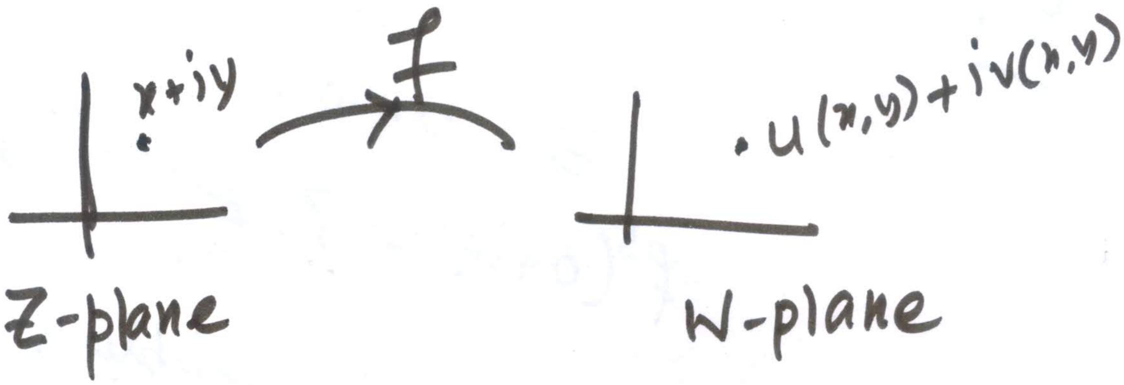
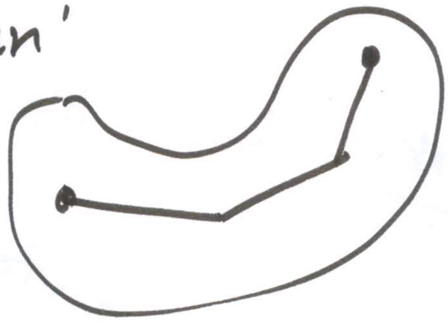
Lec 20

$$f: \mathbb{C} \to \mathbb{C}$$



$$f: D \subseteq \mathbb{C} \to \mathbb{C}$$

'domain'



Differentiable function.

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{h \to 0} \frac{f(z_0 + h) - f(z_0)}{h}$$

$$z = \underset{\mathrm{re}(z)}{\underset{\parallel}{x}} + i \underset{\mathrm{im}(z)}{\underset{\parallel}{y}}$$

where $h = z - z_0$

$h = \mathrm{re}(h) + i\,\mathrm{im}(h)$

Exp.

$$f(z) = \overline{z}.$$

$$z_0 = 0 + i0.$$

$$f'(0+i0) = ?$$ ← does NOT exist !!

NOT differentiable ?

Exp.

$$f(z) = z^n.$$

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{h \to 0} \frac{f(z_0+h) - f(z_0)}{\cancel{z - z_0}\, h}$$

$$= \lim_{h \to 0} \frac{(z_0+h)^n - z_0^n}{\cancel{z - z_0}\, h}$$

$$= n \, z^{n-1}$$

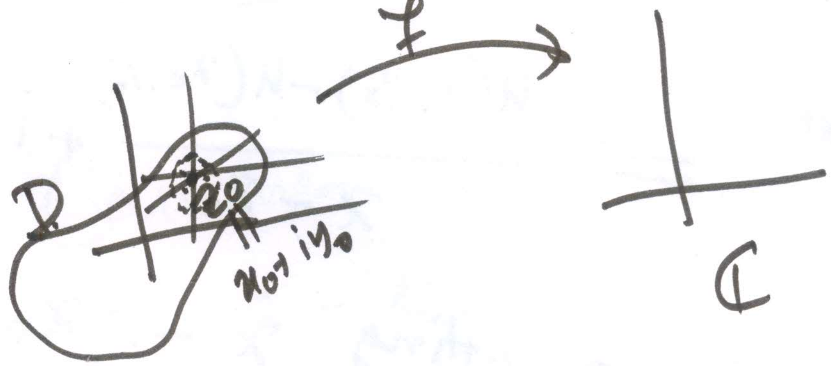
most of the formula for derivative real fns' are valid for 'complex fns'

Analytic/Regular/holomorphic function.

$$f : D \to \mathbb{C}$$

f is said to be analytic in D if it is analytic at every point of D.

f is said to be analytic at a point z_0 if $f(z_0)$ is defined and f is differentiable in some neighborhood of z_0



$$f(z) = u(x,y) + i\, v(x,y)$$

Suppose f is differentiable at z_0.

then $\dfrac{f(z) - f(z_0)}{z - z_0} \to$ a limit

as $z \to z_0 = x_0 + iy_0$