Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology
Spring Semester 2016
Mid Semester Examination

Declaration:

- **Answer without proper justification carries NO marks.**
- **NO query will be entertained during the examination.**

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1. True or False. Justify your answer. (As always, to say 'true' you must prove it, while to say 'false' you must produce an appropriate counterexample.)

   (i) The boundary ∂A of the set $A = \{x \in \mathbb{R} \,|\, 0 \leq x \leq 1\} \subset \mathbb{R}$ is $\{0, 1\}$ when $\mathbb{R}$ is equipped with finite complement topology. [2]

   (ii) Let $f : (X, \tau_x) \to (Y, \tau_y)$ be a continuous function. If x is a limit point of a subset A of X then $f(x)$ is a limit point of $f(A)$ in Y. [3]

   (iii) Let (X, τ) be any topological space and Y a non-empty set. Suppose $f : X \to Y$ is a surjective map. Then, $\{f(U) \,|\, U \in \tau\}$ is a topology on Y. [2]

   (iv) Suppose (X, τ_x) and (Y, τ_y) are topological spaces. Then, $\tau_x \times \tau_y$ is a topology on $X \times Y$, where $\times$ denotes the Cartesian product of sets. [2]

   (v) Let $f : (X, \tau_x) \to (Y, \tau_y)$ be a continuous map. Then $f(U)$ is open in Y for any open set U in X. [3]

   (vi) Any real sequence $f : \mathbb{N} \to \mathbb{R}$ is a continuous function where $\mathbb{N}$ is a subspace of $\mathbb{R}$ with standard topology. [2]

   (vii) Let (X, τ) be any topological space and $A \subset X$. If $p \in Cl(A)$ then p is the limit of a sequence in A. [3]

   (viii) Let X be any topological space, and let $A, B \subset X$ be subsets of X such that $X = A \cup B$. Let $f : A \to Y$ and $g : B \to Y$ be continuous functions such that, for every $x \in A \cap B$, $f(x) = g(x)$. Define $h : A \cup B \to Y$ by

   $$h(x) = \begin{cases} f(x), & \text{if } x \in A \\ g(x), & \text{if } x \in B. \end{cases}$$

   Then h is a continuous function. [3]

   (ix) Let X, Y be topological spaces. If $f : X \to Y$ is a homeomorphism then $f(\partial A) = \partial f(A)$ for every $A \subset X$. [2]

   (x) The subspace topology on the set of rational numbers $\mathbb{Q} \subset (\mathbb{R}, \tau_u)$ is the discrete topology where τ_u denotes the usual topology. [2]

2. Consider the topological space (Y, τ_y) where $Y = \{a, b, c\}$ and $\tau_y = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}\}$. Give an example of a non-constant, continuous function $f : [0,\, 1] \to Y$, where $[0,\, 1] \subset \mathbb{R}$ has the usual topology. [1]

3. Suppose $f : (X, \tau_x) \to (Y, \tau_y)$ is a continuous function which is surjective. Then

(a) If (X, τ_x) is Hausdorff, must (Y, τ_y) always be Hausdorff? Justify your answer. [1]

(b) If (Y, τ_y) is Hausdorff, must (X, τ_x) always be Hausdorff? Justify your answer. [2]

4. Let $\mathbb{R}$ have the standard topology, and define a surjective map $p : \mathbb{R} \to \mathbb{Z}$ as

$$p(x) = \begin{cases} x, & \text{if } x \text{ is an integer} \\ n, & \text{if } x \in (n-1,\, n+1) \text{ and } n \text{ is an odd integer.} \end{cases}$$

Describe the Quotient topology on $\mathbb{Z}$. [2]

All The Best !!