

Lec 16

## Higher order ODE

The general form of a 2<sup>nd</sup> order **LINEAR** ODE is

$$a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y = f(x) \quad (1)$$

- ① If  $f(x) = 0$  then (1) is called homogeneous
- ② otherwise, non-homogeneous.

The general solution  
of the 'homogeneous'  
version of (1) is  
given by

$$y_h(x) = c_1 y_1(x) + c_2 y_2(x)$$

where  $y_1(x)$  &  $y_2(x)$  are  
linearly independent  
solutions of the homogenous  
version

$$a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = 0$$

where  $a_2(x) \neq 0$

Two solutions  $y_1(x)$  &  $y_2(x)$   
are linearly independent (L.I.)

if 
$$c_1 y_1(x) + c_2 y_2(x) = 0$$

implies  $c_1 = c_2 = 0$ .

i.e.  $y_1(x)$  is not a  
scalar multiple of  $y_2(x)$ .

or vice-versa.

Remark: If only solution  
 $y_1(x)$  of a homo. linear 2<sup>nd</sup> order  
ODE is known, another L.I.  
solution can be written as  $y_2(x) = v(x)y_1(x)$   
into the DE. Then solve it to  
derive  $v(x)$ .