

Taylor series representation  
of a function 'f' at  $(a, b) \in D(f)$ ,  
the domain of f.

$$\begin{aligned}
 f(x, y) \cong & f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \\
 & + \frac{1}{2} \left[ f_{xx}(a, b)(x-a)^2 + 2f_{xy}(a, b)(x-a)(y-b) \right. \\
 & \left. + f_{yy}(a, b)(y-b)^2 \right] \\
 & + \alpha_0 (\Delta \rho)^3
 \end{aligned}$$

where  $\Delta \rho = \sqrt{\Delta x^2 + \Delta y^2}$ ,  $\Delta x = x - a$   
 $\Delta y = y - b$

and  $\Delta \rho \rightarrow 0$  when  $\Delta x, \Delta y \rightarrow 0$

→ is a 'good' approximation  
only when  $(x, y)$  is close to  
 $(a, b)$ .

## Lec-12

Taylor series representation  
of a function  $f$  at  $(a, b) \in D(f)$ ,  
the domain of  $f$ .

$$f(x, y) \cong f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \\ + \frac{1}{2} \left[ f_{xx}(a, b)(x-a)^2 + 2f_{xy}(a, b)(x-a)(y-b) \right. \\ \left. + f_{yy}(a, b)(y-b)^2 \right] \\ + \alpha_0 (\Delta \rho)^3$$

where  $\Delta \rho = \sqrt{\Delta x^2 + \Delta y^2}$ ,  $\Delta x = x - a$   
 $\Delta y = y - b$

and  $\Delta \rho \rightarrow 0$  when  $\Delta x, \Delta y \rightarrow 0$

is a 'good' approximation  
only when  $(x, y)$  is close to  
 $(a, b)$ .

If  $(a, b)$  attains a local maxima  
of  $f$ ,  $f(x, y) \leq f(a, b)$  when  $(x, y) \rightarrow (a, b)$

and  $(a, b)$  attains a local minima  
of  $f$  at then  
 $f(x, y) \geq f(a, b)$  when  $(x, y) \rightarrow (a, b)$ .

Let  $xy = b$  and consider the  
f.  $f(x, b)$  then  $f(x, b)$   
has a local max/min at  $x = a$ .  
lly, if  $x = a$  then  $f(a, y)$  has  
a local max/min at  $y = b$ .

Then from Fermat's theorem.

$$\frac{\partial f}{\partial x}(x, b) = 0 \text{ at } x = a$$

$$\Rightarrow \frac{\partial f}{\partial x}(a, b) = 0$$

$$\text{lly, } \frac{\partial f}{\partial y}(x, y) = 0 \text{ at } y = b \Rightarrow \frac{\partial f}{\partial y}(a, b) = 0$$

