L-17

CLASS TEST

NOV-1, 2019

Solving 2nd/higher order ODEs.

ODE

- 1st order
- 2nd/higher order
  - 'reduction'
  - Homogeous ODEs
    - constant coeff.
    - Cauchy-Euler
  - nonHomogeneous ODEs
    - undetermined coeffs.
    - variation of parameters.

Exp. Solve

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = 0$$

Exp. Solve $\frac{d^3y}{dx^3} + y = 0$

Cauchy-Euler eqn.

$$ax^2 \frac{d^2y}{dx^2} + bx^1 \frac{d^1y}{d^1x} + cy = 0, \quad x \neq 0$$

- a homogeneous ode.
- all the terms have the same 'dimension' provided a, b, c all constants.

Even if the coeffs. are NOT constants, it can be solved following a procedure which is used to solve constant coefficient ode.

$$ax^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + cy = 0 \quad \text{—— (1)}$$

If $y(x) = x^m$ is a solution of (1)

then $\frac{dy}{dx} = m x^{m-1}$

$$\frac{d^2y}{dx^2} = m(m-1) x^{m-2}$$

Putting these in eqn (1)

$$m(m-1) a x^{m} + bmx^m + cx^m = 0$$

$$\Rightarrow [m(m-1)a + bm + c] x^m = 0$$