Lec-14.

If you have not checked your mid sem answer script (or want to recheck), drop me an email after vacation.

'ODE'

F(in terms of variables) $= 0$

$F(x, y, y', y'', \ldots, y^{(n)}) = 0$

where y is a function of 'x'.

First order linear ordinary differential equation.

$$\frac{dy}{dx} + p(x)\,y = q(x) \quad \text{---(1)}$$

Solving (1) means, find a fn $y(x)$ which satisfies (1).

Separable Form.

$$\frac{dy}{dx} = f(x)\,g(y) \quad \text{---(*)}$$

$$\frac{dy}{g(y)} = f(x)\,dx$$

and integrate

'Homogeneous' 1st order Linear ODE.

We call a linear 1st order ODE, a homogeneous equation if it is of the form

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right) \quad \text{---(1)}$$

i.e. $\frac{dy}{dx}$ is a homogeneous fn of degree zero.

Then (1) can be reduced to (*)

Plan

Let $\frac{y}{x} = v(x)$

$\Rightarrow y = x v(x)$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Then from (1)

$v + x\frac{dv}{dx} = f(v)$

$\Rightarrow \boxed{\frac{dv}{dx} = \frac{f(v) - v}{x}}$

Exp. Solve $\frac{dy}{dx} = \frac{x^2 + xy}{xy + y^2}$

Using the procedure

$-\frac{1}{2}\int \frac{dv^2}{v^2 - 1} = \int \frac{dx}{x}$

$\Rightarrow -\ln|v| = 2\ln|x| + c_1$

$= \ln c_2 x^2 \quad (c_2 = \ln c_1)$

$\Rightarrow \frac{1}{|v|} = c_2 x^2 \Rightarrow |1 - v^2| = \frac{c_3}{x^2}$ where $c_3 = 1/c_2$