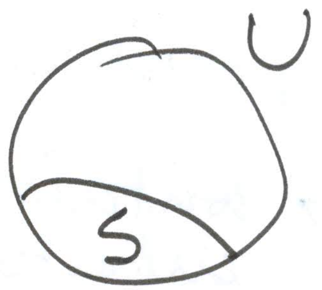
Lecture 2



s.t. $|S| = |U|$

$\Rightarrow U$ is infinite

$\mathbb{Q}$. $\mathbb{R}$

For exp. $\underbrace{|\mathbb{N}| \overset{?}{=} |\mathbb{Q}|}_{\text{true}} \overbrace{\overset{?}{=} |\mathbb{R}|}^{\text{Not true}}$

$\mathbb{Q} = \bigcup_{n \in \mathbb{Z}} A_n$

$\vdots$

$A_{-3} = \dots$

$A_{-2} = \dots$

$A_{-1} = \dots$

$A_1 = \{1, 2, 3, 4, \dots\}$

$A_2 = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \dots\}$

$A_3 = \{\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \dots\}$

$\vdots$

$A_n = \{\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots\}$

$A_0 = \{0\}$

Let $S = \{1, \ldots, m\}$

$\mathcal{P}(S) \equiv$ power set of S

$\mapsto$ contains all subsets of S.

$$|\mathcal{P}(S)| = 2^m$$

Q. How many fns one can define

$$f : S \to \{a, b\}$$

A. Then # of such fns is 2^m.

$S = \{1, 2, 3\}$. $a = 0$, $b = 1$

$f_1(1) = 0$	$f_2(1) = 1$
$f_1(2) = 1$	$f_2(2) = 0$
$f_1(3) = 0$	$f_2(3) = 0$

$f_1 \equiv [0\ 1\ 0]$ $\quad f_2 \equiv [1\ 0\ 0]$

Q. Let $S = \mathbb{N}$

Then how many fns $f : \mathbb{N} \to \{0, 1\}$?

$\#$ of fns $= 2^{\aleph_0}$, when $\aleph_0 = |\mathbb{N}|$

$> \aleph_0$