mathbb{R}^2$, $D = [a,b] \times [c,d]$





$$\lim_{(x,y) \to (a,b) \in \mathbb{R}^2 \boxed{\supseteq D}} f(x,y) = l$$

if (x,y) approches to (a,b) along any path in the domain D then $f(x,y)$ approches to l.

for any/each $\varepsilon > 0$ $\exists$ $\delta > 0$

s.t. when (x, y) ~~line~~ lies inside the circle centered at (a, b) with radius δ then $f(x, y)$ lies in the ε-neighbour of l.

$$\Rightarrow |f(x,y) - l| < \varepsilon \text{ when } 0 < |(x,y)-(a,b)| < \delta$$

$$\Updownarrow$$

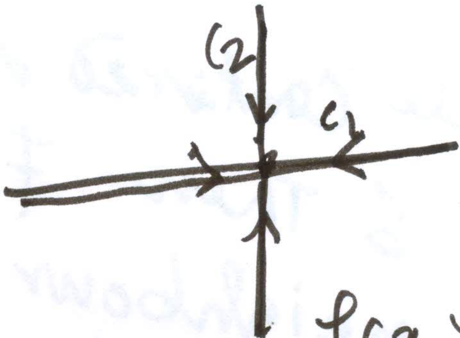
$$0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$$

<u>Meaning of saying limit does not exist.</u>

If we can find a path C_1 along which (x,y) approaches to (a,b) and $f(x,y)$ approaches to L_1, a path C_2 along which (x,y) approaches to (a,b) and $f(x,y)$ approaches to L_2, and

$$L_1 \neq L_2$$

Ex1 $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$ does NOT exist.



C_1: $y=0$, $x\to 0$

$(x,0)\to(0,0)$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } C_1}} f(x,y) = \lim_{\substack{y=0\\ x\to 0}} f(x,y) = \lim_{x\to 0}\frac{x^2}{x^2} = 1 = L_1$$

C_2: $x=0$, $y\to 0$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } C_2}} f(x,y) = \lim_{\substack{x=0\\ y\to 0}} f(x,y) = \lim_{y\to 0} -\frac{y^2}{y^2} = -1 = L_2$$

Note that $L_1 \neq L_2$.