

Ex 18
Ex 1 Find

the general solution
of $y'' + 2y' + y = xe^{-x}$

Solⁿ. $y(x) = C_1 e^{-x} + C_2 x e^{-x}$
 $+ \frac{1}{6} x^3 e^{-x}$.

Ex 2. $x^2 y'' = 3xy' + 4y$
 $= \ln x, x > 0$.

Solⁿ. ~~$y(x) = C_1 x^2 + C_2 x^2 \ln x$~~
 ~~$+ \frac{1}{4} + \frac{1}{4} \ln x, x > 0$~~

$$y_1(x) = x^2, \quad y_2(x) = x^2 \ln x, \quad x > 0$$

$$W(x) = \begin{vmatrix} x^2 & x^2 \ln x \\ 2x & 2x \ln x + x \end{vmatrix}$$

$$= x^3$$

$$y_p(x) = u_1(x) y_1(x) + u_2(x) y_2(x)$$

$$= - \left(\int \frac{x^2 \ln^2 x}{x^3} dx \right) x^2$$

$$+ \left(\int \frac{x^2 \ln x}{x^3} dx \right) x^2 \ln x$$

$$= -x^2 \int \frac{(\ln x)^2}{x^3} dx + x^2 \ln x \int \frac{\ln x}{x^3} dx$$

$$y(x) = C_1 x^2 + C_2 x^2 \ln x + \frac{x^2 (\ln x)^3}{6}$$

D-operator method.

For finding P.I. of
higher order constant coeffs.
ODE.

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_1 \frac{dy}{dx} + a_0 y = g(x)$$

$$D = \frac{d}{dx}, \quad D^i = \frac{d^i}{dx^i}$$

$$(a_n D^n + a_{n-1} D^{n-1} + \dots + a_1 D + a_0) y = g(x)$$

input
↓
output.

machine

$$y = f(x)$$

Q. Given output find input.

