Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology*
Spring Semester 2016
Assignment -04

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1. True or False. Justify your answer.
   (a) Every connected metric space having more than one point is uncountable.
   (b) The set of rational numbers $\mathbb{Q}$ is NOT locally compact wrt the usual topology.
   (c) Arbitrary union of compact sets in a topological space X is compact.
   (d) $\mathbb{Z}$ with the digital line topology is compact.
   (e) If a topological space X is compact and Hausdorff, then X is normal.
   (f) Let $f : (X, \tau_x) \to (Y, \tau_y)$ be a continuous bijective function. Then,
      i. if X is compact then f is a homeomorphism
      ii. if Y is Hausdorff, then f is a homeomorphism.
      iii. if X is compact and Y is Hausdorff, then f is a homeomorphism.

2. Let X be a nonempty set. Let $p \in X$. Define a topology τ on X as follows

$$\tau = \{S \subseteq X : p \in S \,\text{or}\, S = \emptyset\}.$$

   Then show that
   (a) (X, τ) is locally compact
   (b) (X, τ) is compact if and only if X is finite.

3. Let $A_0 = [0, 1]$, the closed interval in $\mathbb{R}$. Let $A_1 \subset A_0$ be obtained by removing the middle third $\left(\frac{1}{3}, \frac{2}{3}\right)$ of A_0. Let A_2 be obtained from A_1 by removing its middle third $\left(\frac{1}{9}, \frac{2}{9}\right)$ and $\left(\frac{7}{9}, \frac{8}{9}\right)$. In general,

$$A_n = A_{n-1} \setminus \bigcup_{k=0}^{\infty} \left(\frac{1+3k}{3^n}, \frac{2+3k}{3^n} \right).$$

   The set

$$C = \bigcap_{n \in \mathbb{Z}_+} A_n$$

   is called the *Cantor set* which is a subspace of $[0, 1]$. Then show that
   (a) C is totally disconnected
   (b) C is compact.

4. Show that the Tube Lemma does not necessarily hold if the compactness of Y is dropped.

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