Lec-15

CLASS TEST - Oct 31

Solving ODE $F\left(x, y, \frac{dy}{dx}, \ldots \frac{d^n y}{dx^n}\right)$

- Order 1 $(n=1)$
- Higher order

Order 1 $(n=1)$

- Separable

  $$\frac{dy}{dx} = f(x)\, g(y)$$

- 'Homogeneous'

  $$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$$

- Differential Form

  $$M(x,y)\,dy + N(x,y)\,dx = 0$$

  (Exact Form) $\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y}$

  (Non-exact Form)
  (Integrating Factor)

- Converting 'non-homogeneous' to 'homogeneous'

  $$\frac{dy}{dx} = \frac{ax + by + c}{\alpha x + \beta y + \gamma}$$

- Bernoulli's Equation.

Exc 1 Solve

$$2xy\,\frac{dy}{dx} = 4x^2 + 3y^2$$

Ans. $y^2 + 4x^2 = cx^3$

where 'c' is a constant.

'Homogeneous'

'Exact' differential 'Form'

$$M(x,y)\,dy + N(x,y)\,dx = 0$$

If we are able to find a $\varphi(x,y)$

s.t. $d\varphi(x,y) = M\,dy + N\,dx$

iff $\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y}$

$\mu(x,y)$ — integrating fn (I.F.)

$$\underbrace{\mu(x,y)\, M(x,y)}_{M_1(x,y)}\, dy + \underbrace{\mu(x,y) N(x,y)}_{N_1(x,y)}\, dx = 0$$

must be exact.

Then $$\frac{\partial M_1}{\partial x} = \frac{\partial N_1}{\partial y}.$$

$$\Downarrow$$

$$M(x,y)\frac{\partial \mu}{\partial x} - N(x,y)\frac{\partial \mu}{\partial y} = \mu(x,y)\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right)$$

If μ is a fn. of 'x' only, then setting $\frac{\partial \mu}{\partial y} = 0$ we obtain

$$\frac{1}{\mu(x)}\frac{\partial \mu}{\partial x} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)} \qquad (1)$$