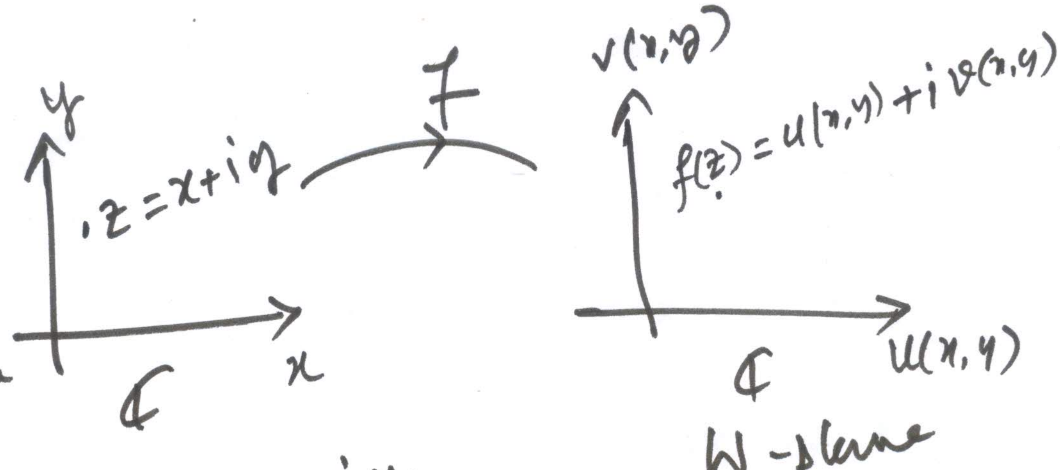
Lec-21



Let $z_0 = x_0 + i y_0$

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{h \to 0} \frac{f(z_0+h) - f(z_0)}{h}, \quad h = \underbrace{h_1}_{re(h_1)} + i \underbrace{h_2}_{im(h_2)}$$

If 'f' is differentiable/analytic at $z_0 = x_0 + i y_0$ then CR (Cauchy-Riemann) Eqns are satisfied.

Besides

$$f'(z_0) = u_x(x_0, y_0) + i\, v_x(x_0, y_0)$$

$$f'(z_0) = v_y(x_0, y_0) - i\, u_y(x_0, y_0)$$

$$\left.\begin{aligned} u_x(x_0, y_0) &= v_y(x_0, y_0) \\ u_y(x_0, y_0) &= -v_x(x_0, y_0) \end{aligned}\right\} \text{check}$$

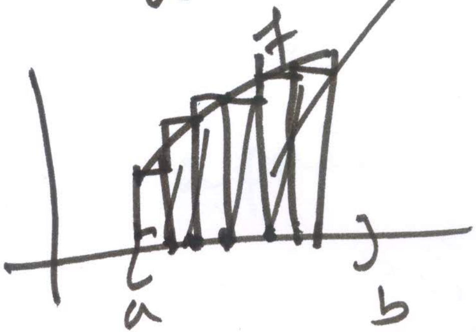
If $u(x,y)$ / $v(x,y)$ is harmonic fn. we can find its conjugate harmonic $v(x,y)$ / $u(x,y)$ such that $f(z) = u(x,y) + i v(x,y)$ is analytic

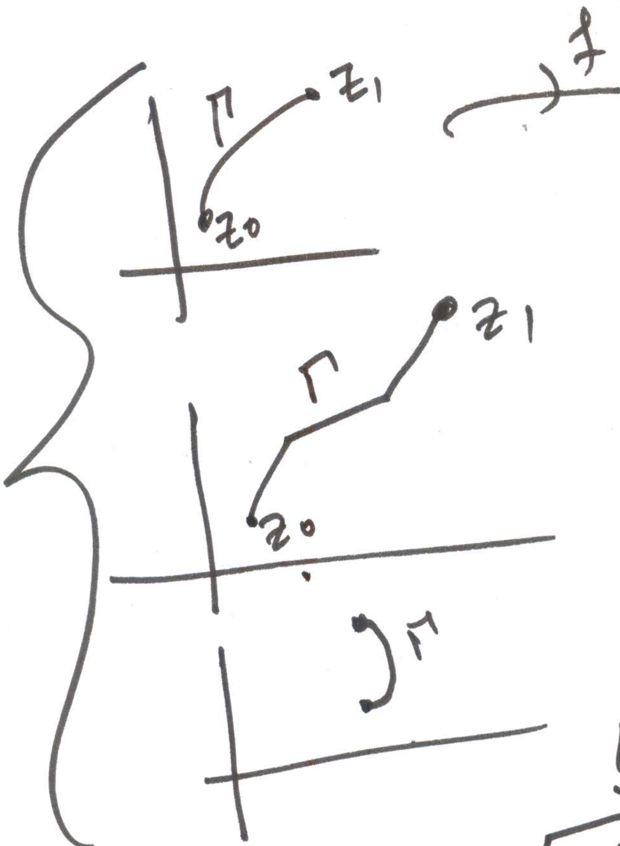
Complex Integration.

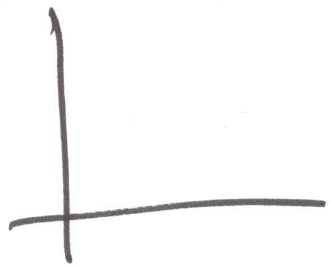
$$\int_a^b f(x)\,dx = \text{some value}$$

when $f: \mathbb{R} \to \mathbb{R}$

$f: [a,b] \subseteq \mathbb{R} \to \mathbb{R}$

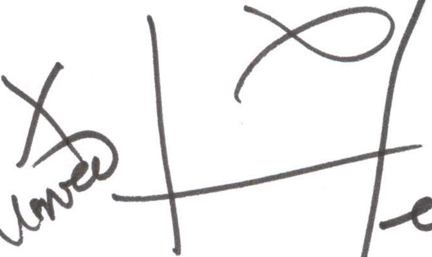






$$\int_\Gamma f(z)\,dz$$

LINE Integral:



Not allowed

path — A non-self-intersecting curve Γ whose end points are NOT coincident is called a path.

$\int_\Gamma f(z)\,dz$ where f is analytic in a given direction along a curve Γ in the complex plane.

If Γ is a path $\int_\Gamma f(z)\,dz$ is called a line integral

<u>Contour</u>. A closed path Γ in the form of a simple (non-self-intersecting) loop is called a contour