CLASS TEST, Nov 01, 2019

Time: 6:15 - 7:15 pm

Sec 9: N343
Sec 10: N344

Syll. - Fns of several variables + ODE.

Lec-19

System of linear first order ODE.

single 'independent' variable $\rightarrow$ 't'
multiple dependent variables $\rightarrow x_1(t), \dots, x_n(t)$

# of dependent variables = # of ODEs.] 'LTI' ≡ Linear Time invariant system.

$$\dot{x}_1(t) = \frac{dx_1}{dt} = a_{11}\, x_1(t) + a_{12}\, x_2(t) + \cdots + a_{1n}\, x_n(t) + f_1(t)$$

$$\dot{x}_2(t) = \frac{dx_2}{dt} = a_{21}\, x_1(t) + a_{22}\, x_2(t) + \cdots + a_{2n}\, x_n(t) + f_2(t)$$

$$\vdots$$

$$\dot{x}_n(t) \quad \frac{dx_n}{dt} = a_{n1}\, x_1(t) + \cdots + a_{nn}\, x_n(t) + f_n(t)$$

method of elliminatia.

Exp.

$$\begin{cases} \dfrac{dy_1}{dt} = 7y_1 - y_2 \\ \dfrac{dy_2}{dt} = 2y_1 + 5y_2 \end{cases}$$

Homogenous or Autonomous

$D = \frac{d}{dt}$

$Dy_1 - 7y_1 + y_2 = 0$ —(1)

$Dy_2 - 2y_1 - 5y_2 = 0$ —(2)

~~From~~ (1)

$(D-7)y_1 + y_2 = 0$ —(3)

$(D-5)y_2 - 2y_1 = 0$ —(4)

From (3). $y_2 = -(D-7)y_1$

Then from (4). $(D^2 - 12D + 37)y_1 = 0$

$y_1(t) = e^{6t}(C_1 \cos t \pm C_2 \sin t)$

then $y_2(t) = 7y_1(t) - Dy_1(t)$

Ex.

$$\frac{d^2y_1}{dt^2} + \frac{dy_1}{dt} - 2y_1 = \sin t$$

$$\frac{dy_2}{dt} + y_2 - 3y_1 = 0$$

i.e. $(D^2 + D - 2)y_1 = \sin t \quad -(1)$

$(D + 1)y_2 - 3y_1 = 0 \quad -(2)$

Solve (1) & put the general solution $y_1(t)$ into (2) then it gives you a linear 1st order eq^n. Then solve this to determine $Y_2(t)$ using the Integrating Fac.

Exp.

$$(5D+4)x - (2D+1)y = e^{-t}$$

$$(D+8)x - 3y = 5e^{-t}.$$

$D = \frac{d}{dt}$, $x(0) = 1$, $y(0) = 2$

Soln.

$$x(t) = c_1 e^{t} + c_2 e^{-2t} + 2e^{-t}$$

$$y(t) = 3c_1 e^{t} + 2c_2 e^{-2t} + 3e^{-t}$$

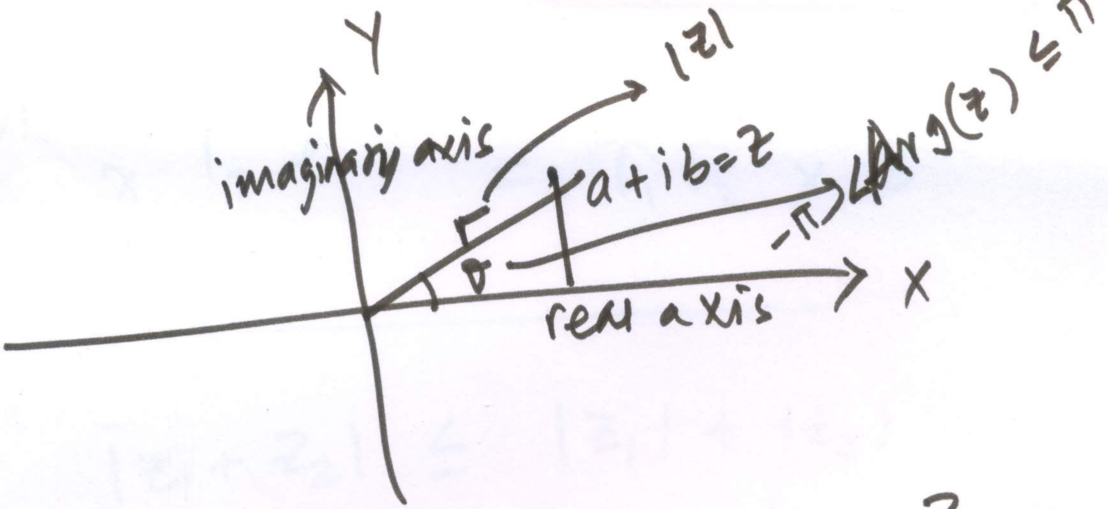
$t = 0$, $x(0) = 1$

$t = 0$, $y(0) = 2$

and calculate the values of c_1 & c_2.

Module - III

functions of complex ~~numbers~~ variables.



$$\mathbb{R}\times\mathbb{R} \neq \mathbb{C} \equiv \{ x+iy \mid x, y \in \mathbb{R} \}$$

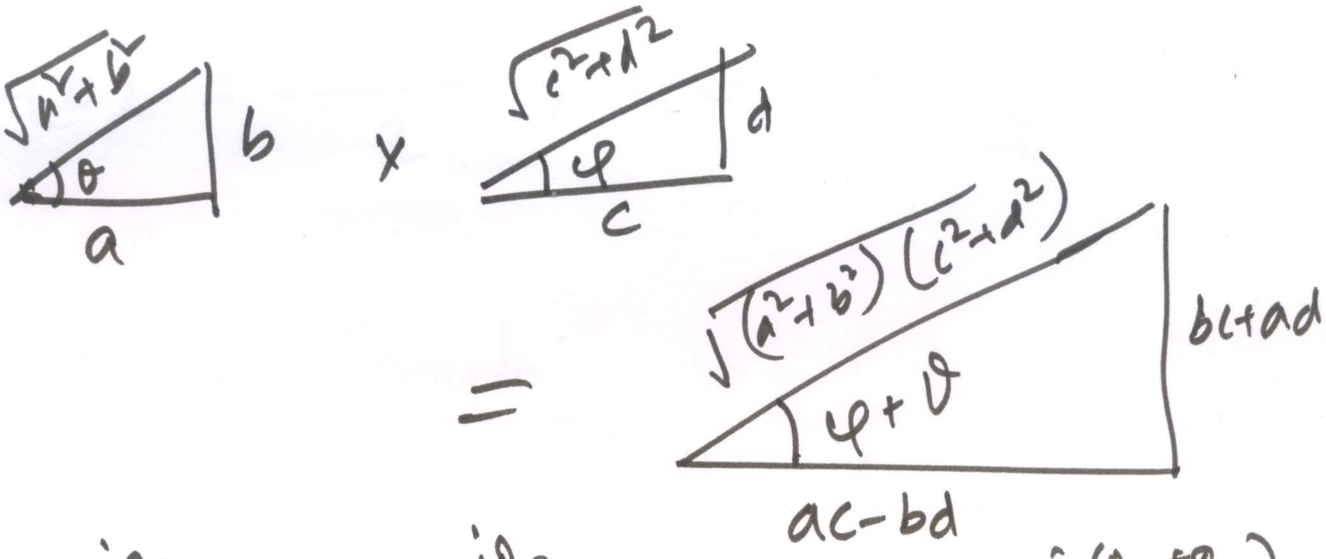
representation of a number which we complex number.

Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

Then addition : $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

multiplication $z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$

$z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$



$$r_1 e^{i\theta_1} \times r_2 e^{i\theta_2} = r_1 r_2 \times e^{i(\theta_1+\theta_2)}$$

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obs.

$$|z_1+z_2| \leq |z_1| + |z_2|$$

$$|Re(z)| \leq |z| \qquad z = re(z) + i\, im(z)$$

$$|im(z)| \leq |z|$$

$$\big|\, |z_1| - |z_2| \,\big| \leq |z_1 - z_2|$$

$$|z_1 - z_2| \geq |z_1| - |z_2|$$

$$|z_1 z_2| = |z_1|\,|z_2|$$

Conjugate of $z = x+iy$ is $\bar{z} = x - iy$.

$$\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2, \quad \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$$

$|z_1| = |\bar{z}_1|$, $|z|^2 = z\bar{z}$, $\frac{1}{z} = \bar{z}/|z|^2$ when $z \neq 0$.

$$\mathrm{re}(z) = \frac{z+\bar{z}}{2}, \quad \mathrm{im}(z) = \frac{z-\bar{z}}{2i}$$

<u>deMoivre's Formula:</u>

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

<u>Pf.</u> $e^{in\theta} = e^{i(n\theta)} = \cos n\theta + i\sin n\theta.$

<u>Obs.</u> (1) $e^{z+2\pi i} = e^z$

$e^{z+2\pi i k} = e^z$ for every integer k.

(2) $\cos x = \dfrac{e^{ix}+e^{-ix}}{2}, \; x \in \mathbb{R}$

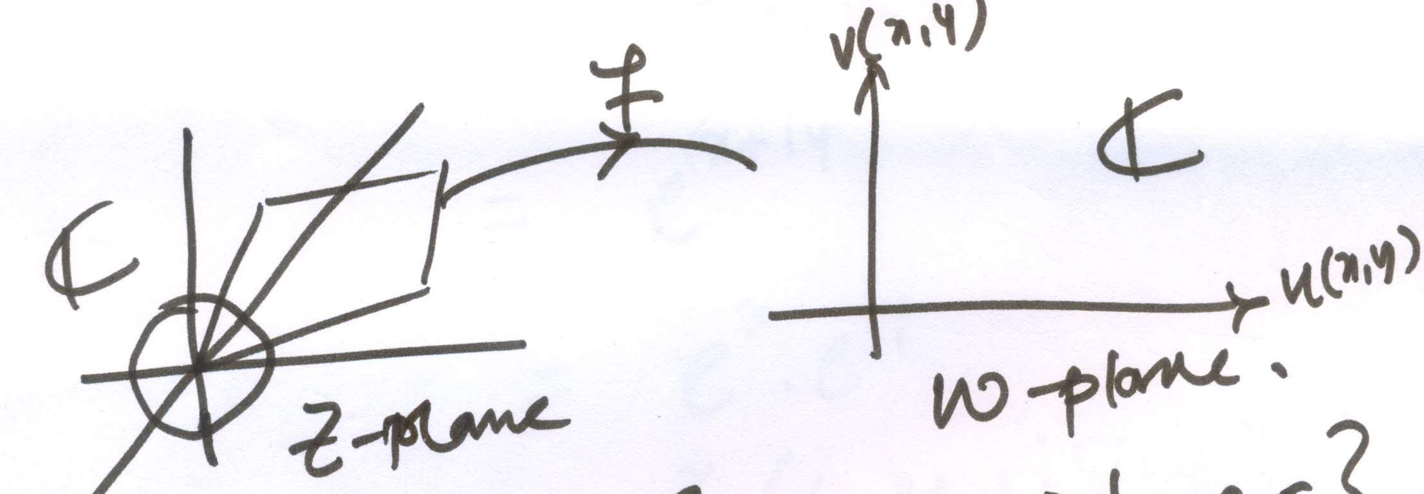
$\sin x = \dfrac{e^{ix}-e^{-ix}}{2i}, \; x \in \mathbb{R}$

$\cos z = \dfrac{e^{iz}+e^{-iz}}{2}, \; z \in \mathbb{C}$

$\sin z = \dfrac{e^{iz}-e^{-iz}}{2i}, \; z \in \mathbb{C}$

Complex valued function.

$$f : \mathbb{C} \longrightarrow \mathbb{C}$$



$$\text{Graph 'f'} = \{ (z, f(z)) \mid z \in \mathbb{C} \}$$

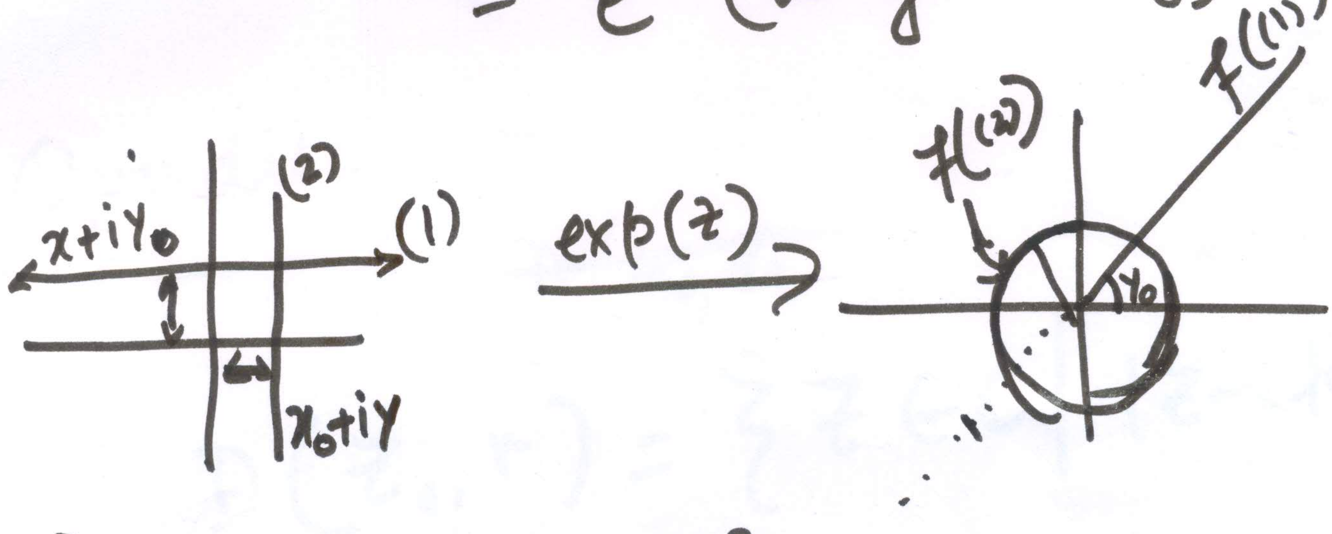
$$w = f(z) \,, \quad z = x + iy$$

$$= \operatorname{re} f(z) + i \operatorname{im} f(z)$$

$$= u(x,y) + i\, v(x,y)$$

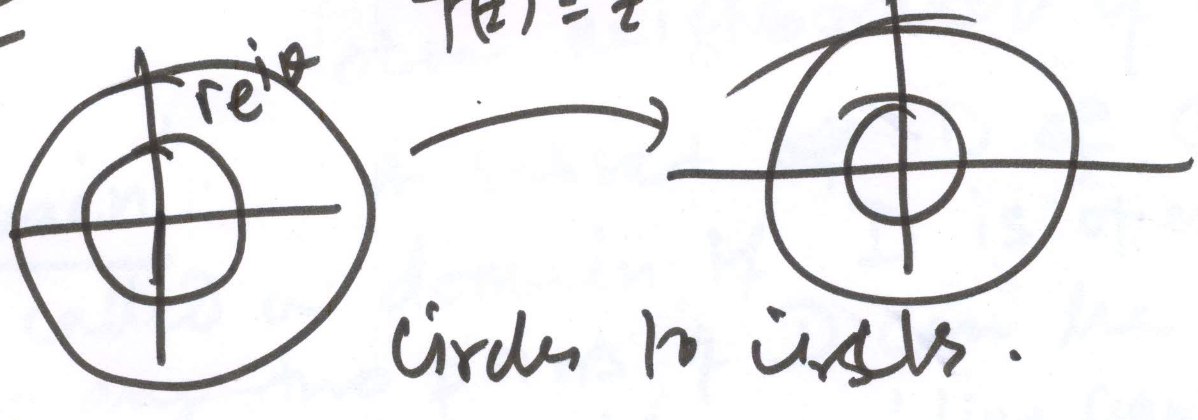
Exp 1

$$f(z) = \exp(z)$$
$$= e^z$$
$$= \exp(x+iy)$$
$$= e^{(x+iy)}$$
$$= e^x \cdot e^{iy}$$
$$= e^x(\cos y + i\sin y)$$



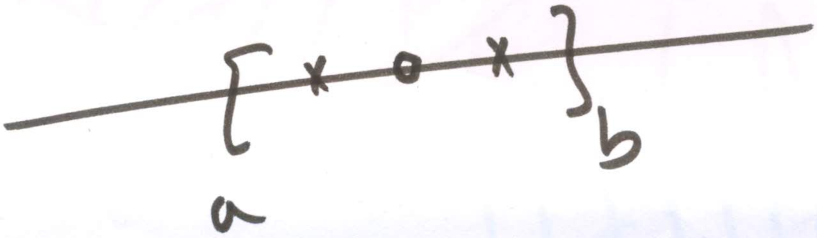
Exp 2.

$f(z) = z^2$

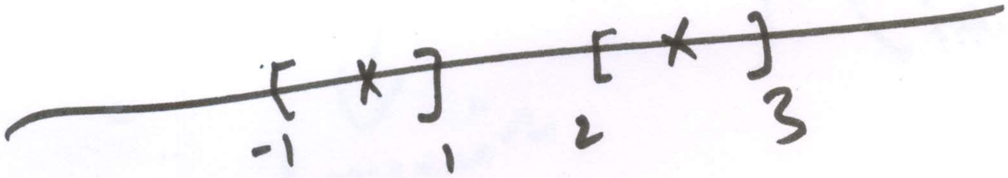


circles to circles.

$$f : [a,b] \longrightarrow \mathbb{R}$$



$$[-1, 1] \cup [2,3]$$



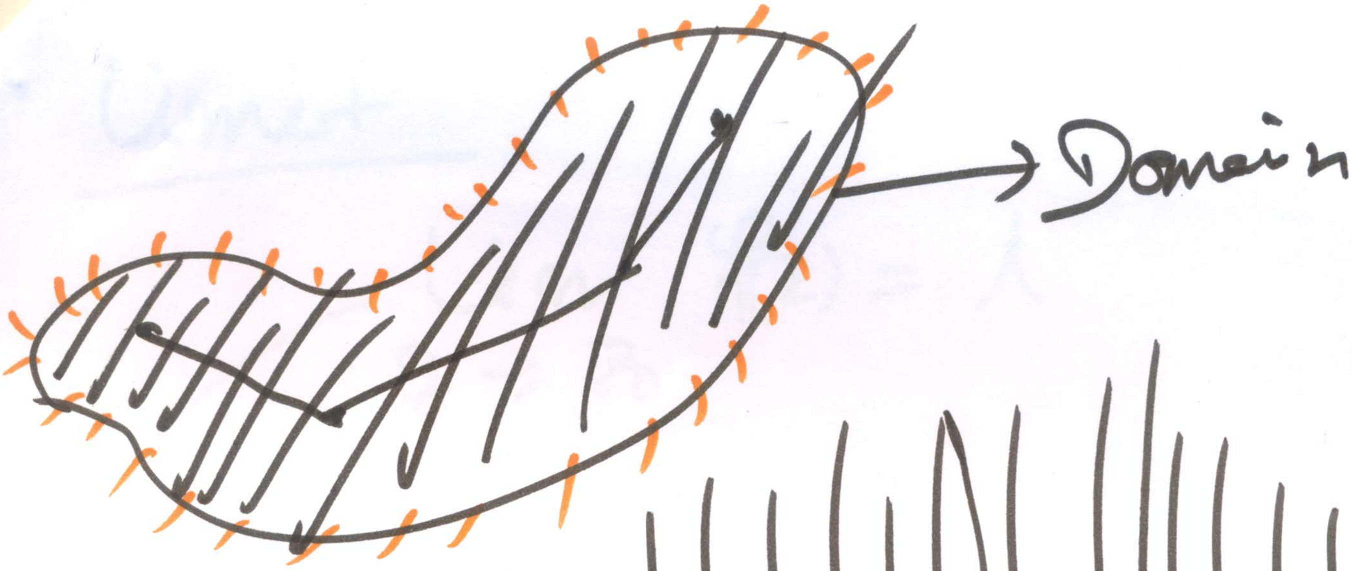
Open disc.

$\cancel{D(z_0, z_0}$

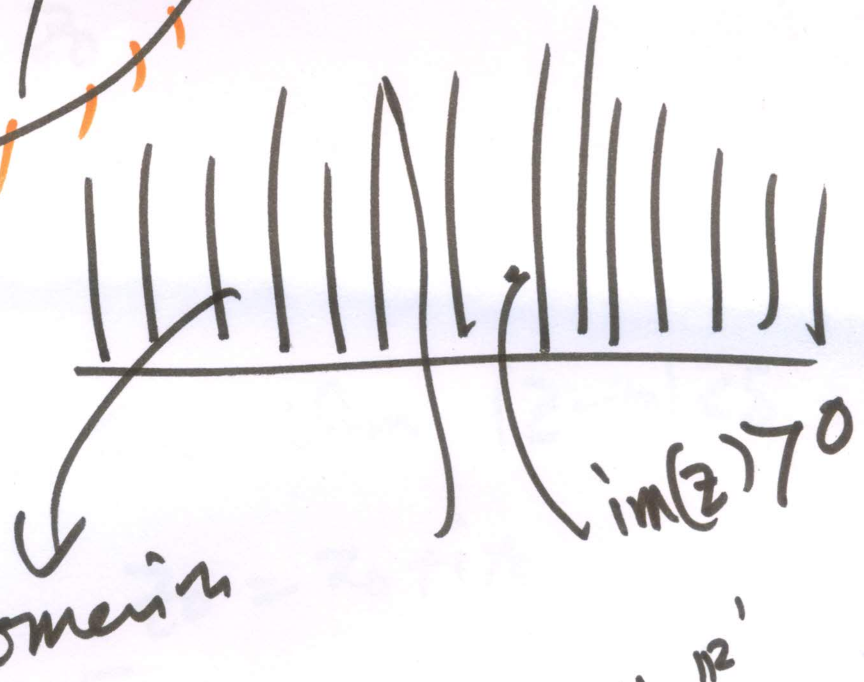
$$D(z_0, r) = \{ z \in \mathbb{C} \mid |z - z_0| < r \}$$

open neighborhood of z_0.

'Domain': A subset ~~of~~ D of $\mathbb{C}$ is called a domain if D is open and any two points of D can be connected by a 'broken' line segments in D



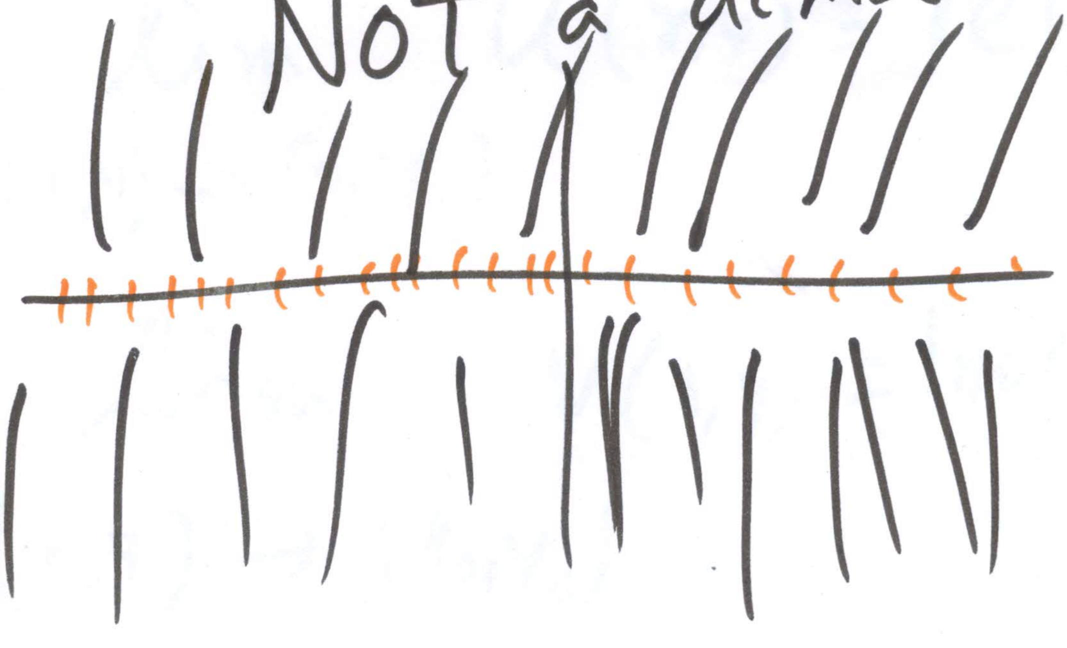
Exp. (1)



← ℂ minus ℝ

Exp (2)

$\mathbb{C} \setminus \mathbb{R}$ is NOT a domain



Limit

$$\lim_{z \to z_0} f(z) = l$$

for every $\varepsilon > 0$ $\exists$ $\delta > 0$

s.t.

$|f(z) - l| < \varepsilon$ when $|z - z_0| < \delta$

$$z_0 = x_0 + iy_0$$

$$\lim_{(x,y) \to (x_0,y_0)} \left[u(x,y) + i\,v(x,y)\right]$$

$$= re(l) + i\,im(l)$$

$$\Leftrightarrow \lim_{(x,y) \to (x_0,y_0)} u(x,y) = re(l)$$

$$\lim_{(x,y) \to (x_0,y_0)} v(x,y) = im(l)$$

Continuity.

f is continuous at z_0

if $\lim_{z \to z_0} f(z) = f(z_0)$.

Differtiability.

$$f'(z_0) = \lim_{h \to 0} \frac{f(z_0+h) - f(z_0)}{h}$$

$h = z - z_0$

$f(z_0+h) - f(z_0) \approx h f'(z_0)$

$\Rightarrow f(z_0+h) = f(z_0) + h f'(z_0)$

$\Rightarrow \boxed{f(z) = f(z_0) + (z - z_0) f'(z_0)}$

'translation + rotation'